Chapter 10
Measurement, Area, and Volume

Before
In previous chapters you’ve . . .
- Written and solved proportions
- Solved problems involving similar figures
- Used the Pythagorean theorem

Now
In Chapter 10 you’ll study . . .
- Classifying triangles and polygons
- Finding areas of parallelograms and trapezoids
- Finding circumferences and areas of circles
- Finding surface areas and volumes of solids

Why?
So you can solve real-world problems about . . .
- box kites, p. 515
- picnic tables, p. 520
- aircraft wings, p. 525
- wrestling, p. 531
- marimba pipes, p. 542
- ant lions, p. 548
- salsa, p. 556
- grain silos, p. 559
Drums When you strike a drum, it vibrates, producing sound. One reason a large drum sounds lower in pitch than a small drum is that the large drum vibrates more slowly. In this chapter, you will find the surface area and volume of objects like drums.

What do you think? The drums in the photo are cylinders. What shape do the top and bottom of a cylinder have? If you were to cut a cylinder’s curved side straight down from top to bottom and flatten it, what shape would you see?
Chapter Prerequisite Skills

PREREQUISITE SKILLS QUIZ

Preparing for Success  To prepare for success in this chapter, test your knowledge of these concepts and skills. You may want to look at the pages referred to in blue for additional review.

1. Vocabulary  Draw a right triangle. Label the hypotenuse and legs.

Solve the equation.  (p. 120)

2. \(15 = 2x - 7\)  
3. \(8 - 3n = 50\)  
4. \(-9 - 4y = 19\)  
5. \(78 = 2p + 12\)

Solve the proportion.  (pp. 275, 280)

6. \(\frac{a}{16} = \frac{5}{4}\)  
7. \(\frac{90}{15} = \frac{r}{34}\)  
8. \(\frac{3}{7} = \frac{31}{z}\)  
9. \(\frac{2}{74} = \frac{96}{m}\)

10. Architecture  A scale drawing of a rectangular wall is 8 inches long and 14 inches high. The drawing has a scale of 1 inch : 3 feet. Find the wall’s dimensions.  (p. 300)

11. Determine whether a triangle with side lengths 20, 37.5, and 42.5 is a right triangle.  (p. 465)

NOTETAKING STRATEGIES

MAIN IDEA WEB When you learn a new concept, you may want to make a web of details surrounding the concept in your notebook.

\[
\tan A = \frac{\text{side opposite } \angle A}{\text{side adjacent to } \angle A} = \frac{a}{b}
\]

Main Idea: A trigonometric ratio is a ratio of the lengths of two sides of a right triangle.

\[
\sin A = \frac{\text{side opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c} \quad \cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{b}{c}
\]

A main idea web will help you in Lesson 10.8.
Triangles

**Review Vocabulary**
- acute triangle, p. 462
- right triangle, p. 462
- obtuse triangle, p. 462
- equiangular triangle, p. 462
- equilateral triangle, p. 463
- isosceles triangle, p. 463
- scalene triangle, p. 463

**Construction** Homes are often built with sloped roofs so that they can shed rain. Such roofs are built using a series of triangular roof trusses. The trusses may include braces that help the roof bear weight, such as the weight of snow.

Recall that you can classify a triangle by its angle measures or by its side lengths. When classified by angle measures, triangles are acute, right, obtuse, or equiangular. When classified by side lengths, triangles are equilateral, isosceles, or scalene.

**Example 1**  
**Classifying a Triangle by Angle Measures**

In the diagram, $\angle DBE = 64^\circ$ and $\angle BDE = \angle BE$. Find $\angle BDE$ and $\angle BE$. Then classify $\triangle BDE$ by its angle measures.

**Solution**
Let $x^\circ$ represent $\angle BDE$ and $\angle BE$.

$m\angle BDE + m\angle BED + m\angle DBE = 180^\circ$

$x^\circ + x^\circ + 64^\circ = 180^\circ$

$2x + 64 = 180$

$2x = 116$

$x = 58$

**Answer** $\angle BDE = \angle BE = 58^\circ$. Because $\angle BDE$, $\angle BDE$, and $\angle BED$ are acute angles, $\triangle BDE$ is an acute triangle.

**Checkpoint**
1. Use the diagram in Example 1. Given that $\angle EDG = 38^\circ$ and the measure of $\angle DEG$ is $38^\circ$ more than $\angle DGE$, find $\angle DGE$ and $\angle DGE$. Then classify $\triangle DEG$ by its angle measures.
Example 2  Finding Unknown Side Lengths

The perimeter of a scalene triangle is 65 centimeters. The length of the first side is twice the length of the second side. The length of the third side is 20 centimeters. Find the lengths of the other two sides.

Solution

Draw the triangle. Let \( x \) and \( 2x \) represent the unknown side lengths. Write an equation for the perimeter \( P \). Then solve for \( x \).

\[
\begin{align*}
P &= 2x + x + 20 & \text{Formula for perimeter} \\
65 &= 2x + x + 20 & \text{Substitute 65 for} \ P. \\
65 &= 3x + 20 & \text{Combine like terms.} \\
45 &= 3x & \text{Subtract 20 from each side.} \\
15 &= x & \text{Divide each side by 3.}
\end{align*}
\]

Answer The length of the second side is 15 centimeters, and the length of the first side is \( 2(15) = 30 \) centimeters.

Checkpoint

2. The perimeter of an equilateral triangle is 42 meters. Find the length of each side.

Example 3  Finding Angle Measures Using a Ratio

The ratio of the angle measures of a triangle is \( 1 : 3 : 5 \). Find the angle measures. Then classify the triangle by its angle measures.

Solution

1) Let \( x^\circ, 3x^\circ\), and \( 5x^\circ \) represent the angle measures. Write an equation for the sum of the angle measures.

\[
\begin{align*}
x^\circ + 3x^\circ + 5x^\circ &= 180^\circ & \text{Sum of angle measures is } 180^\circ. \\
9x &= 180 & \text{Combine like terms.} \\
x &= 20 & \text{Divide each side by 9.}
\end{align*}
\]

2) Substitute 20 for \( x \) in the expression for each angle measure.

\[
(20)^\circ = 20^\circ \quad (3 \cdot 20)^\circ = 60^\circ \quad (5 \cdot 20)^\circ = 100^\circ
\]

Answer The angle measures of the triangle are 20°, 60°, and 100°. So, the triangle is an obtuse triangle.

Checkpoint

3. The ratio of the angle measures of a triangle is \( 3 : 5 : 12 \). Find the angle measures. Then classify the triangle by its angle measures.
Guided Practice

**Vocabulary Check**
1. The ratio of the angle measures of a triangle is $1:1:1$. Find the angle measures. Then classify the triangle by its angle measures.

**Skill Check**

1. The perimeter of an isosceles triangle is 14 meters. The length of one side is 4 meters. The lengths of the other two sides are equal. Find the lengths of the other two sides.
2. The ratio of the angle measures in a triangle is $6:5:4$. Find the angle measures. Then classify the triangle by its angle measures.
3. **Error Analysis** Describe and correct the error in finding the value of $x$ for the triangle shown below.

Practice and Problem Solving

**Homework Help**

- **Example Exercises**
  - 1: 8–13
  - 2: 15–18
  - 3: 19–21

**Find the value of $x$. Then classify the triangle by its angle measures.**

4. 
5. 
6. 
7. 
8. 
9. 
10. 

**Writing** Explain why the sum of the measures of the acute angles of a right triangle is $90^\circ$. 

Lesson 10.1  Triangles 513
Find the unknown side length of the triangle given the perimeter $P$. Then classify the triangle by its side lengths.

15. $P = 49$ in.  
16. $P = 22.5$ yd  
17. $P = 84.3$ cm

18. The perimeter of a triangle is 29 millimeters. The length of the first side is twice the length of the second side. The length of the third side is 5 more than the length of the second side. Find the side lengths of the triangle. Then classify the triangle by its side lengths.

19. **Window** The perimeter of a triangular window is 141 inches. The ratio of the side lengths of the window is $11 : 18 : 18$. Draw and label a diagram of the window. What are the side lengths of the window? Classify the window by its side lengths.

20. The ratio of the angle measures of a triangle is $7 : 16 : 22$. Find the angle measures. Then classify the triangle by its angle measures.

21. The ratio of the side lengths of a triangle is $7 : 24 : 25$. The perimeter of the triangle is 392 inches.
   a. Find the side lengths. Then classify the triangle by its side lengths.
   b. **Analyze** Is the triangle a right triangle? How do you know?

22. **Coordinate Geometry** Plot the points $A(6, 3)$, $B(-3, 9)$, and $C(-3, -3)$ in a coordinate plane. Connect the points to form a triangle. Use the distance formula to find the side lengths. Then classify the triangle by its side lengths.

23. **Extended Problem Solving** You are building a set of nested tables. The surfaces of the tables will be 45°-45°-90° triangles.

   a. **Visual Thinking** Each of the two congruent edges of the surface of the smallest table has a length of 24 inches. Make and label a scale drawing of the surface of the smallest table.

   b. **Calculate** The ratio of an edge length of the surface of the smallest table to a corresponding edge length of the largest table is $1 : 2$. Find the length of each of the two congruent edges of the surface of the largest table.

   c. **Calculate** The ratio of an edge length of the surface of the middle-sized table to a corresponding edge length of the surface of the smallest table is $3 : 2$. Find the length of each of the two congruent edges of the surface of the middle-sized table.

   d. For each table surface, find the length of the third edge to the nearest inch. Make and label scale drawings of the surfaces of the two larger tables.
24. **Winged Box Kite** The design for a winged box kite uses four triangular pieces of cloth. To cut out one of these pieces, you fold a piece of cloth in half and pin a pattern on the cloth, as shown. You cut along $\overline{AB}$ and $\overline{BC}$. Then you remove the pattern and unfold the cloth.

a. How many square inches of cloth do you need for each unfolded triangle? Round your answer to the nearest square inch.

b. **Apply** You need to attach wooden dowels to the congruent shorter sides of each of the 4 unfolded triangles. You can buy dowels that are 50 inches long. How many do you need to buy? Explain.

25. **Critical Thinking** The perimeter of an isosceles triangle is 17 centimeters. The length of one side is 5 centimeters. Your friend claims that there is not enough information to find the other two side lengths of the triangle. Is your friend correct? Explain your reasoning. Include diagrams in your answer.

26. **Triangle Inequality** The triangle inequality theorem states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side. Using this theorem, determine if the given side lengths form a triangle. Explain your reasoning.

   a. 4, 5, 10    b. 4, 5, 9    c. 4, 5, 7

27. **Challenge** The triangle shown is an equilateral triangle. Make 6 copies of the triangle. Put the 6 equilateral triangles together so that they all share a vertex and do not overlap. The figure formed is a regular hexagon. Find the sum of the measures of the angles of the regular hexagon. Explain your reasoning.

28. **Writing** Is it possible to have a triangle whose angle measures are in the ratio $3:4:5$ and whose side lengths are in the same ratio? Explain.

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**Mixed Review**

Write the sine and cosine ratios for both acute angles of the triangle.

*(Lesson 9.8)*

29.  

30.  

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**Standardized Test Practice**

31. **Multiple Choice** The ratio of the side lengths of a triangle is $3:3:4$. Classify the triangle by its side lengths.

   A. Scalene   B. Equilateral   C. Isosceles   D. Acute

32. **Multiple Choice** The ratio of the angle measures of a triangle is $1:2:3$. Classify the triangle by its angle measures.

   F. Acute   G. Obtuse   H. Right   I. Equiangular
## 10.2 Polygons and Quadrilaterals

**Vocabulary**
- polygon, p. 516
- regular polygon, p. 516
- convex, p. 516
- concave, p. 516
- pentagon, p. 516
- hexagon, p. 516
- heptagon, p. 516
- octagon, p. 516
- trapezoid, p. 517
- parallelogram, p. 517
- rhombus, p. 517
- diagonal of a polygon, p. 518

**Reading Geometry**
The name $n$-gon refers to a polygon that has $n$ sides. For example, a 15-gon is a polygon that has 15 sides.

### Before
You classified triangles.

### Now
You'll classify polygons and quadrilaterals.

### Why?
So you can find the length of a side of a clock face, as in Ex. 21.

A **polygon** is a closed plane figure whose sides are segments that intersect only at their endpoints. In a **regular polygon**, all the sides have the same length and all the angles have the same measure.

<table>
<thead>
<tr>
<th>Polygons</th>
<th>Regular polygons</th>
<th>Not polygons</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Polygon" /></td>
<td><img src="image" alt="Regular Polygon" /></td>
<td><img src="image" alt="Not Polygon" /></td>
</tr>
</tbody>
</table>

A polygon is **convex** if a segment joining any two interior points lies completely within the polygon. A polygon that is not convex is called **concave**.

You already know that a 3-sided polygon is a triangle and a 4-sided polygon is a quadrilateral. Below are names of other polygons.

<table>
<thead>
<tr>
<th>Polygons</th>
<th>Pentagon</th>
<th>Hexagon</th>
<th>Heptagon</th>
<th>Octagon</th>
<th>$n$-gon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of sides</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>$n$</td>
</tr>
</tbody>
</table>

### Example 1
**Identifying and Classifying Polygons**

Tell whether the figure is a polygon. If it is a polygon, classify it and tell whether it is convex or concave. If not, explain why.

- **a.**
  - The keyhole is not a polygon because the top part of the keyhole is round.

- **b.**
  - The stop sign is an 8-sided polygon. So it is an octagon. It is convex and regular.
**Checkpoint**

Tell whether the figure is a polygon. If it is a polygon, classify it and tell whether it is *convex* or *concave*. If not, explain why.

1. 

2. 

3. 

**Quadrilaterals** Some quadrilaterals have special names based on whether they have parallel or congruent sides and whether they have right angles.

<table>
<thead>
<tr>
<th>Quadrilaterals</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Trapezoid</strong></td>
<td><img src="TrapezoidDiagram.png" alt="Diagram" /></td>
</tr>
<tr>
<td>A trapezoid is a quadrilateral with exactly 1 pair of parallel sides.</td>
<td></td>
</tr>
<tr>
<td><strong>Parallelogram</strong></td>
<td><img src="ParallelogramDiagram.png" alt="Diagram" /></td>
</tr>
<tr>
<td>A parallelogram is a quadrilateral with both pairs of opposite sides parallel.</td>
<td></td>
</tr>
<tr>
<td><strong>Rhombus</strong></td>
<td><img src="RhombusDiagram.png" alt="Diagram" /></td>
</tr>
<tr>
<td>A rhombus is a parallelogram with 4 congruent sides.</td>
<td></td>
</tr>
<tr>
<td><strong>Rectangle</strong></td>
<td><img src="RectangleDiagram.png" alt="Diagram" /></td>
</tr>
<tr>
<td>A rectangle is a parallelogram with 4 right angles.</td>
<td></td>
</tr>
<tr>
<td><strong>Square</strong></td>
<td><img src="SquareDiagram.png" alt="Diagram" /></td>
</tr>
<tr>
<td>A square is a parallelogram with 4 right angles and 4 congruent sides.</td>
<td></td>
</tr>
</tbody>
</table>

**Example 2** *Classifying Quadrilaterals*

Classify the quadrilateral.

a.  

The quadrilateral is a parallelogram because both pairs of opposite sides are parallel.

b.  

The quadrilateral is a parallelogram with 4 right angles. So, it is a rectangle.
**Angle Measures in Quadrilaterals** A **diagonal of a polygon** is a segment that joins two vertices that are not adjacent. You can use a diagonal of a quadrilateral to show that the sum of the angle measures in a quadrilateral is 360°.

1. Draw diagonal $\overline{FI}$, which divides quadrilateral $FGHI$ into two triangles.

2. The sum of the angle measures in each triangle is 180°.

3. The sum of the angle measures in a quadrilateral is $180° + 180° = 360°$.

---

**Example 3**  
**Finding an Unknown Angle Measure**

Find the value of $x$.

![Diagram](image)

\[ x° + (2x + 1)° + 90° + 68° = 360° \]

- **Sum of angle measures in quadrilateral is 360°**.
- **Combine like terms**.
- **Subtract 159 from each side**.
- **Divide each side by 3**.

\[
egin{align*}
3x + 159 &= 360 \\
3x &= 201 \\
x &= 67
\end{align*}
\]

---

**Guided Practice**

**Vocabulary Check**

1. How are a trapezoid and a parallelogram different from each other?

**Skill Check**

Tell whether the figure is a polygon. If it is a polygon, classify it and tell whether it is **convex** or **concave**. If not, explain why.

2.  
3.  
4.

In Exercises 5 and 6, use the quadrilateral shown.

5. Classify the quadrilateral.

6. Find the value of $y$.

![Diagram](image)
Practice and Problem Solving

Tell whether the figure is a polygon. If it is a polygon, classify it and tell whether it is convex or concave. If not, explain why.

7. 
8. 
9. 

10. Error Analysis Describe and correct the error in solving the following problem.

A quadrilateral has 4 congruent sides, and the opposite sides of the quadrilateral are parallel. Sketch and classify the quadrilateral.

Classify the quadrilateral.

11. 
12. 
13. 

Copy and complete the statement using always, sometimes, or never.

14. A square is ___ a rectangle. 
15. A square is ___ a rhombus. 
16. A rhombus is ___ a square. 
17. A trapezoid is ___ a parallelogram.

Find the value of x.

18. 
19. 
20. 

21. Extended Problem Solving The Allen-Bradley Clock Tower in Milwaukee, Wisconsin, has four faces. Each face is a regular octagon. The perimeter of one octagonal face is approximately 133 feet.

a. Calculate Find the length of one side of one of the octagonal faces.

b. Visual Thinking Your friend says that you can find the area of one of the clocks by dividing one of the octagons into 8 congruent triangles. Sketch a regular octagon and show how to divide it into 8 congruent triangles.

c. Critical Thinking What additional information would you need in part (b) to find the area of the clock face? Assume that you had this information. What would your next steps be?

22. For the trapezoid shown, the ratio \( m\angle A : m\angle C \) is 2 : 1. Write and solve an equation to find the value of x.
23. In mathematics, a kite is a special type of quadrilateral. Two pairs of sides are congruent, but opposite sides are not congruent. Exactly one pair of opposite angles are congruent. In kite $ABCD$ shown, the measure of $\angle A$ is twice the measure of $\angle C$, and $\angle B$ has a measure of $114^\circ$. Find the measures of $\angle A$, $\angle C$, and $\angle D$.

24. **Challenge** Use the figure shown to find $m\angle WXY$ and $m\angle XYZ$. Explain your reasoning.

![Diagram of a kite with labeled angles]

**Mixed Review** Solve the linear system by graphing. *(Lesson 8.8)*

25. $y = x - 5$
   $y = 2x + 1$

26. $y = -3x + 7$
   $y = 3x + 4$

27. $x + y = 6$
   $2x - 8y = -11$

28. Find the midpoint of the segment with endpoints $(-7, 5)$ and $(4, -20)$. *(Lesson 9.5)*

29. The ratio of the angle measures of a triangle is $2 : 3 : 7$. Find the angle measures. Then classify the triangle by its angle measures. *(Lesson 10.1)*

**Standardized Test Practice**

30. **Extended Response** The top of the picnic table shown has the shape of a regular polygon.
   
   **a.** Sketch and classify the polygon. Is it convex or concave?
   
   **b.** Draw a single segment that divides the polygon in your sketch into two trapezoids.
   
   **c.** Find the sum of the measures of the angles of the polygon.

![Image of a hexagonal picnic table]

---

**Brain Game**

**Toothpick Task**

Move exactly two toothpicks in the figure below to make 4 congruent squares instead of 5. Each toothpick must be used as a side of a square.

![Diagram of toothpick puzzle]
Areas of Parallelograms and Trapezoids

**Vocabulary**
- base of a parallelogram, p. 521
- height of a parallelogram, p. 521
- bases of a trapezoid, p. 522
- height of a trapezoid, p. 522

**BEFORE**
You classified polygons. You’ll find the areas of parallelograms and trapezoids. So you can compare the areas of two parking lots, as in Ex. 26.

**Now**

The **base of a parallelogram** is the length of any one of its sides. The **height of a parallelogram** is the perpendicular distance between the side whose length is the base and the opposite side. The diagrams below show how to change a parallelogram into a rectangle with the same base, height, and area as the parallelogram.

1. Start with any parallelogram.
2. Cut to form a right triangle and a trapezoid.
3. Move the triangle to form a rectangle.

Notice that the area of the rectangle above is the product of the base $b$ and the height $h$. The diagram suggests the formula below.

**Area of a Parallelogram**

**Words** The area $A$ of a parallelogram is the product of the base $b$ and the height $h$.

**Algebra** $A = bh$

**Numbers** $A = 8 \cdot 6 = 48 \text{ m}^2$

**Example 1** Finding the Area of a Parallelogram

The base of a parallelogram is 5 inches. The height is twice the base. Find the area of the parallelogram.

1. Find the height.
   
   $h = 2b$
   
   $= 2(5)$
   
   $= 10 \text{ in.}$

2. Find the area.
   
   $A = bh$
   
   $= 5(10)$
   
   $= 50 \text{ in.}^2$

**Answer** The parallelogram has an area of 50 square inches.
**Trapezoids** The **bases of a trapezoid** are the lengths of its parallel sides. The **height of a trapezoid** is the perpendicular distance between the sides whose lengths are the bases. The diagram below shows how two congruent trapezoids with height \( h \) and bases \( b_1 \) and \( b_2 \) can be put together to form a parallelogram with base \( b_1 + b_2 \) and height \( h \).

![Diagram of trapezoids and parallelogram]

Notice the area of the parallelogram is twice the area of either trapezoid. This result suggests the formula below.

---

**Area of a Trapezoid**

**Words** The area \( A \) of a trapezoid is one half the product of the sum of the bases, \( b_1 \) and \( b_2 \), and the height, \( h \).

**Algebra** \( A = \frac{1}{2}(b_1 + b_2)h \)

**Numbers** \( A = \frac{1}{2}(5 + 7)4 = 24 \text{ cm}^2 \)

---

**Example 2** **Finding the Area of a Trapezoid**

**Quilts** The diagram shows one of the trapezoids in a quilt design. Find the area of the trapezoid.

**Solution**

\[
A = \frac{1}{2}(b_1 + b_2)h \\
= \frac{1}{2}(4 + 9)2.5 \\
= 16.25
\]

**Simplify.**

**Answer** The trapezoid has an area of 16.25 square centimeters.

---

**Checkpoint**

Find the area of the parallelogram or trapezoid.

1. \[
\begin{array}{c}
\text{5 ft} \\
\text{6 ft}
\end{array}
\]

2. \[
\begin{array}{c}
\text{3 m} \\
\text{8.5 m} \\
\text{9 m}
\end{array}
\]

3. \[
\begin{array}{c}
\text{16 in.} \\
\text{13 in.} \\
\text{22 in.}
\end{array}
\]
Example 3  Finding an Unknown Length

The height of a trapezoid is 6 meters. One of its bases is 8 meters. The area of the trapezoid is 54 square meters. Find the other base.

\[ A = \frac{1}{2}(b_1 + b_2)h \quad \text{Write formula for area of a trapezoid.} \]

\[ 54 = \frac{1}{2}(8 + b_2)6 \quad \text{Substitute 54 for } A, 8 \text{ for } b_1, \text{ and 6 for } h. \]

\[ 54 = 3(8 + b_2) \quad \text{Multiply.} \]

\[ 54 = 24 + 3b_2 \quad \text{Distributive property} \]

\[ 30 = 3b_2 \quad \text{Subtract 24 from each side.} \]

\[ 10 = b_2 \quad \text{Divide each side by 3.} \]

Answer  The other base is 10 meters.

Example 4  Using Area of Trapezoids

Desk  You are building an L-shaped desk for your room. The dimensions of the desktop are shown. Find the area of the desktop.

Solution

1. Divide the desktop into two trapezoids, A and B, as shown.

2. Find the sum of the areas of trapezoids A and B.

   Area of trapezoid A = \( \frac{1}{2}(b_1 + b_2)h \)

   \[ = \frac{1}{2}(5 + 8)3 = \frac{39}{2} = 19\frac{1}{2} \]

   Area of trapezoid B = \( \frac{1}{2}(b_1 + b_2)h \)

   \[ = \frac{1}{2}(4 + 9)3 = \frac{39}{2} = 19\frac{1}{2} \]

3. Add the areas.

   Area of trapezoid A + Area of trapezoid B = \( 19\frac{1}{2} + 19\frac{1}{2} = 39 \)

Answer  The total area of the desktop is 39 square feet.

Checkpoint

4. One base of a trapezoid is 9 feet, and the height is 4 feet. The area of the trapezoid is 28 square feet. Find the other base.
Guided Practice

Vocabulary Check
1. Sketch a trapezoid and label its bases and height. State the formula for finding its area.

2. The height of a parallelogram is 22 inches. The base is one half of the height. Find the area of the parallelogram.

Skill Check
Find the area of the trapezoid.

3. \( \frac{9 \text{ ft}}{12 \text{ ft}} \)
4. \( \frac{35 \text{ in.}}{70 \text{ in.}} \)
5. \( \frac{88 \text{ m}}{62 \text{ m}} \)

Find the unknown base or height of the parallelogram.
6. \( A = 40 \text{ in.}^2, b = 25 \text{ in.}, h = ? \)
7. \( A = 300 \text{ m}^2, b = ?, h = 20 \text{ m} \)

Find the unknown base or height of the trapezoid.
8. \( A = 12 \text{ ft}^2, b_1 = 2 \text{ ft}, b_2 = ?, h = 3 \text{ ft} \)
9. \( A = 240 \text{ m}^2, b_1 = 16 \text{ m}, b_2 = 8 \text{ m}, h = ? \)

10. Track Uniform You are sewing a red stripe on the front of a track uniform. As shown, the stripe is a parallelogram. What is the area of the stripe?

Practice and Problem Solving

Find the area of the parallelogram.

11. \( \frac{5 \text{ in.}}{14 \text{ in.}} \)
12. \( \frac{8 \text{ yd}}{9.5 \text{ yd}} \)
13. \( \frac{8.3 \text{ mm}}{11.5 \text{ mm}} \)

Find the area of the trapezoid.

14. \( \frac{14 \text{ ft}}{12 \text{ ft}} \)
15. \( \frac{3.2 \text{ m}}{3.6 \text{ m}} \)
16. \( \frac{7.5 \text{ cm}}{10.2 \text{ cm}} \)
17. The base of a parallelogram is 10 meters. The height is one fourth of the base. Find the area of the parallelogram.

18. The height of a trapezoid is 2 feet. One of the bases is three times the height, and the other base is four times the height. Find the area of the trapezoid.

**Find the unknown measure of the parallelogram.**

19. \( A = 2025 \text{ m}^2 \)  
20. \( A = 71.5 \text{ in}^2 \)  
21. \( A = 1 \text{ yd}^2 \)

**Find the unknown measure of the trapezoid.**

22. \( A = 192.5 \text{ cm}^2 \)  
23. \( A = 1800 \text{ ft}^2 \)  
24. \( A = 16.555 \text{ mm}^2 \)

25. **Aircraft Wings** A wing of each aircraft described has the shape of a trapezoid. Find the area of the wing.
   
   a. An F-18 wing has bases of 6 feet and 15 feet and height of 13 feet.
   
   b. A Boeing 747 wing has bases of 13.3 feet and 54.3 feet and height of 81.3 feet.

26. **Parking Lot** Two parking lots each have space for 5 cars, as shown in the diagrams below.

   **Parking Lot A**  
   **Parking Lot B**

   a. Find the base of each figure formed by the 5 parking spaces.
   
   b. Find the area of each figure formed by the 5 parking spaces.
   
   c. **Compare** Which parking lot covers less area to park 5 cars?

**Coordinate Geometry** In Exercises 27 and 28, plot the points in a coordinate plane. Connect the points so that they form a polygon. Identify the polygon and find its area.

27. \((-2, -3), (-2, 0), (2, 3), (2, -4)\)  
28. \((-1, 3), (4, 3), (2, -1), (-3, -1)\)

29. **Writing** What happens to the area of a trapezoid if you double its height? if you double both its bases? if you double the height and both bases?
Find the area of the figure.

30. 

31. 

32. **Summer Camp** This summer at camp, you can stay in room A or room B with one roommate. Which room will give you and your roommate more space?

Room A

Room B

33. **Picture Frame** You have a 4 inch by 6 inch picture that you want to have framed. You want the frame to be 2 inches wide. A wooden frame can be made from four trapezoids, as shown. Find the areas of the bottom and side trapezoids. Then find the ratio of the area of the bottom trapezoid to the area of the side trapezoid.

34. **Challenge** You form a rhombus by putting two equilateral triangles with side length \(2n\) together, as shown. Write an expression for the area of the rhombus in terms of \(n\). Explain your reasoning.

**Mixed Review**

For an account that earns interest compounded annually, find the balance of the account. Round to the nearest cent.  
(Lesson 7.7)

35. \(P = $1200, r = 5\%, t = 3\) years \hspace{0.5cm} 36. \(P = $8550, r = 3.5\%, t = 20\) years

Approximate the square root to the nearest integer.  
(Lesson 9.1)

37. \(\sqrt{40}\) \hspace{0.5cm} 38. \(\sqrt{587}\) \hspace{0.5cm} 39. \(\sqrt{10.2}\) \hspace{0.5cm} 40. \(\sqrt{0.725}\)

41. Find the value of \(x\) in the quadrilateral shown.  
(Lesson 10.2)

**Standardized Test Practice**

42. **Multiple Choice** The height of a parallelogram is 13.5 feet. The base is four times the height. What is the area of the parallelogram?

A. 45.5625 ft\(^2\) \hspace{0.5cm} B. 54 ft\(^2\) \hspace{0.5cm} C. 182.25 ft\(^2\) \hspace{0.5cm} D. 729 ft\(^2\)

43. **Short Response** Is it possible for two parallelograms to have the same area but not be congruent? Explain why or why not.
10.4 Investigating Circles

**Goal**
Compare the circumferences and the diameters of circles.

**Materials**
- compass
- metric ruler
- paper and pencil
- string

The *diameter* of a circle is the distance across the circle through its center. The *circumference* of a circle is the distance around the circle.

**Investigate**

**Find the ratio of circumference to diameter of a circle.**

1. Draw a circle of any size using a compass.

2. Lay a string around the circle. Mark the point where the string completes the circle. Straighten the string, and measure its length with a ruler.

![cm measurement scale]

The circumference is about 9.4 centimeters.

3. Measure the diameter of the circle with a ruler. Make sure the ruler goes through the center of the circle.

4. Write the ratio of the circumference to the diameter for the circle. Write the ratio as a decimal.

\[
\frac{\text{circumference}}{\text{diameter}} = \frac{9.4}{3} = 3.13
\]

The diameter is 3 centimeters.

**Draw Conclusions**

1. **Conjecture** Repeat Steps 1–4 of the activity for two circles of different diameters. What do you notice about the ratios in Step 4?

**Predict the circumference of a circle with the given diameter \(d\).**

2. \(d = 4\) feet
3. \(d = 2\) meters
4. \(d = 0.75\) inch
5. \(d = 12\) feet
A circle consists of all points in a plane that are the same distance from a fixed point called the center. The distance between the center and any point on the circle is the radius. The distance across the circle through the center is the diameter.

The circumference of a circle is the distance around the circle. For any circle, the ratio of its circumference to its diameter is an irrational number that is approximately equal to 3.14 or \( \frac{22}{7} \). The Greek letter \( \pi \) (pi) is used to represent this ratio.

**Circumference of a Circle**

**Words** The circumference \( C \) of a circle is the product of \( \pi \) and the diameter \( d \), or twice the product of \( \pi \) and the radius \( r \).

**Algebra** \[ C = \pi d \quad C = 2\pi r \]

**Example 1** Finding the Circumference of a Circle

**Meteor Crater** Scientists have identified the faint outline of part of an ancient meteor crater on the coast of Mexico. The rest of the approximately circular crater lies underwater. The crater’s diameter is about 170 kilometers. Approximate the distance around the crater to the nearest kilometer.

**Solution**

\[ C = \pi d \]

\[ = 3.14(170) \]

\[ = 533.8 \]

**Answer** The distance around the crater is about 534 kilometers.
Example 2  Finding the Radius of a Circle

The circumference of a circle is 70 inches. Find the radius of the circle to the nearest inch.

\[ C = 2\pi r \quad \text{Write formula for circumference of a circle.} \]
\[ 70 = 2(3.14)r \quad \text{Substitute 70 for } C \text{ and } 3.14 \text{ for } \pi. \]
\[ 70 \approx 6.28r \quad \text{Multiply.} \]
\[ 11.1 = r \quad \text{Divide each side by } 6.28. \text{ Use a calculator.} \]

Answer  The radius of the circle is about 11 inches.

Checkpoint  

1. The diameter of a circle is 28 feet. Find the circumference of the circle to the nearest foot.  
2. The circumference of a circle is 186 centimeters. Find the radius of the circle to the nearest centimeter.

Area of a Circle

Words  The area \( A \) of a circle is the product of \( \pi \) and the square of the radius \( r \).

Algebra  \[ A = \pi r^2 \]

Example 3  Finding the Area of a Circle

Find the area of the circle to the nearest square foot.

1. Find the radius.
   \[ r = \frac{d}{2} = \frac{26}{2} = 13 \]
2. Find the area.
   \[ A = \pi r^2 \quad \text{Write formula for area of a circle.} \]
   \[ = 3.14(13)^2 \quad \text{Substitute 3.14 for } \pi \text{ and } 13 \text{ for } r. \]
   \[ = 530.7 \quad \text{Simplify.} \]

Answer  The area of the circle is about 531 square feet.

Checkpoint  

3. The diameter of a circle is 14 inches. Find the area of the circle to the nearest square inch.  
4. Critical Thinking  One circle has a diameter of 12 centimeters. Another circle has a radius of 7 centimeters. Which circle has a greater area? Explain your reasoning.
Example 4  Finding the Radius of a Circle

The area of a circle is 72 square millimeters. Find the radius of the circle to the nearest millimeter.

\[ A = \pi r^2 \]  Write formula for area of a circle.
\[ 72 = (3.14)r^2 \]  Substitute 72 for \( A \) and 3.14 for \( \pi \).
\[ 22.9 = r^2 \]  Divide each side by 3.14.
\[ \sqrt{22.9} = r \]  Take positive square root of each side.
\[ 4.8 \approx r \]  Use a calculator to approximate square root.

Answer  To the nearest millimeter, the radius is 5 millimeters.

Checkpoint

Find the unknown measure. Round to the nearest whole number.

5. \( A = 1567 \text{ in}^2 \)
   \[ r = ? \]

6. \( A = 59 \text{ ft}^2 \)
   \[ d = ? \]

7. \( A = 197 \text{ cm}^2 \)
   \[ d = ? \]

8. Critical Thinking  How is finding the diameter of a circle when given its area different from finding the radius when given its area?

Example 5  Finding the Area of a Figure

Norman Windows  A Norman window from Congress Hall in Philadelphia consists of a rectangle and a half circle, as shown. Find the area of the window to the nearest square foot.

Solution

1) Find the area of the rectangle.
   \[ A = lw = 8.25(4) = 33 \]

2) For the window shown, the radius of the half circle is half the width of the window, or 2 feet.
   \[ A = \frac{1}{2}\pi r^2 \]  Write formula for area of a half circle.
   \[ \approx \frac{1}{2}(3.14)(2)^2 \]  Substitute 3.14 for \( \pi \) and 2 for \( r \).
   \[ = 6.28 \]  Simplify.

3) Find the total area.
   Total area \( \approx 33 + 6.28 = 39.28 \)

Answer  The area of the Norman window is about 39 square feet.
Guided Practice

Vocabulary Check
1. The diameter of a circle is 5 centimeters. What is the radius?

2. Copy and complete: The ratio of the _?_ of a circle to its diameter is equal to $\pi$.

Skill Check
3. Find the circumference and area of the circle shown. Use $\frac{22}{7}$ for $\pi$. Round your answers to the nearest whole number.

4. The circumference of a circle is 22 meters. Find the radius of the circle to the nearest tenth of a meter.

5. The area of a circle is 87 square feet. Find the diameter of the circle to the nearest foot.

Guided Problem Solving
6. **Wrestling** Use the diagram of the square wrestling mat shown. The circle is the part of the mat used for competition. What is the area of the part that is _not_ used for competition?
   1. Find the area of the entire mat.
   2. Find the area of the circle used for competition to the nearest square meter.
   3. Subtract the area of the circle from the area of the mat.

Practice and Problem Solving

**Homework Help**

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Find the circumference of the circle. Use $3.14$ or $\frac{22}{7}$ for $\pi$. Round to the nearest whole number.

7. \[ \text{18 in.} \]
8. \[ \text{22 m} \]
9. \[ \text{42 cm} \]

10. \[ \text{70 yd} \]
11. \[ \text{32 mm} \]
12. \[ \text{44 ft} \]

For a circle with the given circumference $C$, find the radius and diameter of the circle. Round to the nearest whole number.

13. $C = 37 \text{ m}$
14. $C = 25 \text{ cm}$
15. $C = 51 \text{ in.}$

Lesson 10.4 Circumference and Area of a Circle
16. **Error Analysis** Describe and correct the error in finding the approximate area of a circle with a diameter of 20 feet.

\[
A = \pi r^2 \\
= 3.14(20)^2 \\
= 1256 \text{ ft}^2
\]

Find the area of the circle. Use 3.14 or \(\frac{22}{7}\) for \(\pi\). Round to the nearest whole number.

17. \[\text{8 in.}\]
18. \[\text{14 ft}\]
19. \[\text{28 cm}\]
20. \[\text{46 mm}\]
21. \[\text{33 m}\]
22. \[\text{52 yd}\]

For a circle with the given area \(A\), find the radius and diameter of the circle. Round to the nearest whole number.

23. \(A = 254 \text{ m}^2\)  
24. \(A = 615 \text{ cm}^2\)  
25. \(A = 1109 \text{ in}^2\)

26. **Extended Problem Solving** A signal from a walkie-talkie can be received up to 1 mile away. A signal from a CB radio can be received up to 5 miles away.

a. **Calculate** Over how great an area can a walkie-talkie transmit a signal? Express your answer in terms of \(\pi\).

b. **Calculate** Over how great an area can a CB radio transmit a signal? Express your answer in terms of \(\pi\).

c. **Compare** Write a ratio to compare the area of CB radio reception to the area of walkie-talkie reception.

27. **Round Barn** The Ryan barn in Annawan Township, Illinois, is a round barn built in 1910. The floor has a diameter of 85 feet. What is the area of the floor to the nearest square foot?

28. **Pantheon** The circular floor in the Pantheon in Rome has an area of about 1473 square meters. What is the diameter of the floor to the nearest tenth of a meter?

29. **Centrifuge Training** Astronauts train for space flight in a centrifuge, which consists of a rotating arm with a cab at the outer end of the arm. The arm, which has a length of 58 feet, is revolved about the center of the centrifuge. An astronaut sits in the cab, which is then rotated 50 times per minute. To the nearest hundred feet, how far does the astronaut travel in one minute?
30. Draw a rectangle that is 6 units by 5 units on graph paper. Use a compass to draw a half circle with a radius of 3 units on a longer side of the rectangle. Find the area of the figure to the nearest whole number.

31. **Predict** You double the radius of a circle. Predict what will happen to the circle’s circumference and what will happen to its area. Test your prediction for a few circles. Use a different radius for each circle. Then predict how doubling a circle’s diameter will affect its circumference and area. Test your prediction for a few circles with different diameters.

32. In the diagram, the diameter of the large circle is 18 meters. All four small circles are the same size. Find the area of one small circle and the area of the large circle in terms of $\pi$. Then find the ratio of the area of a small circle to the area of the large circle.

33. **Challenge** An air traffic control radar screen is a circle with a diameter of 24 inches.

   a. What is the area of the screen to the nearest square inch?
   
   b. The radar screen is set to have a scale of 6 inches : 25 nautical miles. To the nearest square nautical mile, what is the area of the circular region covered by the radar?

34. **Writing** A half circle is drawn on each side of a right triangle as shown. What is the relationship among the areas of the 3 half circles? Explain your reasoning.

---

**Mixed Review**

41. **Evaluate the expression. (Lesson 1.3)**

\[ 35. \quad 64 \div 16 + 6 \div 2 \quad 36. \quad 3 \cdot 14 + 8 \quad 37. \quad 7 \cdot 5 + 3 \cdot 9 + 5 \]

42. **Find the slope of the line through the given points. (Lesson 8.4)**

\[ 38. \quad (6, 1), (3, 4) \quad 39. \quad (-5, -3), (-5, 6) \quad 40. \quad (3, -6), (-1, -6) \]

**Find the area of the parallelogram or trapezoid. (Lesson 10.3)**

\[ 41. \quad \text{Area of parallelogram} \quad 42. \quad \text{Area of trapezoid} \quad 43. \quad \text{Area of trapezoid} \]

**Standardized Test Practice**

44. **Multiple Choice** The diameter of a circle is 22 meters. What is the approximate area?

   A. 35 m$^2$  \quad B. 69 m$^2$  \quad C. 380 m$^2$  \quad D. 1520 m$^2$

45. **Short Response** The base of the sundial shown is a square with a circle inside it. To the nearest square inch, what is the area of the part of the base that is not within the circle? Explain your answer.
1. Find the value of \( x \) for the triangle shown. Then classify the triangle by its angles.

2. The perimeter of an equilateral triangle is 219 feet. Find the lengths of the sides.

Tell whether the figure is a polygon. If it is a polygon, classify it and tell whether it is convex or concave. If not, explain why.

3. 

4. 

5. 

Find the area of the parallelogram, trapezoid, or circle to the nearest square unit. Use 3.14 or \( \frac{22}{7} \) for \( \pi \).

6. 

7. 

8. 

9. 

10. 

11. 

Three farmers inherit farmland that is divided into six fields. In the drawing, the green fields are trapezoids, the yellow fields are parallelograms, and the blue field is a rectangle. The farmers want to divide up the fields so that each farmer has the same area of land. They don't want to change the shape of any field. How can they distribute the fields fairly?
Solids

Classifying Solids

A **solid** is a three-dimensional figure that encloses a part of space. The polygons that form the sides of a solid are called **faces**.

<table>
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<th>Four Types of Solids</th>
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<td><strong>Prism</strong> is a solid formed by polygons. Prisms have two congruent bases that lie in parallel planes. The other faces are rectangles.</td>
</tr>
<tr>
<td><strong>Pyramid</strong> is a solid formed by polygons. The base can be any polygon, and the other faces are triangles.</td>
</tr>
<tr>
<td><strong>Cylinder</strong> is a solid with two congruent circular bases that lie in parallel planes.</td>
</tr>
<tr>
<td><strong>Cone</strong> is a solid with one circular base.</td>
</tr>
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</table>

**Example** Classify the solid as a **prism**, **pyramid**, **cylinder**, or **cone**.

- **a.**
  
  ![Soup Can](image)

  The soup can has two congruent circular bases. It is a **cylinder**.

- **b.**
  
  ![Gift Box](image)

  All sides of the gift box are rectangles. Any two opposite sides can be considered bases. The gift box is a **rectangular prism**.

- **c.**
  
  ![Ice Cream Cone](image)

  The ice cream novelty has one circular base. It is a **cone**.

**Solution**

- **a.** The soup can has two congruent circular bases. It is a cylinder.
- **b.** All sides of the gift box are rectangles. Any two opposite sides can be considered bases. The gift box is a rectangular prism.
- **c.** The ice cream novelty has one circular base. It is a cone.
Counting Faces, Edges, and Vertices

The faces of a prism or a pyramid meet in segments called edges. Edges meet at points called vertices. (The singular form of vertices is vertex.)

Example Count the number of faces, edges, and vertices in a triangular pyramid.

![Diagram showing 4 faces, 6 edges, and 4 vertices]

Sketching a Solid

You can sketch a solid so that it appears to be three-dimensional.

Example Sketch a triangular prism.

1. Sketch two congruent triangles.
2. Use segments to connect corresponding vertices.
3. Make any “hidden” lines dashed.

![Sketching a solid]

Test your knowledge of solids by solving these problems.

Checkpoint

Classify each solid as a prism, pyramid, cylinder, or cone. If the solid is a prism or a pyramid, count the number of faces, edges, and vertices.

1. 2. 3.

Sketch the solid.

4. Rectangular prism 5. Square pyramid 6. Cone
10.5 Nets and Surface Area

**Goal**
Use the net of a rectangular prism to find the prism’s surface area.

**Materials**
- cereal box
- ruler
- scissors

The *surface area* of a solid is the sum of the areas of all of its surfaces.

**Investigate**

**Find the surface area of a cereal box.**

1. Cut along the edges of the box until you can flatten it as shown below. The resulting shape is called a net. Measure the length of each edge of the net using a ruler.

2. Find the area of the net.

   First find the area of each face.
   - **Area of front or back:** \((7.5)(10.75) = 80.625 \text{ in.}^2\)
   - **Area of top or bottom:** \((7.5)(2.5) = 18.75 \text{ in.}^2\)
   - **Area of each side:** \((2.5)(10.75) = 26.875 \text{ in.}^2\)

   Then find the sum of the areas of the faces.
   \[
   80.625 + 80.625 + 18.75 + 18.75 + 26.875 + 26.875 = 252.5 \text{ in.}^2
   \]

   The surface area of the cereal box shown is 252.5 square inches.

**Draw Conclusions**

1. **Critical Thinking** How can you find the surface area of a box without making a net? Explain.

2. **Writing** Describe the net of the cylindrical oatmeal container shown. Make a sketch of the net. Find the surface area of the container and explain your reasoning.
Surface Areas of Prisms and Cylinders

Vocabulary
- surface area, p. 538
- net, p. 538
- lateral face of a prism, p. 539
- lateral area of a prism, p. 539
- lateral surface of a cylinder, p. 540
- lateral area of a cylinder, p. 540

**Pizza Box** The **surface area** of a solid is the sum of the areas of its faces. The pizza box shown has the shape of a rectangular prism. What is its surface area?

In Example 1, a **net** is used to find the surface area of the pizza box. A **net** is a two-dimensional representation of a solid. The surface area of a solid is equal to the area of its net.

**Example 1 Using a Net to Find Surface Area**

The net at the right represents the pizza box shown above. (Any flaps or foldovers to hold the box together have been ignored.) Use the net to find the surface area of the pizza box.

**Solution**

1) Find the area of each face.

   - **Area of top or bottom:** $16 \cdot 16 = 256 \text{ in}^2$
   - **Area of each side:** $16 \cdot 2 = 32 \text{ in}^2$

2) Find the sum of the areas of the faces.

   $$256 + 256 + 32 + 32 + 32 + 32 = 640 \text{ in}^2$$

**Answer** The surface area of the pizza box is 640 square inches.
**Surface Areas of Prisms** The lateral faces of a prism are the faces that are not bases. The lateral area of a prism is the sum of the areas of the lateral faces. The surface area of a prism is the sum of the areas of the bases and the lateral area. In the diagram, \( P \) is the base perimeter.

\[
\text{Surface area} = 2 \cdot \text{Base area} + \text{Lateral area} = 2B + Ph
\]

---

**Study Strategy**

In this book, every prism is a right prism, which means that the edges connecting the bases are perpendicular to the bases.

---

**Surface Area of a Prism**

**Words** The surface area \( S \) of a prism is the sum of twice the base area \( B \) and the product of the base perimeter \( P \) and the height \( h \).

**Algebra** \( S = 2B + Ph \)

**Numbers** \( S = 2(6 \cdot 4) + [2(6) + 2(4)]10 = 248 \) square units

---

**Example 2** Using a Formula to Find Surface Area

Find the surface area of the prism.

The bases of the prism are right triangles.

\[
S = 2B + Ph = 2\left(\frac{1}{2} \cdot 6 \cdot 8\right) + (6 + 8 + 10)(18) = 480
\]

**Answer** The surface area of the prism is 480 square centimeters.

---

**Checkpoint**

Find the surface area of the prism.

1.

2.
**Surface Areas of Cylinders** The curved surface of a cylinder is called the **lateral surface**. The **lateral area** of a cylinder is the area of the lateral surface. The surface area of a cylinder is the sum of the areas of the bases and the product of the base circumference and the height. In the diagram below, \( C \) represents the base circumference.

\[
\text{Surface area} = 2 \cdot \text{Base area} + \text{Lateral area} = 2B + Ch
\]

**Surface Area of a Cylinder**

**Words** The surface area \( S \) of a cylinder is the sum of twice the base area \( B \) and the product of the base circumference \( C \) and the height \( h \).

**Algebra**

\[
S = 2B + Ch = 2\pi r^2 + 2\pi rh
\]

**Numbers**

\[
S = 2\pi(4)^2 + 2\pi(4)(10) = 352 \text{ square units}
\]

---

**Example 3** *Using a Formula to Find Surface Area*

**Racquetball** Find the surface area of the container of racquetballs. Round to the nearest square inch.

**Solution**

The radius is one half of the diameter, so \( r = 1.25 \) inches.

\[
S = 2\pi r^2 + 2\pi rh
\]

\[
= 2\pi(1.25)^2 + 2\pi(1.25)(5)
\]

\[
= 15.625\pi
\]

\[
= 49.1
\]

**Answer** The surface area of the container of racquetballs is about 49 square inches.
Guided Practice

Vocabulary Check
1. What formula would you use to find the surface area of a triangular prism?

2. How do you find the area of the bases of a cylinder? How do you find the lateral area of a cylinder?

Skill Check
3. Draw a net for the solid. Then find the surface area.

4. Draw a net for the solid. Then find the surface area.

5. Error Analysis Describe and correct the error in finding the surface area of the prism.

\[ S = 2B + Ph \]
\[ = 2(5 \cdot 4) + (5 + 12 + 13)(4) \]
\[ = 160 \text{ square centimeters} \]

Practice and Problem Solving

Draw a net for the solid. Then find the surface area. Round to the nearest whole number.

6.

7.

8.

9.

10.

11.

12. Cans A can of vegetables is in the shape of a cylinder. The diameter of the can is 7 centimeters, and the height is 11 centimeters. Find the surface area of the can. Round to the nearest square centimeter.
13. **Marimba Pipes**  Find the lateral area of the marimba pipe. Round to the nearest square inch.

![Marimba Pipe](image)

14. **Painting**  You are going to paint the platforms shown below for your school’s theater production. All sides of each platform must be painted yellow. What total area must the paint cover?

![Platform Images](image)

15. **In the Real World**  

**Marimbas**  The rectangular keys of a marimba are always made of rosewood. The dimensions of the longest key are $20 \frac{1}{4}$ in. by $3 \frac{1}{4}$ in. The dimensions of the shortest key are $7 \frac{5}{8}$ in. by $1 \frac{5}{8}$ in. What is the ratio of the area of the longest key to the area of the shortest key?

The solids shown are composed of prisms and half cylinders. Find the surface area of the solid. Round to the nearest whole number.

16. 

![Solid Images](image)

17. **Critical Thinking**  A cylinder has a radius of $r$ inches and a height of $h$ inches. Suppose the radius and height of the cylinder are both doubled.

a. How does the area of the base of the new cylinder compare with the area of the base of the original cylinder? Explain.

b. How does the circumference of the new cylinder compare with the circumference of the original cylinder? Explain.

c. How does the surface area of the new cylinder compare with the surface area of the original cylinder? Explain.

18. **Visual Thinking**  When a plane intersects a solid, the intersection of the plane and the solid forms a **cross section**.

a. A plane intersects a cylinder parallel to the bases of the cylinder, as shown. What shape does the cross section have? To the nearest square meter, what is the area of the cross section?

b. Another plane intersects the cylinder perpendicular to the bases and passes through the centers of the bases. What shape does the cross section have? What is the area of the cross section?
19. **Mailbox Sliders** Some snowboarding parks have a feature shaped like an elongated mailbox and called a mailbox slider. Snowboarders use their boards to jump onto the mailbox slider and slide along its top surface. A mailbox slider is composed of a half cylinder and a rectangular prism. Find the surface area of the mailbox slider shown. Round your answer to the nearest square inch.

20. **Measurement** Find an object that is shaped like a cylinder. Measure its radius and height, then find its surface area. Describe the procedure you used.

21. The diameter of a base of a cylinder is twice the height of the cylinder.
   a. Write a formula in terms of $h$ for the surface area of the cylinder.
   b. The surface area of the cylinder is $64\pi$ square units. Find the radius and height of the cylinder.

22. **Challenge** Draw a cylinder and label the radius $r$ and the height $h$. Assume a plane slices the cylinder in half by passing through the center of each base. Write a formula for the surface area of a half cylinder.

**Mixed Review**

23. The perimeter of a triangle is 37 feet. The length of the first side is 4 more than the length of the second side. The length of the third side is equal to the length of the second side. Find the lengths of each side. Then classify the triangle by its side lengths. (Lesson 10.1)

**Find the area of the parallelogram or trapezoid.** (Lesson 10.3)

24. [Diagram of a parallelogram with sides 6 in and 15 in.]

25. [Diagram of a trapezoid with bases 26 m and 12.5 m, height 5 m.]

**Find the circumference of the circle given the diameter $d$ or the radius $r$.** Use $3.14$ or $\frac{22}{7}$ for $\pi$. Round to the nearest whole number. (Lesson 10.4)

26. $d = 30$ m  
27. $d = 84$ in.  
28. $r = 78$ ft  
29. $r = 102$ cm

30. **Multiple Choice** What is the approximate surface area of the cylinder shown?
   A. 35 yd$^2$  
   B. 57 yd$^2$  
   C. 68 yd$^2$  
   D. 80 yd$^2$

31. **Short Response** The length of a base of a rectangular prism is twice the width of the base. The height of the prism is 3 times the width of a base. The surface area is 792 square centimeters. What are the dimensions of the prism?
Recall that the base of a pyramid is a polygon, and the lateral faces of the pyramid are triangles with a common vertex. The **height of a pyramid** is the perpendicular distance between the base and this common vertex.

In this lesson, all pyramids are **regular pyramids**. In a **regular pyramid**, the base is a regular polygon and all of the lateral faces of the pyramid are congruent isosceles triangles. The **slant height of a pyramid** is the height of any of these triangular lateral faces.

In this lesson, the variable $h$ represents the height of the pyramid, and the variable $l$ represents the slant height.

**Example 1**

**Finding the Slant Height of a Pyramid**

**Architecture** The Pyramid at California State University, Long Beach, has a height of 192 feet. The base of the pyramid is a square with a side length of 345 feet. What is the slant height of the pyramid to the nearest foot?

**Solution**

Notice that the slant height $l$ of the pyramid is the hypotenuse of a right triangle. The length of one leg of this triangle is 192 feet. The length of the other leg is $\frac{345}{2} = 172.5$ feet. Use the Pythagorean theorem to find the slant height.

\[
192^2 + (172.5)^2 = l^2 \\
36,620.25 = l^2 \\
\sqrt{36,620.25} = l \\
258.1 = l
\]

**Answer** The slant height of the Pyramid is about 258 feet.
**Surface Areas of Regular Pyramids** You can use a net of a pyramid with a square base to find a formula for its surface area. Let \( l \) be the slant height of the pyramid, and let \( s \) be the side length of the square base. The lateral area of the pyramid is the sum of the areas of the 4 triangular faces. In the diagram below, the area of 1 triangular face is \( \frac{1}{2}sl \), and \( P \) is the perimeter of the square base.

\[
\begin{align*}
\text{Surface area} & = \text{Base area} + \text{Lateral area} \\
& = B + 4 \left( \frac{1}{2}sl \right) \\
& = B + \frac{1}{2}(4sl) \\
& = B + \frac{1}{2}Pl
\end{align*}
\]

**Study Strategy**

In simplifying the expression for the lateral area of a regular pyramid, you can substitute \( P \) for \( 4s \) because the product of the number of triangular faces and the side length of the base equals the perimeter of the base.

**Study Strategy**

The formula \( S = B + \frac{1}{2}Pl \) can be used to find the surface area of any regular pyramid, not just a square pyramid.

**Example 2**

**Finding the Surface Area of a Regular Pyramid**

Find the surface area of the regular pyramid.

1. Find the perimeter and area of the base.
   \[
   P = 4(10) = 40 \text{ cm} \\
   B = 10^2 = 100 \text{ cm}^2
   \]

2. Find the surface area.
   \[
   S = B + \frac{1}{2}Pl \\
   = 100 + \frac{1}{2}(40)(13) \\
   = 360
   \]

**Answer** The surface area of the pyramid is 360 square centimeters.
**Surface Areas of Cones** The point on a cone directly above the center of its base is called the **vertex** of the cone. The distance between the vertex and center of the base is the **height of the cone**. The **slant height of the cone** is the distance between the vertex and any point on the edge of the base. To find the surface area of a cone, you need to know the radius \( r \) of the circular base and the slant height \( l \).

\[
\text{Surface area} = \text{Base area} + \text{Lateral area} = B + \pi rl
\]

---

**Surface Area of a Cone**

**Words** The surface area \( S \) of a cone is the sum of the base area \( B \) and the product of \( \pi \), the base radius \( r \), and the slant height \( l \).

**Algebra** \( S = B + \pi rl = \pi r^2 + \pi rl \)

---

**Example 3** Finding the Surface Area of a Cone

Find the surface area of the cone. Round to the nearest square inch.

\[
S = \pi r^2 + \pi rl
\]

Write formula for surface area of a cone.

\[
= \pi(7)^2 + \pi(7)(24)
\]

Substitute 7 for \( r \) and 24 for \( l \).

\[
= 217\pi \approx 681.7
\]

Simplify. Then evaluate using a calculator.

**Answer** The surface area of the cone is about 682 square inches.

---

**Checkpoint**

Find the surface area of the pyramid or cone. Round to the nearest whole number.

1. \[
\text{Base area} = 3.9 \text{ m}^2
\]

2. \[
\text{Base area} = \frac{1}{3}bh = \frac{1}{3}(8 \text{ cm})(15 \text{ cm}) = 40 \text{ cm}^2
\]
Guided Practice

Vocabulary Check
1. Explain the difference between the height and the slant height of a pyramid.

2. What part of the formula \( S = \pi r^2 + \pi rl \) gives you the base area of a cone? Which part gives you the lateral area?

Skill Check
Find the surface area of the pyramid or cone. Round to the nearest whole number.

3.

4.

5.

Guided Problem Solving
6. What is the surface area of the cone? Round to the nearest square centimeter.

1) Find the slant height of the cone.

2) Find the surface area of the cone to the nearest square centimeter.

Practice and Problem Solving

Homework Help

Find the slant height of the pyramid or cone. Round to the nearest whole number.

7.

8.

9.

Use the net to sketch the solid and find its surface area. Round to the nearest whole number.

10.

11.

12.

13. Writing Which is greater, a pyramid’s height or its slant height? Explain your reasoning.
Find the surface area of the regular pyramid.

14. \( \begin{align*}
10 \text{ ft} \\
5 \text{ ft} \\
5 \text{ ft} \\
B = 10.8 \text{ ft}^2
\end{align*} \)

15. \( \begin{align*}
72 \text{ mm} \\
42 \text{ mm} \\
42 \text{ mm}
\end{align*} \)

16. \( \begin{align*}
19.3 \text{ cm} \\
15.5 \text{ cm} \\
15.5 \text{ cm}
\end{align*} \)

Find the surface area of the cone. Round to the nearest whole number.

17. \( \begin{align*}
8.5 \text{ m} \\
3.25 \text{ m}
\end{align*} \)

18. \( \begin{align*}
12 \text{ ft} \\
18 \text{ ft}
\end{align*} \)

19. \( \begin{align*}
7.8 \text{ in.} \\
14.2 \text{ in.}
\end{align*} \)

20. **Ant Lions** Ant lions are insects that dig cone-shaped pits that they use to trap ants for food. Find the surface area of the sloping walls of an ant lion pit. Round to the nearest square inch. *(Hint: You need to find lateral area.)*

Find the surface area of the pyramid or cone. Round to the nearest whole number.

21. \( \begin{align*}
25 \text{ in.} \\
30 \text{ in.} \\
15 \text{ in.}
\end{align*} \)

22. \( \begin{align*}
64 \text{ cm} \\
60 \text{ cm}
\end{align*} \)

23. \( \begin{align*}
4.1 \text{ yd} \\
5.8 \text{ yd}
\end{align*} \)

**Spheres** In Exercises 24–27, use the following information to find the surface area of the specified objects. Round to the nearest square unit.

A *sphere* is a solid formed by all points in space that are the same distance from a fixed point (the center). The formula for the surface area of a sphere is \( S = 4\pi r^2 \), where \( r \) is the radius.

24. The radius of a spherical soap bubble is 0.5 inch.

25. The diameter of Europa, one of Jupiter’s moons, is about 1950 miles.

26. The diameter of a table tennis ball is 40 millimeters.

27. The diameter of a large exercise ball is 2.5 feet.

28. **Pyramid** The unusual building shown in the photo includes a square pyramid turned upside down. The side length of the base of the pyramid is 74 feet, and the height of the pyramid is 48 feet. What is the surface area of the pyramid?

29. **Critical Thinking** The side length of the base of a square pyramid is 8 feet. The diameter of the base of a cone is 8 feet. The height of both solids is 3 feet. Sketch both solids. Which one has the greater surface area?
30. **Visual Thinking** You can cut the lateral surface of a cone into congruent wedges. You can rearrange these wedges to form a figure that resembles a parallelogram as shown. The more wedges you cut, the more closely the shape will resemble a parallelogram.

![Diagram of cone and parallelogram]

a. Write expressions for the height and the base of the parallelogram in terms of \( r \) and \( l \).

b. Use the expressions from part (a) to write a formula for the area of the parallelogram in terms of \( r \) and \( l \). How is this formula related to the formula for the surface area of a cone?

The solids shown are composed of cones, cylinders, and pyramids. Find the surface area of the solid. Round to the nearest whole number.

31.  

32.  

33. **Challenge** The surface area of a cone is \( 90\pi \) square inches. The base radius of the cone is 5 inches. Find the slant height of the cone.

---

**Mixed Review**  
**Simplify the expression** *(Lesson 9.2)*

34. \( \sqrt{48} \)  
35. \( \sqrt{288} \)  
36. \( \frac{\sqrt{40}}{9} \)  
37. \( \sqrt{\frac{24}{121}} \)

38. Find the unknown side lengths of the triangle. Give exact answers. *(Lesson 9.6)*  

39. The base radius of a cylinder is 5 centimeters, and the height is 12 centimeters. Find the surface area of the cylinder. Round your answer to the nearest square centimeter. *(Lesson 10.5)*

40. **Multiple Choice** What is the approximate surface area of a cone that has a height of 14.5 meters and base diameter of 10 meters?  
   A. \( 140 \text{ m}^2 \)  
   B. \( 306 \text{ m}^2 \)  
   C. \( 319 \text{ m}^2 \)  
   D. \( 795 \text{ m}^2 \)

41. **Short Response** Two pyramids have congruent bases. The slant height of the first pyramid is one third of the slant height of the second pyramid. Is the surface area of the first pyramid one third of the surface area of the second pyramid? Explain your reasoning.
10.7 Building and Sketching Solids

**Goal**
Build, sketch, and find the volume of solids using unit cubes and dot paper.

**Materials**
- unit cubes
- dot paper

In this activity, you will build or draw solids given the top, side, and front views. Assume that there are no missing blocks in views that are not shown.

**Investigate**

**Use the three views of a solid to build the solid using unit cubes. Find the volume of the solid.**

1. The top view gives information about the bottom layer of the solid. There are 9 unit cubes on the bottom layer.

2. The side view shows that there are two layers in the solid.

3. The front view shows you how to form the two layers of cubes. The middle row of cubes is missing from the top layer. The volume of the solid is 15 cubic units.

**Draw Conclusions**

1. Use the three views of a solid to build the solid using unit cubes. Which solid has a greater volume?

   **a.**
   \[
   \text{Top} \quad \text{Side} \quad \text{Front}
   \]

   **b.**
   \[
   \text{Top} \quad \text{Side} \quad \text{Front}
   \]
Use the three views of a solid to draw the solid using dot paper. Find the volume of the solid.

1. On dot paper, draw a set of three axes that form 120° angles.

2. Draw a unit cube where the three axes intersect.

3. Draw the cubes you can see from the front.

4. Draw the cubes you can see from the side.

5. Check that the top view matches what you have drawn. The solid has a volume of 5 cubic units.

Draw Conclusions

2. Use the three views of each solid to draw the solids using dot paper. Which solid has a greater volume?

a. Top Side Front

b. Top Side Front
Volumes of Prisms and Cylinders

**Before**
You found surface areas of prisms and cylinders.

**Now**
You'll find the volumes of prisms and cylinders.

**Why?**
So you can compare the volumes of two suitcases, as in Ex. 14.

Recall that the volume of a solid is the amount of space the solid occupies. Volume is measured in cubic units. For the prism below, the base area is 8 square units. To find the volume, you can imagine unit cubes filling the prism. There are 3 layers of unit cubes, so the volume is 8 \(\times\) 3 = 24 cubic units.

![Volume of a prism diagram]

\[
\text{Volume of a prism} = \text{Base area} \times \text{Height}
\]

**Volume of a Prism**

**Words** The volume \(V\) of a prism is the product of the base area \(B\) and the height \(h\).

**Algebra** \(V = Bh\)

**Example 1** Finding the Volume of a Prism

Find the volume of the prism shown.
The bases of the prism are triangles, so use the formula for the area of a triangle to find \(B\).

\[
V = Bh
\]

\[
= \frac{1}{2}(8)(6)(13)
\]

\[
= 312
\]

**Answer** The volume of the prism is 312 cubic centimeters.
Cylinders  The formula for the volume of a cylinder is like the formula for the volume of a prism.

\[
\text{Volume of a cylinder} = \text{Base area} \times \text{Height}
\]

**Study Strategy**

The formula for the volume of a cylinder is obtained by substituting \( \pi r^2 \) for \( B \). So, the formula \( V = Bh \) becomes \( V = \pi r^2 h \).

**Volume of a Cylinder**

**Words**  The volume \( V \) of a cylinder is the product of the base area \( B \) and the height \( h \).

**Algebra**  \( V = Bh = \pi r^2 h \)

**Example 2  Finding the Volume of a Cylinder**

**Swimming Pool**  Find the capacity (in gallons) of the swimming pool shown. Round to the nearest whole number. (Use the fact that 1 ft\(^3\) = 7.481 gal.)

**Solution**

1. The radius is one half of the diameter. So, \( r = 9 \).
   \[
   V = \pi r^2 h \quad \text{Write formula for volume of a cylinder.}
   = \pi(9)^2(4) \quad \text{Substitute 9 for } r \text{ and 4 for } h.
   = 324 \pi \quad \text{Simplify.}
   \]

2. Use a conversion factor that converts cubic feet to gallons.
   \[
   \frac{324 \pi \text{ ft}^3 \cdot 7.481 \text{ gal}}{1 \text{ ft}^3} = 7614.7 \text{ gal} \quad \text{Evaluate. Use a calculator.}
   \]

**Answer**  The capacity of the swimming pool is about 7615 gallons.

**Checkpoint**

Find the volume of the prism or cylinder.

1. 
   \[
   \text{Volume} = 3 \times 10 \times 8 = 240 \text{ m}^3
   \]

2. 
   \[
   \text{Volume} = 4 \times 6 \times 9 = 216 \text{ yd}^3
   \]

3. 
   \[
   \text{Volume} = 5 \times 15 \times \pi \times 9 = 675 \pi \text{ ft}^3
   \]

4. **Critical Thinking**  Which solid has a greater volume, a prism with bases that are squares with side length 8 units and a height of 11 units or a cylinder with a diameter of 11 units and a height of 8 units? Which solid has the greater surface area?
Example 3  Finding the Volume of a Solid

The solid shown is composed of a rectangular prism and two half cylinders. Find the volume of the solid. Round to the nearest cubic foot.

Solution

1) Find the area of a base. Each end of a base is a half circle with a radius of 1 foot. Together, the ends form a complete circle.

\[ B = \text{Area of rectangle} + \text{Area of circle} = lw + \pi r^2 \]

Use formulas for area of a rectangle and area of a circle.

\[ = 3(2) + \pi(1)^2 \]

Substitute 3 for \( l \), 2 for \( w \), and 1 for \( r \).

\[ = 6 + \pi \]

Simplify. Leave in terms of \( \pi \).

2) \[ V = Bh \]

Write formula for volume of a prism.

\[ = (6 + \pi)1.5 \]

Substitute \( 6 + \pi \) for \( B \) and 1.5 for \( h \).

\[ = 9 + 1.5\pi \]

Use distributive property.

\[ = 13.71 \]

Evaluate. Use a calculator.

Answer  The volume of the solid shown is about 14 cubic feet.

10.7 Exercises

Guided Practice

Vocabulary Check  1. Copy and complete: The volume of a solid is measured in \_ units.

2. Explain how to find the volume of any prism.

Skill Check  Find the volume of the prism or cylinder. Round to the nearest whole number.

3. \[ \text{10 cm} \]

4. \[ \text{5 in.} \]

5. \[ \text{8 ft} \]

6. Error Analysis  Explain why the following calculation would not give the correct volume for the prism: \( V = Bh = 6 \cdot 9 \cdot 10 \).
Find the volume of the prism or cylinder. Round to the nearest whole number.

7. \[11 \text{ m} \times 2 \text{ m} \times 4 \text{ m}\]
8. \[6 \text{ ft} \times 4 \text{ ft} \times 9 \text{ ft}\]
9. \[18 \text{ cm} \times 9 \text{ cm} \times 22 \text{ cm}\]
10. \[7 \text{ in.} \times 8 \text{ in.} \times \text{ unknown}\]
11. \[3 \text{ mm} \times 2 \text{ mm} \times \text{ unknown}\]
12. \[17 \text{ yd} \times 6 \text{ yd} \times \text{ unknown}\]

13. **Mailing** You are mailing a gift box that is 17 inches by 14 inches by 10 inches. You want to put it in a larger box and surround it with foam packing. The larger box is 20 inches by 17 inches by 13 inches. How many cubic inches of foam packing do you need?

14. **Suitcases** Tell which suitcase holds more.

**Suitcase A**
- 12 in.
- 30 in.
- 22 in.

**Suitcase B**
- 14 in.
- 30 in.
- 21 in.

15. **Candles** The red candle costs $7.05, the blue candle costs $7.80, and the green candle costs $10.55.

\[3 \text{ in.} \times 6 \text{ in.}\]
\[3 \text{ in.} \times 9 \text{ in.}\]
\[4 \text{ in.} \times 4 \text{ in.}\]

a. How much wax is used in each candle? Round to the nearest cubic inch.

b. Find the ratio of the cost of a candle to the volume of wax used in the candle.

c. **Interpret and Apply** Which candle is the best buy? Explain.

Find the unknown dimension. Round to the nearest whole number.

16. \[V = 210 \text{ cm}^3\]
17. \[V = 301 \text{ in.}^3\]
18. \[V = 254 \text{ m}^3\]
The solids shown are composed of prisms, cylinders, and half cylinders. Find the volume of the solid. Round to the nearest whole number.

19. 

20. 

21. The radius of a cylinder is 3 units, and the height is 4 units. 
   a. What is the volume of the cylinder? 
   b. What is the volume when the radius is doubled? When the height is doubled? When both the radius and height are doubled? 
   c. **Compare** For each cylinder in part (b), compare its volume to the volume of the cylinder in part (a). What do you notice in each case? 

22. **Salsa** You made 10 quarts of salsa. You are putting the salsa in jars with a diameter of 3 inches and a height of 5.5 inches. How many full jars of salsa will you have? (Use the fact that \(1 \text{ in.}^3 = 0.017 \text{ qt}\).) 

23. **Challenge** Copy the regular hexagonal prism shown. A base of the prism can be divided into 6 equilateral triangles. The height of each equilateral triangle is \(2\sqrt{3}\). Find the base area. Then find the volume of the prism. 

**Mixed Review**

Determine whether the triangle with the given side lengths is a right triangle. (*Lesson 9.3*)

24. 8, 11, 14 
25. 8, 15, 17 
26. 2, 4.8, 5.2 

27. For \(\triangle ABC\), find \(\tan A\) and \(\tan B\). (*Lesson 9.7*)

Find the surface area of the pyramid or cone. Round to the nearest whole number. (*Lesson 10.6*)

28. 
29. 
30. 

**Standardized Test Practice**

31. **Multiple Choice** What is the volume of a rectangular prism with a length of 16 inches, a height of 4 inches, and a width of 12 inches?
   A. 48 in.\(^3\)  
   B. 768 in.\(^3\)  
   C. 1810 in.\(^3\)  
   D. 2413 in.\(^3\)  

32. **Multiple Choice** What is the approximate volume of a cylinder with a diameter of 18 meters and a height of 3 meters?
   F. 54 m\(^3\)  
   G. 243 m\(^3\)  
   H. 763 m\(^3\)  
   I. 3054 m\(^3\)
10.7 Surface Area and Volume

**Goal** Use a spreadsheet to compare the surface areas and volumes of solids.

**Example**

Efficient packaging uses the least amount of material for the greatest volume. Compare the ratios of volume to surface area of the rectangular prisms below. Which prism is a more efficient package?

![Diagram of two rectangular prisms: Package A with dimensions 5 in. x 2 in. x 3 in. and Package B with dimensions 4 in. x 3 in. x 3 in.]

Use a spreadsheet to compare the ratios of volume to surface area. The prism with the greater ratio is the more efficient package.

1. **Label columns for length, width, height, surface area, volume, and ratio in row 1. Enter the dimensions of package A and the formulas for surface area, volume, and ratio in row 2 as shown.**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>l</td>
<td>w</td>
<td>h</td>
<td>Surface area</td>
<td></td>
<td>Volume Ratio</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>(=2^{<em>}A2^{</em>}B2)</td>
<td></td>
<td>(=A2^{<em>}B2^{</em>}C2)</td>
</tr>
</tbody>
</table>

2. **Enter the dimensions of package B in row 3. Use the Fill down feature to calculate the surface area, volume, and ratio of the second prism.**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>l</td>
<td>w</td>
<td>h</td>
<td>Surface area</td>
<td></td>
<td>Volume Ratio</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td></td>
<td>62</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td></td>
<td>66</td>
<td>36</td>
</tr>
</tbody>
</table>

Because \(0.55 > 0.48\), package B is more efficient than package A.

**Draw Conclusions**

1. **Compare** Find the ratio of volume to surface area for a cube that is 5 inches long, 5 inches wide, and 5 inches high. Is the cube more efficient or less efficient than the packages in the example above?

2. **Critical Thinking** A product is packaged in cylinders. Package A has a radius of 2 inches and a height of 6 inches. Package B has a radius of 3 inches and a height of 4 inches. Which package is more efficient?
Volumes of Pyramids and Cones

**Review Vocabulary**

volume, p. 789

**BEFORE**

You found the volumes of prisms and cylinders.

**Now**

You’ll find the volumes of pyramids and cones.

**WHY?**

So you can find the volume of a composter, as in Ex. 15.

Consider a prism and a pyramid that have the same base area and the same height. If you completely fill the pyramid with sand and pour the sand into the prism, you’ll find that the sand fills one third of the prism. You can conclude that the volume of the pyramid is one third of the volume of the prism. The same relationship holds for a cylinder and a cone with the same base area and the same height.

**Volume of a Pyramid or a Cone**

**Words**

The volume $V$ of a pyramid or a cone is one third of the product of the base area $B$ and the height $h$.

**Algebra**

$V = \frac{1}{3} Bh$

**Example 1**

**Finding the Volume of a Pyramid**

The base of a pyramid is a square. The side length of the square is 24 feet. The height of the pyramid is 9 feet. Find the volume of the pyramid.

$V = \frac{1}{3} Bh$

Write formula for volume of a pyramid.

$= \frac{1}{3}(24^2)(9)$

Substitute $24^2$ for $B$ and 9 for $h$.

$= 1728$

Simplify.

**Answer**

The volume of the pyramid is 1728 cubic feet.
Example 2  Finding the Volume of a Cone

Find the volume of the cone shown. Round to the nearest cubic millimeter.

The radius is one half of the diameter, so \( r = 6.75 \).

\[
V = \frac{1}{3} \pi r^2 h
\]

Write formula for volume of a cone.

\[
= \frac{1}{3} \pi (6.75)^2 (10)
\]

Substitute 6.75 for \( r \) and 10 for \( h \).

\[
\approx 477.1
\]

Evaluate. Use a calculator.

Answer The volume of the cone is about 477 cubic millimeters.

Example 3  Finding the Volume of a Solid

Silos  The grain silo shown is composed of a cylinder and a cone. Find the volume of the silo to the nearest cubic foot.

Solution

1. Find the volume of the cylindrical section. The radius is one half of the diameter, so \( r = 9 \).

\[
V = \pi r^2 h
\]

Write formula for volume of a cylinder.

\[
= \pi (9)^2 (29) = 2349\pi
\]

Substitute values. Then simplify.

2. Find the volume of the conical section.

\[
V = \frac{1}{3} \pi r^2 h
\]

Write formula for volume of a cone.

\[
= \frac{1}{3} \pi (9)^2 (7) = 189\pi
\]

Substitute 9 for \( r \) and 7 for \( h \). Then simplify.

3. Find the sum of the volumes.

\[
2349\pi + 189\pi = 2538\pi = 7973.4
\]

Answer The volume of the silo is about 7973 cubic feet.

In the Real World

Silos  Suppose you have planted 360 acres of corn on your family farm and expect to produce about 140 bushels of shelled corn per acre. A bushel is a unit of volume equal to about 1.25 cubic feet. How many silos, each the size of the one in Example 3, would it take to store your entire crop?

Checkpoint

Find the volume of the pyramid or cone. Round to the nearest whole number.

1. \[
\begin{align*}
&10 \text{ cm} \\
&14 \text{ cm}
\end{align*}
\]

2. \[
\begin{align*}
&9 \text{ m} \\
&3.5 \text{ m}
\end{align*}
\]

3. \[
\begin{align*}
&12 \text{ ft} \\
&24 \text{ ft}
\end{align*}
\]

Lesson 10.8  Volumes of Pyramids and Cones
Summary: Surface Areas and Volumes of Solids

Prism
- Surface Area: $S = 2B + Ph$
- Volume: $V = Bh$

Cylinder
- Surface Area: $S = 2\pi r^2 + 2\pi rh$
- Volume: $V = \pi r^2h$

Pyramid
- Surface Area: $S = B + \frac{1}{2}Pl$
- Volume: $V = \frac{1}{3}Bh$

Cone
- Surface Area: $S = \pi r^2 + \pi rl$
- Volume: $V = \frac{1}{3}\pi r^2h$

Guided Practice

Vocabulary Check
1. What formula can you use to find the volume of a pyramid?
2. How is the formula for the volume of a cone related to the formula for the volume of a cylinder?

Skill Check
Find the volume of the pyramid or cone. Round to the nearest whole number.

3. Pyramid with base $5$ in., height $3$ in., and slant height $4$ in.

4. Cone with radius $5$ in. and height $15$ in.

5. Error Analysis
- Describe and correct the error in finding the volume of the solid shown.

\[ V = \frac{1}{3}Bh \]
\[ \frac{1}{3} \pi (8)^2 (7) \]
\[ = 469 \text{ cm}^3 \]
Practice and Problem Solving

Find the volume of the pyramid or cone. Round to the nearest whole number.

6. \( \text{Vol} = \frac{1}{3} \pi r^2 h \)

7. \( \text{Vol} = \frac{1}{3} \pi r^2 h \)

8. \( \text{Vol} = \frac{1}{3} \pi r^2 h \)

9. \( \text{Vol} = \frac{1}{3} \pi r^2 h \)

10. \( \text{Vol} = \frac{1}{3} \pi r^2 h \)

11. \( \text{Vol} = \frac{1}{3} \pi r^2 h \)

12. The diameter of a cone-shaped paper cup is 8 centimeters, and the height is 10 centimeters. The radius of another cone-shaped paper cup is 3 centimeters, and the height is 11 centimeters.

   a. **Predict** Which cup do you predict will hold more water? Explain your prediction.

   b. **Compare** Find the volume of each paper cup to the nearest tenth of a cubic centimeter. Which cup holds more water?

The solid in Exercise 13 is composed of a cylinder and a cone. The solid in Exercise 14 is a cube with a cone-shaped hole in it. Find the volume of the solid. Round your answer to the nearest whole number.

13.

14.

15. **Extended Problem Solving** The composter shown turns biodegradable materials like leaves and grass into fertilizer.

   a. Find the volume of the cylindrical portion of the composter in terms of \( \pi \).

   b. Find the volume of the top cone and the volume of the bottom cone in terms of \( \pi \).

   c. **Apply** Find the total volume of the composter to the nearest cubic inch.

   d. **Writing** When assembling the composter, you can adjust the height and the radius. Which has the greater effect on the volume of the composter, changing the height or changing the radius? Explain.
16. **Compare** The radius of cone A is 3 inches, and its height is 7 inches. The radius of cone B is 4 inches, and its height is 6 inches. Create a spreadsheet to compare the volumes and surface areas of the cones. Which cone has a greater ratio of volume to surface area?

17. **Paperweight** A solid crystal paperweight is in the shape of a cube that has an edge length of 6 centimeters. A triangular pyramid is cut from one corner of the cube. The base area of the pyramid is about 7.64 square centimeters, and the height of the pyramid is about 1.7 centimeters. Find the volume of the paperweight to the nearest hundredth of a cubic centimeter.

Find the unknown dimension of the pyramid or cone. Round to the nearest whole number.

18. \( V = 12.4\pi \text{ ft}^3 \)

19. \( V = 1452 \text{ mm}^3 \)

20. **Funnel** Most funnels consist of a cone whose tip is removed. The cone is then attached to a narrow cylinder. In the diagram below, the small cone inside the cylinder shows the portion of the large cone that has been cut off.

   a. Find the volume of the large cone and the volume of the small cone to the nearest hundredth of a cubic inch.
   
   b. Calculate the difference between the volume of the large cone and the volume of the small cone.
   
   c. Find the volume of the cylinder to the nearest hundredth of a cubic inch.
   
   d. Use your results from part (b) and part (c) to find the total volume of the funnel to the nearest tenth of a cubic inch.

**Spheres** In Exercises 21–23, find the volume \( V \) of the spherical object using the formula \( V = \frac{4}{3}\pi r^3 \), where \( r \) is the radius. Round to the nearest cubic unit.

21. The diameter of a pearl is 8.3 millimeters.

22. The radius of a women’s basketball is 4.5 inches.

23. The diameter of an inflatable beach ball is 3 feet.

24. **Challenge** Find the volume of the cone shown. Round to the nearest hundredth of a cubic meter.
Mixed Review

25. Given that \( \triangle ABC \sim \triangle ADE \), find \( DE \). (Lesson 6.5)

[Diagram of triangles]

Algebra Basics Identify the slope and \( y \)-intercept of the line. (Lesson 8.5)

26. \( y = -5x + 2 \)  27. \( y = \frac{3}{2}x - 1 \)  28. \( y = 13 \)  29. \( 2y = 9 \)

30. What is the volume of the cylinder shown? Round to the nearest cubic inch. (Lesson 10.7)

[Diagram of a cylinder]

31. What is the volume of a rectangular prism with a length of 8 inches, a width of 4 inches, and a height of 4 inches? (Lesson 10.7)

32. Multiple Choice A cone has a height of 3 feet. The radius of the base is 4 feet. What is the approximate volume of the cone?
   A. 12.6 ft\(^3\)  B. 37.7 ft\(^3\)  C. 50.3 ft\(^3\)  D. 150.8 ft\(^3\)

33. Multiple Choice The base area of a triangular pyramid is 12 square centimeters. The height of the pyramid is 5 centimeters. What is the volume of the pyramid?
   F. 20 cm\(^3\)  G. 30 cm\(^3\)  H. 60 cm\(^3\)  I. 120 cm\(^3\)

34. Short Response A marble monument is in the shape of a square pyramid. The side length of the base is 5 feet. The height of the pyramid is 5 feet. Find the volume of the pyramid. Use the fact that 1 cubic foot of marble weighs about 170 pounds. To the nearest pound, how much does the monument weigh? Explain your reasoning.

Standardized Test Practice

Brain Game

Thinking About Sculptures

Each sculpture below is a prism, a cylinder, a cone, or a pyramid. Find the volume of each sculpture in terms of \( a \) and \( b \). Then compare the coefficients of the five volume expressions to order the volumes from least to greatest. The letters associated with the sculptures will spell out the last name of the artist who created a famous statue called \( The \ Thinker \).
Chapter Review

Vocabulary Review

polygons: pentagon, hexagon, heptagon, octagon, p. 516
quadrilaterals: trapezoid, parallelogram, rhombus, p. 517
diagonal of a polygon, p. 518
base, height of a parallelogram, p. 521
bases, height of a trapezoid, p. 522
circles: center, radius, diameter, p. 528
circumference, p. 528
surface area, p. 538
net, p. 538
lateral face of a prism, p. 539
lateral area of a prism, p. 539
lateral surface of a cylinder, p. 540
lateral area of a cylinder, p. 540
height, slant height of a pyramid, p. 544
regular pyramid, p. 544
height, slant height of a cone, p. 546

1. When finding the area of a trapezoid, what lengths do you need to know?
2. Describe convex and concave polygons. Tell how they are alike and how they are different.
3. Describe how to find the surface area of a prism using a net.
4. Tell how the slant height of a cone is different from the height of the cone.

10.1 Triangles

Goal
Find unknown angle measures and classify triangles.

Example
Find the value of \( y \). Then classify the triangle by its angle measures.

\[
23^\circ + (7y + 5)^\circ + y^\circ = 180^\circ \\
8y + 28 = 180 \\
8y = 152 \\
y = 19 \\
7y + 5 = 7(19) + 5 = 138
\]

The triangle has 1 obtuse angle, so it is an obtuse triangle.

5. \( \frac{66^\circ}{(4y - 6)^\circ} \) 
6. \( \frac{1}{2}y - 3^\circ \) 
7. \( \frac{33^\circ}{(y + 3)^\circ} \)
10.2 Polygons and Quadrilaterals

**Goal**
Classify polygons and quadrilaterals.

**Example**
Tell whether the figure is a polygon. If it is, classify it. If not, explain why.

The figure is a 6-sided polygon. So, it is a hexagon. It is convex and regular.

8. 9. 10.

10.3 Areas of Parallelograms and Trapezoids

**Goal**
Find the areas of parallelograms and trapezoids.

**Example**
Find the area of the trapezoid.

\[ A = \frac{1}{2}(b_1 + b_2)h \]

Write formula for area of a trapezoid.

\[ = \frac{1}{2}(3 + 7)(1.5) \]

Substitute 3 for \( b_1 \), 7 for \( b_2 \), and 1.5 for \( h \).

\[ = 7.5 \text{ cm}^2 \]

Simplify.


14. The diameter of a circle is 16 inches. Find the circumference and area of the circle. Round your answers to the nearest whole number.

10.4 Circumference and Area of a Circle

**Goal**
Find circumferences and areas of circles.

**Example**
Find the area of the circle to the nearest square inch. Use 3.14 for \( \pi \).

\[ A = \pi r^2 \]

Write the area of a circle.

\[ \approx 3.14(9)^2 \]

Substitute 3.14 for \( \pi \) and 9 for \( r \).

\[ \approx 254 \text{ in}^2 \]

Simplify.

14. The diameter of a circle is 16 inches. Find the circumference and area of the circle. Round your answers to the nearest whole number.
10.5 Surface Areas of Prisms and Cylinders

Goal

Find the surface areas of prisms and cylinders.

Example

Find the surface area of the prism.

The bases of the prism are right triangles.

\[ S = 2B + Ph \]

\[ = 2 \left( \frac{1}{2} \cdot 5 \cdot 12 \right) + (5 + 12 + 13)12 \]

\[ = 420 \text{ ft}^2 \]

The bases of the prism are right triangles.

\[ S = 2B + Ph \]

\[ = 2 \left( \frac{1}{2} \cdot 5 \cdot 12 \right) + (5 + 12 + 13)12 \]

\[ = 420 \text{ ft}^2 \]

Find the surface area of the prism or cylinder to the nearest square inch.

15. 16. 17.

10.6 Surface Areas of Pyramids and Cones

Goal

Find the surface areas of pyramids and cones.

Example

Find the surface area of the cone to the nearest square meter.

\[ S = \pi r^2 + \pi rl \]

\[ = \pi (9)^2 + \pi (9)(41) \]

\[ = 1414 \text{ m}^2 \]

Find the surface area of the regular pyramid or cone to the nearest square foot.

18. 19. 20.
10.7 Volumes of Prisms and Cylinders

Goal
Find the volumes of prisms and cylinders.

Example
Find the volume of the cylinder to the nearest cubic centimeter.

The radius is one half of the diameter, so \( r = 10 \).

\[
V = \pi r^2 h
\]

Write formula for volume of a cylinder.

\[
= \pi (10)^2 (24)
\]

Substitute 10 for \( r \) and 24 for \( h \).

\[
= 2400 \pi
\]

Simplify.

\[
= 7539.8
\]

Evaluate. Use a calculator.

The volume of the cylinder is about 7540 cubic centimeters.

Find the volume of the prism or cylinder. Round to the nearest whole number.

21.

22.

23.

10.8 Volumes of Pyramids and Cones

Goal
Find the volumes of pyramids and cones.

Example
Find the volume of the pyramid.

\[
V = \frac{1}{3} Bh
\]

Write formula for volume of a pyramid.

\[
= \frac{1}{3} (5^2)(3)
\]

Substitute \( 5^2 \) for \( B \) (because the base is a square) and 3 for \( h \).

\[
= 25
\]

Simplify.

The volume of the pyramid is 25 cubic meters.

Find the volume of the pyramid or cone. Round to the nearest whole number.

24.

25.

26.
1. The perimeter of a triangle is 53 inches. The length of one side is 15 inches. The other two sides are congruent. Find their lengths.

2. The ratio of the angle measures of a triangle is 1 : 3 : 8. Find the angle measures. Then classify the triangle by its angle measures.

3. Tell whether the figure shown is a polygon. If it is, classify it. If not, explain why.

**Find the area of the parallelogram or trapezoid.**

4. [Diagram of a parallelogram with sides 20 m, 12 m, and 10 m]

5. [Diagram of a trapezoid with bases 8 ft and 5 ft, and height 5 ft]

6. [Diagram of a trapezoid with bases 4 m and 8 m, and height 14 m]

7. **Rug** A circular rug has a diameter of 15 feet. Find the area of the rug to the nearest square foot. Then find the circumference of the rug to the nearest foot. Use 3.14 for \( \pi \).

**Find the surface area of the pyramid, cylinder, or cone. Round to the nearest whole number.**

8. [Diagram of a cylinder with radius 8 mm and height 18 mm]

9. [Diagram of a triangular pyramid with base 15 yd, height 12 yd, and slant height 10 yd]

10. [Diagram of a square pyramid with base 12 ft, height 10 ft, and slant height 10 ft]

11. **Doorstop** You are decorating a doorstop to use for your bedroom door. The doorstop is a triangular prism. In order to buy paint for the doorstop, you need to know its surface area. What is the surface area of the doorstop?

**Find the volume of the prism, pyramid, or cone. Round to the nearest whole number.**

12. [Diagram of a rectangular prism with dimensions 2.5 cm, 4 cm, and 4.5 cm]

13. [Diagram of a cone with base diameter 15 in. and height 7 in.]

14. [Diagram of a triangular prism with base 6 ft, height 8 ft, and slant height 8 ft]

15. **Container** You use a plastic container to hold pasta salad for your lunch. The container is a cylinder with a diameter of 5 inches and a height of 3 inches. Find its volume to the nearest cubic inch.
1. The perimeter of a triangle is 20 feet. The length of a side is 4 feet. The lengths of the other two sides are equal. What are the lengths of the other two sides?
   A. 4 ft    B. 8 ft    C. 10 ft    D. 16 ft

2. What is the value of \( x \) in the quadrilateral shown?
   F. 11    G. 94
   H. 105    I. 285

3. The height of a parallelogram is 20 meters. The base is one fifth of the height. What is the area of the parallelogram?
   A. 4 m\(^2\)    B. 40 m\(^2\)    C. 80 m\(^2\)    D. 100 m\(^2\)

4. What is the approximate circumference of the circle?
   F. 6 ft    G. 12 ft
   H. 24 ft    I. 48 ft

5. The area of a circle is 64 square inches. What is its approximate diameter?

6. What is the surface area of the rectangular prism?
   F. 92 in.\(^2\)    G. 104 in.\(^2\)
   H. 160 in.\(^2\)    I. 184 in.\(^2\)

7. What is the volume of the prism in Exercise 6?
   A. 92 in.\(^3\)    B. 104 in.\(^3\)
   C. 160 in.\(^3\)    D. 184 in.\(^3\)

8. The height of a cylinder is 10 inches, and the diameter of the base is 6 inches. What is the approximate volume of the cylinder?
   F. 117 in.\(^3\)    G. 283 in.\(^3\)
   H. 360 in.\(^3\)    I. 1130 in.\(^3\)

9. What is the approximate volume of the cone?
   A. 9.42 mm\(^3\)    B. 18.84 mm\(^3\)
   C. 28.26 mm\(^3\)    D. 56.52 mm\(^3\)

10. **Short Response** A base of a trapezoid is 16 feet, and the height is 3 feet. The area of the trapezoid is 36 square feet. Find the length of the other base.

11. **Extended Response** The lampshade shown below can be described as part of a cone.

   a. The right triangles shown with the cone are similar. Find the value of \( x \). Explain how you found your answer.
   b. Find the lateral areas of the large and small cones to the nearest square inch.
   c. Find the lateral area of the lampshade.