Linear Functions

How can you use math to describe the steepness of a bike trail?

In previous chapters you’ve . . .
- Located points in a coordinate plane
- Written and solved equations and inequalities

In Chapter 8 you’ll study . . .
- Representing relations and functions
- Finding and interpreting slopes of lines
- Writing and graphing linear equations in two variables
- Graphing and solving systems of linear equations
- Graphing linear inequalities in two variables

So you can solve real-world problems about . . .
- hurricanes, p. 389
- transportation, p. 401
- horseback riding, p. 409
- robotics, p. 416
- marathons, p. 424
- rivers, p. 430
- video, p. 440

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### Bike Trails

The steepness of a bike trail, such as the Slickrock Trail in Utah, can be described by the ratio of the change in elevation to the horizontal distance traveled. In this chapter, you will use **slope** to compare the vertical change to the horizontal change between two points on a line in a coordinate plane.

### What do you think?

Suppose one bike trail rises 15 feet over a horizontal distance of 100 feet. Another trail rises 5 feet over a horizontal distance of 40 feet. Which trail do you think is steeper? Why?
Chapter Prerequisite Skills

**PREREQUISITE SKILLS QUIZ**

**Preparing for Success** To prepare for success in this chapter, test your knowledge of these concepts and skills. You may want to look at the pages referred to in blue for additional review.

1. **Vocabulary** Describe the difference between an equation and an inequality.

2. **School Trip** You are saving money for a school trip. You have saved $156. This is $62 less than the trip costs. How much money does the trip cost? (p. 91)

Solve the equation. Check your solution. (p. 125)

3. \[4(7 - 2t) = 4\]

4. \[-18 = 3(w - 1)\]

5. \[11y + 9 - 5y = -15\]

6. \[29 + 4(f + 2) = -7\]

Write an inequality represented by the graph. (p. 138)

7. \[\begin{array}{cccccc}
-3 & -2 & -1 & 0 & 1 & 2 & 3 \\
\end{array}\]

8. \[\begin{array}{cccccccc}
-7 & -6 & -5 & -4 & -3 & -2 \\
\end{array}\]

Solve the inequality. Graph your solution. (pp. 138, 144)

9. \[x + 14 < 25\]

10. \[-5y \leq 150\]

11. \[\frac{n}{3} \geq 11\]

12. \[m - 6 > 21\]

**NOTETAKING STRATEGIES**

**Note Worthy** You will find a notetaking strategy at the beginning of each chapter. Look for additional notetaking and study strategies throughout the chapter.

**USING COLOR** You may find it helpful to use color to identify important pieces of information and show how they are related to each other.

\[5x + 6 + 9x + 2\]

\[= 5x + 6 + 9x + 2\] Use color to identify like terms.

\[= 5x + 9x + 6 + 2\] Group like terms.

\[= 14x + 8\] Combine like terms.

In Lesson 8.4, you can use color when finding slopes of lines.
Relations and Functions

**Vocabulary**
- relation, p. 385
- domain, p. 385
- range, p. 385
- input, p. 385
- output, p. 385
- function, p. 386
- vertical line test, p. 387

**Before\ Now\ Why?**
You graphed ordered pairs. You’ll use graphs to represent relations and functions. So you can show the growth of a bird over time, as in Ex. 26.

**Alligators** The table below shows the ages and lengths of five alligators.

<table>
<thead>
<tr>
<th>Age (years), x</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (in.), y</td>
<td>32</td>
<td>59</td>
<td>65</td>
<td>69</td>
<td>96</td>
</tr>
</tbody>
</table>

You can represent the relationship between age and length using the ordered pairs (x, y):

(2, 32), (4, 59), (5, 65), (5, 69), (7, 96)

The ordered pairs form a relation. A relation is a pairing of numbers in one set, called the domain, with numbers in another set, called the range. Each number in the domain is an input. Each number in the range is an output. For a relation represented by ordered pairs, the inputs are the x-coordinates and the outputs are the y-coordinates.

**Example 1 \ Identifying the Domain and Range**

a. Identify the domain and range of the relation given above.

b. Identify the domain and range of the relation represented by the table below, which shows one alligator’s length at different ages.

<table>
<thead>
<tr>
<th>Age (years), x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (in.), y</td>
<td>23</td>
<td>36</td>
<td>47</td>
<td>61</td>
<td>73</td>
</tr>
</tbody>
</table>

**Solution**

a. The domain of the relation is the set of all inputs, or x-coordinates. The range is the set of all outputs, or y-coordinates.

- **Domain:** 2, 4, 5, 7
- **Range:** 32, 59, 65, 69, 96

b. The relation consists of the ordered pairs (1, 23), (2, 36), (3, 47), (4, 61), and (5, 73). The domain and range are shown below.

- **Domain:** 1, 2, 3, 4, 5
- **Range:** 23, 36, 47, 61, 73

**Checkpoint**

Identify the domain and range of the relation.

1. (0, 1), (2, 4), (3, 7), (5, 4)
2. (−1, 2), (−3, −1), (6, 0), (−1, 4)
Representing Relations  In addition to using ordered pairs or a table to represent a relation, you can also use a graph or a mapping diagram.

**Example 2  Representing a Relation**

Represent the relation \((-1, 1), (2, 0), (3, 1), (3, 2), (4, 5)\) as indicated.

a. A graph

\[\begin{array}{c|c|c|c|c|c}
\hline
& 1 & 2 & 3 & 4 & 5 \\
\hline
\hline
1 & & & & & \\
2 & & & & & \\
3 & & & & & \\
4 & & & & & \\
5 & & & & & \\
\hline
\end{array}\]

b. A mapping diagram

\[\begin{array}{c|c}
\hline
\text{Input} & \text{Output} \\
\hline
-1 & 0 \\
2 & 1 \\
3 & 2 \\
4 & 5 \\
\hline
\end{array}\]

Functions  A relation is a **function** if for each input there is exactly one output. In this case, the output is a function of the input.

**Example 3  Identifying Functions**

Tell whether the relation is a function.

a. The relation at the top of page 385, consisting of the ordered pairs (age, length) for five different alligators:

\[(2, 32), (4, 59), (5, 65), (5, 69), (7, 96)\]

b. The relation in part (b) of Example 1, consisting of the ordered pairs (age, length) for one alligator at different times:

\[(1, 23), (2, 36), (3, 47), (4, 61), (5, 73)\]

Solution

a. The relation is not a function because the input 5 is paired with two outputs, 65 and 69. This makes sense, as two alligators of the same age do not necessarily have the same length.

b. The relation is a function because every input is paired with exactly one output. This makes sense, as a single alligator can have only one length at a given point in time.

**Checkpoint**

Represent the relation as a graph and as a mapping diagram. Then tell whether the relation is a function. Explain your reasoning.

3. \((0, 3), (1, 2), (2, -1), (4, 4), (5, 4)\)  
4. \((-2, -1), (0, 2), (2, 3), (-2, -4)\)
**Vertical Line Test** When a relation is represented by a graph, you can use the vertical line test to tell whether the relation is a function. The vertical line test says that if you can find a vertical line passing through more than one point of the graph, then the relation is not a function. Otherwise, the relation is a function.

**Study Strategy**

In part (b) of Example 4, notice why the vertical line test works. Because the vertical line at \( x = 3 \) intersects the graph twice, the input 3 must be paired with two outputs, 1 and \(-2\). So, the graph does not represent a function.

**Example 4** Using the Vertical Line Test

a. In the graph below, no vertical line passes through more than one point. So, the relation represented by the graph is a function.

b. In the graph below, the vertical line shown passes through two points. So, the relation represented by the graph is not a function.

---

**8.1 Exercises**

*More Practice, p. 810*

**Guided Practice**

**Vocabulary Check**

1. Copy and complete: A relation is a(n) ___ if for each input there is exactly one output.

2. Draw a mapping diagram that represents a relation with domain \(-1, 0, 2\) and range \(1, 4\). Is only one answer possible? Explain.

**Skill Check**

**Identify the domain and range of the relation.**

3. (0, 0), (1, 2), (2, 4), (3, 6), (4, 8)  
4. (2, 5), (−5, 2), (1, 5), (2, −3), (7, 5)

**Represent the relation as a graph and as a mapping diagram. Then tell whether the relation is a function. Explain your reasoning.**

5. (1, 2), (1, 5), (2, 4), (3, 3), (4, 1)  
6. (−4, 2), (2, 3), (4, 8), (0, 3), (−2, 2)

**7. Error Analysis** Describe and correct the error in the given statement.

The relation (1, −5), (2, −5), (3, 6), (4, 11) is not a function because the inputs 1 and 2 are both paired with the output −5.
Identify the domain and range of the relation.

8. \((-2, 5), (-1, 2), (0, 4), (1, -9)\)
9. \((7, 3), (7, 6), (7, 9), (3, 3), (3, 6)\)

10. \[
\begin{array}{ccccccc}
\hline
x & 4 & 2 & -3 & 4 & -4 & \\
y & 0 & -1 & 0 & -1 & 0 & \\
\hline
\end{array}
\]

11. \[
\begin{array}{ccccccc}
\hline
x & 1.5 & 1.5 & 2.8 & 2.8 & 6.5 & \\
y & 4.3 & 6.5 & 4.3 & 3.9 & 0.2 & \\
\hline
\end{array}
\]

12. Copy and complete using always, sometimes, or never: A relation is \_a function._

Represent the relation as a graph and as a mapping diagram. Then tell whether the relation is a function. Explain your reasoning.

13. \((1, 2), (2, 1), (3, 0), (3, 4), (4, 3)\)
14. \((0, 4), (2, 0), (6, -4), (-4, 2), (8, 0)\)

15. \[
\begin{array}{ccccccc}
\hline
x & -2 & -1 & 0 & 1 & 2 & \\
y & -3 & -3 & -3 & -3 & -3 & \\
\hline
\end{array}
\]

16. \[
\begin{array}{ccccccc}
\hline
x & 0 & -5 & -10 & 5 & -10 & \\
y & 15 & -10 & -5 & -15 & 20 & \\
\hline
\end{array}
\]

17. **Height** The height of a person is measured every year from the age of 1 year to the age of 50 years.
   
   a. Do the ordered pairs (age, height) represent a function? Explain.
   
   b. **Critical Thinking** Would you expect the ordered pairs (height, age) to represent a function? Why or why not?

Tell whether the relation represented by the graph is a function.

18. [Graph]
19. [Graph]
20. [Graph]

21. **Basketball** The table shows the numbers of games played and points scored by each starting player on the New Jersey Nets basketball team during the team’s 2001–2002 regular season.

<table>
<thead>
<tr>
<th>Player</th>
<th>Games played, (x)</th>
<th>Points scored, (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Todd MacCulloch</td>
<td>62</td>
<td>604</td>
</tr>
<tr>
<td>Kenyon Martin</td>
<td>73</td>
<td>1086</td>
</tr>
<tr>
<td>Keith Van Horn</td>
<td>81</td>
<td>1199</td>
</tr>
<tr>
<td>Kerry Kittles</td>
<td>82</td>
<td>1102</td>
</tr>
<tr>
<td>Jason Kidd</td>
<td>82</td>
<td>1208</td>
</tr>
</tbody>
</table>

a. Identify the domain and range of the relation given by the ordered pairs \((x, y)\).

b. Draw a mapping diagram for the relation.

c. Is the relation a function? Explain.
22. **Hurricanes** In 1995, a total of 32 regular weather advisories were issued during the storm that became Hurricane Opal. The graph shows the wind speed inside Opal at the time of each advisory.

![Wind Speeds During Hurricane Opal graph]

<table>
<thead>
<tr>
<th>Wind Speed (mi/h)</th>
<th>Advisory number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>14</td>
<td>28</td>
</tr>
<tr>
<td>16</td>
<td>32</td>
</tr>
</tbody>
</table>


b. **Estimation** An ocean storm is considered a hurricane if its wind speed is at least 74 miles per hour. For which advisories did Opal qualify as a hurricane?

23. **Writing** Suppose a relation is represented as a set of ordered pairs and as a mapping diagram. Which representation more clearly shows whether or not the relation is a function? Explain.

24. **Extended Problem Solving** A skydiver uses an altimeter to track altitude so that he or she knows when to open the parachute. The altimeter determines altitude by measuring changes in atmospheric pressure. The graph below shows how pressure varies with altitude as a skydiver falls from 12,000 feet to ground level. (The elevation of the ground is assumed to be 0 feet with respect to sea level.)

![Skydiver Pressure graph]

<table>
<thead>
<tr>
<th>Pressure (lb/ft²)</th>
<th>Altitude (thousands of feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2200</td>
<td>0</td>
</tr>
<tr>
<td>2000</td>
<td>2</td>
</tr>
<tr>
<td>1800</td>
<td>4</td>
</tr>
<tr>
<td>1600</td>
<td>6</td>
</tr>
<tr>
<td>1400</td>
<td>8</td>
</tr>
<tr>
<td>1200</td>
<td>10</td>
</tr>
<tr>
<td>1000</td>
<td>12</td>
</tr>
</tbody>
</table>

a. As a skydiver falls, does the atmospheric pressure increase or decrease? Does the reading on the skydiver’s altimeter increase or decrease?

b. **Writing** Describe the domain and range of the relation represented by the graph.

c. Is the relation a function? Explain.

d. **Interpret and Apply** Some altimeters can sound an alarm warning a skydiver to open the parachute when the altitude falls to a certain level. If the alarm is set to go off at an altitude of 3000 feet, approximately what atmospheric pressure will trigger the alarm?
25. **Challenge**  To form the *inverse* of a relation represented by a set of ordered pairs, you switch the coordinates of each ordered pair. For example, the inverse of the relation $(1, 2), (3, 4), (5, 6)$ is $(2, 1), (4, 3), (6, 5)$. Give an example of a relation that is a function, but whose inverse is *not* a function.

26. **Birds**  The brown-headed cowbird does not raise its own offspring. It lays eggs in the nests of other bird species, which then hatch the eggs and raise the young. A scientist investigated whether the growth of a young cowbird is affected by the species of bird that raises it. The scientist’s results for two bird species are shown below.

<table>
<thead>
<tr>
<th>Cowbird Raised by Red-Eyed Vireo</th>
<th>Cowbird Raised by Blue-Gray Gnatcatcher</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cowbird age (days)</td>
<td>Cowbird age (days)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Cowbird mass (grams)</td>
<td>Cowbird mass (grams)</td>
</tr>
<tr>
<td>1.7</td>
<td>2.2</td>
</tr>
<tr>
<td>4.5</td>
<td>5.2</td>
</tr>
<tr>
<td>10.7</td>
<td>10.1</td>
</tr>
<tr>
<td>20.0</td>
<td>15.5</td>
</tr>
<tr>
<td>28.3</td>
<td>19.3</td>
</tr>
<tr>
<td>33.0</td>
<td>21.2</td>
</tr>
<tr>
<td>35.0</td>
<td>22.0</td>
</tr>
<tr>
<td>35.7</td>
<td>22.3</td>
</tr>
</tbody>
</table>

a. For each table, draw a graph for the relation given by the ordered pairs (age, mass). Draw both graphs in the same coordinate plane, and use a different color for each graph.

b. **Interpret**  Compare the graphs from part (a). How is a cowbird’s growth when raised by a red-eyed vireo like its growth when raised by a blue-gray gnatcatcher? How is its growth different?

**Mixed Review**

Evaluate the expression when $x = -5$ and $y = -7$. *(Lessons 1.5-1.7)*

27. $x + y$  
28. $y - x + 10$  
29. $2x^2y$  
30. $3x - 4y$

Tell whether the given value of the variable is a solution of the equation. *(Lesson 2.4)*

31. $x + 11 = 3; x = -8$  
32. $-17 - a = -23; a = -6$  
33. $-6m = -84; m = 13$  
34. $\frac{-144}{u} = 12; u = -12$

For an account that earns simple annual interest, find the interest and the balance of the account. *(Lesson 7.7)*

35. $P = 850, r = 3\%, t = 6$ years  
36. $P = 4200, r = 5\%, t = 7.5$ years

**Standardized Test Practice**

37. **Extended Response**  The table shows the amount charged for standard ground shipping by an online electronics store.

<table>
<thead>
<tr>
<th>Total cost of merchandise</th>
<th>Shipping cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>$.01–$25.00</td>
<td>$5.95</td>
</tr>
<tr>
<td>$25.01–$50.00</td>
<td>$7.95</td>
</tr>
<tr>
<td>$50.01–$75.00</td>
<td>$9.95</td>
</tr>
<tr>
<td>$75.01–$100.00</td>
<td>$11.95</td>
</tr>
<tr>
<td>Over $100.00</td>
<td>$13.95</td>
</tr>
</tbody>
</table>


8.2 Linear Equations in Two Variables

Vocabulary
- equation in two variables, p. 391
- solution of an equation in two variables, p. 391
- graph of an equation in two variables, p. 392
- linear equation, p. 392
- function form, p. 393

**BEFORE**
- You solved equations in one variable.

**NOW**
- You’ll find solutions of equations in two variables.

**WHY?**
- So you can find the speed of a platypus, as in Ex. 41.

**Volcanoes** The Hawaiian volcano Mauna Loa has erupted many times. In 1859, lava from the volcano traveled 32 miles to the Pacific Ocean at an average speed of 4 miles per hour. In Example 2, you’ll see how to use an equation in two variables to describe the flow of the lava toward the ocean.

An example of an equation in two variables is $2x - y = 5$. A solution of an equation in $x$ and $y$ is an ordered pair $(x, y)$ that produces a true statement when the values of $x$ and $y$ are substituted into the equation.

**Example 1** Checking Solutions

Tell whether the ordered pair is a solution of $2x - y = 5$.

**a.** $(1, -3)$

**Solution**

$$2x - y = 5$$

$2(1) - (-3) \not= 5$

$5 \neq 5$

**Answer** $(1, -3)$ is not a solution of $2x - y = 5$.

**b.** $(4, 7)$

$$2x - y = 5$$

$2(4) - 7 \not= 5$

$1 \neq 5$

**Answer** $(4, 7)$ is a solution of $2x - y = 5$.

**Checkpoint**

Tell whether the ordered pair is a solution of $3x + 2y = -8$.

1. $(0, 4)$
2. $(-2, -1)$
3. $(4, -12)$
4. $(10, -19)$
Example 2  Finding Solutions

For the 1859 Mauna Loa eruption described on page 391, the lava’s distance \( d \) (in miles) from the ocean \( t \) hours after it left the volcano can be approximated by the equation \( d = 32 - 4t \).

a. Make a table of solutions for the equation.

b. How long did it take the lava to reach the ocean?

Solution

a. Substitute values of \( t \) into the equation \( d = 32 - 4t \), and find values of \( d \). The table shows that the following ordered pairs are solutions of the equation:

<table>
<thead>
<tr>
<th>( t )</th>
<th>Substitution</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( d = 32 - 4(0) )</td>
<td>32</td>
</tr>
<tr>
<td>1</td>
<td>( d = 32 - 4(1) )</td>
<td>28</td>
</tr>
<tr>
<td>2</td>
<td>( d = 32 - 4(2) )</td>
<td>24</td>
</tr>
</tbody>
</table>

b. Find the value of \( t \) when \( d = 0 \).

\[ 0 = 32 - 4t \]  \quad Substitute 0 for \( d \) in the equation \( d = 32 - 4t \).
\[ -32 = -4t \]  \quad Subtract 32 from each side.
\[ 8 = t \]  \quad Divide each side by \(-4\).

Answer  It took the lava about 8 hours to reach the ocean.

In the Real World
Volcanoes  The temperature of lava from a Hawaiian volcano is about 1160°C. You can use the equation \( F = 1.8C + 32 \) to convert a Celsius temperature \( C \) to a Fahrenheit temperature \( F \). What is the lava’s temperature in degrees Fahrenheit?

Example 3  Graphing a Linear Equation

Graph \( y = 2x - 1 \).

1) Make a table of solutions.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( -2 )</th>
<th>( -1 )</th>
<th>( 0 )</th>
<th>( 1 )</th>
<th>( 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>(-5)</td>
<td>(-3)</td>
<td>(-1)</td>
<td>(1)</td>
<td>(3)</td>
</tr>
</tbody>
</table>

2) List the solutions as ordered pairs.

\((-2, -5), (-1, -3), (0, -1), (1, 1), (2, 3)\)

3) Graph the ordered pairs, and note that the points lie on a line. Draw the line, which is the graph of \( y = 2x - 1 \).

Graph the equation.

5. \( y = 2x \)
6. \( y = -x + 3 \)
7. \( y = 3x - 4 \)
8. \( y = \frac{1}{2}x + 1 \)
**Horizontal and Vertical Lines** The graph of the equation \( y = b \) is the horizontal line through \((0, b)\). The graph of the equation \( x = a \) is the vertical line through \((a, 0)\).

**Example 4**

**Graphing Horizontal and Vertical Lines**

**Graph \( y = 3 \) and \( x = -2 \).**

**a.** The graph of the equation \( y = 3 \) is the horizontal line through \((0, 3)\).

**b.** The graph of the equation \( x = -2 \) is the vertical line through \((-2, 0)\).

**Equations as Functions** In Examples 3 and 4, the vertical line test shows that \( y = 2x - 1 \) and \( y = 3 \) are functions, while \( x = -2 \) is not a function. In general, a linear equation is a function unless its graph is a vertical line. An equation that is solved for \( y \) is in **function form**. You may find it helpful to write an equation in function form before graphing it.

- *Not function form:* \( 3x + y = 7 \)
- *Function form:* \( y = -3x + 7 \)

**Example 5**

**Writing an Equation in Function Form**

Write \( x + 2y = 6 \) in function form. Then graph the equation.

To write the equation in function form, solve for \( y \):

\[
x + 2y = 6 \\
2y = -x + 6 \\
y = -\frac{1}{2}x + 3
\]

Multiply each side by \(\frac{1}{2}\)

To graph the equation, use its function form to make a table of solutions. Graph the ordered pairs \((x, y)\) from the table, and draw a line through the points.

<table>
<thead>
<tr>
<th>(x)</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

**Checkpoint**

9. Graph \( y = -1 \) and \( x = 4 \). Tell whether each equation is a function.

10. Write \( 2x - 3y = 3 \) in function form. Then graph the equation.
### Guided Practice

#### Vocabulary Check
1. Copy and complete: An equation whose graph is a line is called a(n) ___.
2. Is the equation $x = 4y + 3$ in function form? Explain.

#### Skill Check
Tell whether the ordered pair is a solution of $y = 5x - 7$.
3. $(2, 3)$  
4. $(0, -6)$  
5. $(4, 14)$  
6. $(-3, -22)$

Graph the equation.
7. $y = x - 4$  
8. $x = -1$  
9. $y = 2$  
10. $3x + 2y = -2$

#### Guided Problem Solving
11. **Spacecraft**
   In 1997, the Pathfinder spacecraft landed on Mars. It contained a robotic vehicle named Sojourner that could roam up to 500 meters from the lander. The distance $d$ (in meters) that Sojourner could travel in $t$ hours is given by $d = 24t$. How long would it take Sojourner to reach its maximum distance from the lander?

   1. Copy and complete the table using the given equation.
   
<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
</table>

   2. Use your completed table to graph $d = 24t$.
   3. Find the point on the graph whose $d$-coordinate is 500, and estimate the $t$-coordinate of this point. How much time would it take Sojourner to reach its maximum distance from the lander?

### Practice and Problem Solving

Tell whether the ordered pair is a solution of the equation.
12. $y = x - 3; (1, -4)$  
13. $y = -4x + 9; (3, -3)$
14. $x - 2y = 8; (-6, -7)$  
15. $3x - 5y = -1; (9, 5)$

Graph the equation. Tell whether the equation is a function.
16. $y = -x$  
17. $y = 2x - 3$  
18. $y = 1$  
19. $x = -4$
20. $y = \frac{3}{2}x + 1$  
21. $y = -5$  
22. $x = 3$  
23. $y = -5x + 2$

Write the equation in function form. Then graph the equation.
24. $y - x = -1$  
25. $2x + y = 1$  
26. $3x - y = 5$  
27. $8x + 2y = -4$
28. $x - 3y = -9$  
29. $3x + 4y = 0$  
30. $5x - 2y = 6$  
31. $2x + 3y = 12$
32. **Converting Weights**  The formula \( y = 2000x \) converts a weight \( x \) in tons to a weight \( y \) in pounds. The largest known blue whale weighed 195 tons. Find the weight of the whale in pounds.

33. **Converting Volumes**  The formula \( y = 0.001x \) converts a volume \( x \) in milliliters to a volume \( y \) in liters. A juice can has a volume of 355 milliliters. Find the volume of the can in liters.

34. **Converting Areas**  The formula \( y = 2.59x \) converts an area \( x \) in square miles to an approximate area \( y \) in square kilometers. The state of Iowa has an area of 56,276 square miles. Find this area in square kilometers. Round your answer to the nearest thousand square kilometers.

**Find the value of \( a \) that makes the ordered pair a solution of the equation.**

35. \( y = 2x + 5; (-1, a) \)

36. \( y = -3x - 1; (a, 5) \)

37. \( 4x - 7y = 19; (-4, a) \)

38. \( 6x + 5y = 21; (a + 2, -3) \)

39. **Extended Problem Solving**  The fork length of a shark is the distance from the tip of the shark’s snout to the fork of its tail, as shown.

The table lists equations giving the fork length \( f \) as a function of the total length \( t \) for three species of sharks, where both \( f \) and \( t \) are measured in centimeters.

<table>
<thead>
<tr>
<th>Species</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bigeye thresher</td>
<td>( f = 0.560t + 17.7)</td>
</tr>
<tr>
<td>Scalloped hammerhead</td>
<td>( f = 0.776t - 0.313)</td>
</tr>
<tr>
<td>White shark</td>
<td>( f = 0.944t - 5.74)</td>
</tr>
</tbody>
</table>

a. To the nearest centimeter, approximate the fork length of each given species of shark if the shark’s total length is 250 centimeters.

b. **Interpret**  For each species of shark, what percent of the total length does the fork length represent if the shark is 250 centimeters long? Round your answers to the nearest percent.

c. **Writing**  Which species of shark do you think has the longest tail relative to its body size? Explain your reasoning.

40. **Volcanoes**  The Hawaiian-Emperor chain of volcanoes is shown at the left. The age \( a \) (in millions of years) of a volcano in the chain can be approximated by \( a = 0.0129d - 2.25 \), where \( d \) is the volcano’s distance (in kilometers) from Kilauea, measured along the chain.

a. Suiko is 4794 kilometers from Kilauea, measured along the chain. Approximate the age of Suiko to the nearest tenth of a million years.

b. Midway is about 27.7 million years old. Approximate Midway’s distance along the chain from Kilauea to the nearest ten kilometers.
41. **Platypuses** The platypus is an animal with a broad flat tail, webbed feet, and a snout like a duck’s bill. Although a platypus spends much of its time in the water, it can also walk on land. The diagram below shows one complete stride of a walking platypus.

The stride frequency $f$ is the number of strides per second the platypus takes. It can be approximated by the equation $f = 2.13s + 1.19$, where $s$ is the speed of the platypus in meters per second.

- **a.** Solve the given equation for $s$ to obtain an equation that gives speed as a function of stride frequency.

- **b.** **Apply** Use the equation from part (a) to approximate the speed of a platypus that takes 3 strides per second. Round your answer to the nearest tenth of a meter per second.

42. **Challenge** In this exercise, you will investigate the graph of $y = x^2$.

- **a.** Copy and complete the table of solutions for $y = x^2$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3$</td>
<td>?</td>
</tr>
<tr>
<td>$-2$</td>
<td>?</td>
</tr>
<tr>
<td>$-1$</td>
<td>?</td>
</tr>
<tr>
<td>$0$</td>
<td>?</td>
</tr>
<tr>
<td>$1$</td>
<td>?</td>
</tr>
<tr>
<td>$2$</td>
<td>?</td>
</tr>
<tr>
<td>$3$</td>
<td>?</td>
</tr>
</tbody>
</table>

- **b.** Graph $y = x^2$ by plotting the points from the table and drawing a smooth curve that passes through all the points.

- **c.** Is $y = x^2$ a linear equation? Is $y = x^2$ a function? Explain.

### Mixed Review

**Solve the equation. Check your solution.** *(Lesson 3.1)*

43. $2x + 5 = -7$
44. $5c - 8 = 27$
45. $4 - 3w = 16$
46. $\frac{n}{6} + 2 = 9$

**Find the percent of the number.** *(Lesson 7.1)*

47. 25% of 12
48. 90% of 80
49. 75% of 140
50. 38% of 500

**Identify the domain and range of the relation.** *(Lesson 8.1)*

51. $(-2, 1), (0, 2), (2, 3), (4, 4)$
52. $(5, 0), (-7, 8), (-7, 3), (5, 3)$
53. $(6, 4), (6, -2), (6, 9), (6, 1)$
54. $(1, 1), (2, 8), (3, 27), (4, 64)$

### Standardized Test Practice

**55. Multiple Choice** Which ordered pair is *not* a solution of $5x - 4y = 7$?

- A. $(-9, -13)$
- B. $(-5, -9)$
- C. $(7, 7)$
- D. $(11, 12)$

**56. Multiple Choice** The graph of which equation is shown?

- F. $x = -2$
- G. $y = -2$
- H. $x = 2$
- I. $y = 2$
8.2 Graphing Linear Equations

**Goal** Use a graphing calculator to graph linear equations.

**Example**

Use a graphing calculator to solve the following problem.

A pool charges $6 for a summer membership, plus $1.25 per visit. The equation \( C = 6 + 1.25v \) gives your cost \( C \) if you visit \( v \) times. How many times can you visit if you have $30 to spend on swimming?

1. Rewrite the equation using \( x \) and \( y \).
   \[
   C = 6 + 1.25v \quad \text{Write original equation.}
   
   y = 6 + 1.25x \quad \text{Substitute} \ x \text{ for} \ v \text{ and} \ y \text{ for} \ C.
   
2. Enter the equation.
   **Keystrokes**
   \[
   \begin{align*}
   y &= 6 + 1.25x \\
   \end{align*}
   \]

3. Press \( \text{WINDOW} \) to set the borders of the graph. Set the cursor increment to 1 unit: \( \Delta X = 1 \).
   
   Press \( \text{GRAPH} \) to graph the equation. Press \( \text{TRACE} \) and move the cursor along the graph using \( \leftarrow \) and \( \rightarrow \).

**Answer** The graph shows that you can visit 19 times for $29.75.

**Tech Help**

If the cursor moves out of view as you trace, press \( \text{ENTER} \) to redraw the screen with the cursor in the center.

**Online Resources**

• Keystroke Help

**Draw Conclusions**

Use a graphing calculator to graph the equation. Find the unknown value in the ordered pair. (Use \( \Delta X = 0.1 \).)

1. \( y = 5 - x; (1.8, \ ?) \)
2. \( y = x - 5; (\ ?, -2.2) \)
3. \( y = -2.5x + 6; (3.2, \ ?) \)
4. \( y = -0.5x + 4; (\ ?, 5.2) \)
5. **Video Games** A video game store has a $15 membership fee and rents games for $3.25 each. Use a graphing calculator to graph \( C = 15 + 3.25g \), which gives your cost \( C \) if you rent \( g \) games. How many games can you rent if you have $45 to spend?
Using Intercepts

**Before**

You graphed using tables of solutions.

**Now**

You’ll use x- and y-intercepts to graph linear equations.

**Why?**

So you can find how much food to buy for a barbecue, as in Ex. 9.

You can graph a linear equation quickly by recognizing that only two points are needed to draw a line. It is often convenient to choose points where the line crosses the axes.

The x-coordinate of a point where a graph crosses the x-axis is an **x-intercept**. The y-coordinate of a point where a graph crosses the y-axis is a **y-intercept**. The graph shown has an x-intercept of −6 and a y-intercept of 4.

### Finding Intercepts

To find the x-intercept of a line, substitute 0 for y in the line’s equation and solve for x.

To find the y-intercept of a line, substitute 0 for x in the line’s equation and solve for y.

### Example 1

**Finding Intercepts of a Graph**

Find the intercepts of the graph of $3x - 2y = 6$.

To find the x-intercept, let $y = 0$ and solve for $x$.

1. $3x - 2y = 6$ \text{ Write original equation.}
2. $3x - 2(0) = 6$ \text{ Substitute 0 for } y.
3. $3x = 6$ \text{ Simplify.}
4. $x = 2$ \text{ Divide each side by 3.}

To find the y-intercept, let $x = 0$ and solve for $y$.

1. $3x - 2y = 6$ \text{ Write original equation.}
2. $3(0) - 2y = 6$ \text{ Substitute 0 for } x.
3. $-2y = 6$ \text{ Simplify.}
4. $y = -3$ \text{ Divide each side by } -2.

**Answer** The x-intercept is 2, and the y-intercept is $-3$.

**Watch Out**

The intercepts of a graph are numbers, not ordered pairs. In Example 1, for instance, the x-intercept is 2, not $(2, 0)$. Similarly, the y-intercept is $-3$, not $(0, -3)$.
Example 2 Using Intercepts to Graph a Linear Equation

Graph the equation $3x - 2y = 6$ from Example 1.

The $x$-intercept is 2, so plot the point $(2, 0)$. The $y$-intercept is $-3$, so plot the point $(0, -3)$.

Draw a line through the two points.

✓ Checkpoint

Find the intercepts of the equation’s graph. Then graph the equation.

1. $x - 2y = -2$
2. $4x + 3y = 12$
3. $y = -2x - 8$

Example 3 Writing and Graphing an Equation

Canoeling You are canoeing along a 12 mile stretch of river. You travel 4 miles per hour when paddling and 2 miles per hour when drifting. Write and graph an equation describing your possible paddling and drifting times for the trip. Give three possible combinations of paddling and drifting times.

Solution

1. To write an equation, let $x$ be the paddling time and let $y$ be the drifting time (both in hours). First write a verbal model.

<table>
<thead>
<tr>
<th>Padding distance</th>
<th>Drifting distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Padding rate $\times$ Padding time</td>
<td>Drifting rate $\times$ Drifting time = Total distance</td>
</tr>
</tbody>
</table>

Then use the verbal model to write the equation.

$4x + 2y = 12$

2. To graph the equation, find and use the intercepts.

Find $x$-intercept: $4x + 2y = 12$

$4x + 2(0) = 12$

$4x = 12$

$x = 3$

Find $y$-intercept: $4x + 2y = 12$

$4(0) + 2y = 12$

$2y = 12$

$y = 6$

3. Three points on the graph are $(0, 6), (2, 2),$ and $(3, 0)$. So, you can either not paddle at all and drift for 6 hours, or paddle for 2 hours and drift for 2 hours, or paddle for 3 hours and not drift at all.
Guided Practice

Vocabulary Check

1. Copy and complete: For the line that passes through the points (0, −7) and (3, 0), the _ _ is −7 and the _ _ is 3.

2. Describe how you can find the x- and y-intercepts of a line by using the line’s equation.

Skill Check

Identify the x-intercept and the y-intercept of the line.

3. [Graph of a line with x-intercept at (1, 0) and y-intercept at (0, 3)]

4. [Graph of a line with x-intercept at (2, 0) and y-intercept at (0, 1)]

5. [Graph of a line with x-intercept at (−1, 0) and y-intercept at (0, 2)]

Draw the line with the given intercepts.

6. x-intercept: 4  
y-intercept: 5

7. x-intercept: −6  
y-intercept: 3

8. x-intercept: −1  
y-intercept: −2

9. Shopping  You are in charge of buying food for a barbecue. You have budgeted $30 for ground beef and chicken. Ground beef costs $3 per pound, and chicken costs $5 per pound. Write an equation describing the possible amounts of ground beef and chicken that you can buy. Use intercepts to graph the equation.

Practice and Problem Solving

Find the intercepts of the equation’s graph. Then graph the equation.

10. 5x + y = 5

11. x − 2y = 4

12. 3x − 2y = −6

13. 4x + 5y = −20

14. 4x + 3y = 24

15. 2x − 3y = −18

16. y = 2x − 4

17. y = −x + 7

18. y = 3x + 9

19. Animal Nutrition  Your beagle is allowed to eat 800 Calories of food each day. You buy canned food containing 40 Calories per ounce and dry food containing 100 Calories per ounce.

   a. Write an equation describing the possible amounts of canned and dry food that you can feed your beagle each day.

   b. Use intercepts to graph the equation from part (a).

   c. Apply  Give three possible combinations of canned and dry food that you can feed your beagle each day.
20. **Transportation** At the start of a trip, you fill up your car’s fuel tank with gas. After you drive for \( x \) hours, the amount \( y \) (in gallons) of gas remaining is given by the equation \( y = 18 - 2x \).
   
a. Find the \( x \)-intercept and the \( y \)-intercept of the given equation’s graph. Use the intercepts to graph the equation.
   
b. **Interpret** What real-life quantities do the \( x \)- and \( y \)-intercepts represent in this situation?
   
c. After how many hours of driving do you have only \( \frac{1}{4} \) tank of gas left?

**Find the intercepts of the equation’s graph. Then graph the equation.**

21. \( 1.9x - 1.9y = 3.8 \)  
22. \( 2.1x + 3.5y = 10.5 \)  
23. \( y = 1.5x + 6 \)

24. \( y = \frac{2}{7}x - 2 \)  
25. \( \frac{1}{2}x + \frac{1}{4}y = \frac{3}{2} \)  
26. \( y = \frac{7}{3}x - \frac{7}{2} \)

27. **Critical Thinking** Write an equation of a line that has no \( x \)-intercept and an equation of a line that has no \( y \)-intercept. Describe the graph of each equation.

28. **Visual Thinking** For a certain line, the \( x \)-intercept is negative and the \( y \)-intercept is positive. Does the line slant **upward** or **downward** from left to right? Sketch a graph to justify your answer.

29. **Extended Problem Solving** At a flight school, pilots-in-training can rent single-engine airplanes for $60 per hour and twin-engine airplanes for $180 per hour. The flight school’s goal is to take in $9000 in rental fees each month.
   
a. Write an equation describing the number of hours per month each type of plane should be rented if the flight school is to meet its goal.
   
b. Use intercepts to graph the equation from part (a).
   
c. **Estimation** During one month, the twin-engine planes are rented for 30 hours. Use your graph to estimate how many hours the single-engine planes must be rented if the flight school is to meet its goal.
   
d. **Reasonableness** Check your answer to part (c) by writing and solving an equation.

30. **Geometry** The rectangle shown has a perimeter of 16 inches.
   
\[
\begin{array}{c}
\text{In the Real World} \\
\text{Pilot’s License} \quad \text{To get a private pilot’s license, a pilot-in-training must have 40 hours of flight time in a single-engine plane. What is the total cost of this much flight time at the flight school in Exercise 29?}
\end{array}
\]

\[
\begin{array}{c}
\text{y} \\
\text{x}
\end{array}
\]

a. Write an equation describing the possible values of \( x \) and \( y \).
   
b. Use intercepts to graph the equation from part (a).
   
c. Give three pairs of whole-number values of \( x \) and \( y \) that could represent side lengths of the rectangle.
   
d. **Critical Thinking** Does either the \( x \)-intercept or the \( y \)-intercept represent a possible side length of the rectangle? Explain.

31. **Number Sense** Consider the equation \( 4x + 6y = c \). Find three values of \( c \) for which both the \( x \)-intercept and the \( y \)-intercept are integers. How are your values of \( c \) related to the coefficients of \( x \) and \( y \) in the given equation?
32. **Fitness** You use a combination of running and walking to complete a race \(d\) miles long. Your running speed is \(r\) miles per hour and your walking speed is \(w\) miles per hour. Let \(x\) be your running time and let \(y\) be your walking time (both in hours). Then \(rx + wy = d\).

   a. The table below shows equations of the form \(rx + wy = d\). In each column, one of the values \(r, w,\) or \(d\) increases while the other two values stay the same. Draw a coordinate plane for each column, and graph the equations in the column on that plane.

   \[
   \begin{array}{|c|c|c|}
   \hline
   \text{\(r\) increases.} & \text{\(w\) increases.} & \text{\(d\) increases.} \\
   3x + 2y = 18 & 9x + 2y = 18 & 6x + 2y = 12 \\
   6x + 2y = 18 & 9x + 3y = 18 & 6x + 2y = 18 \\
   9x + 2y = 18 & 9x + 6y = 18 & 6x + 2y = 24 \\
   \hline
   \end{array}
   \]

   b. What happens to the graph of \(rx + wy = d\) when running speed increases while walking speed and racing distance stay the same?

   c. What happens to the graph of \(rx + wy = d\) when walking speed increases while running speed and racing distance stay the same?

   d. What happens to the graph of \(rx + wy = d\) when racing distance increases while running speed and walking speed stay the same?

33. **Challenge** For the graph of \(y = ax + b\) where \(a \neq 0\), show that the \(x\)-intercept is \(-\frac{b}{a}\) and the \(y\)-intercept is \(b\). Use these results to find the intercepts of the graph of \(y = 3x + 12\).

### Mixed Review

**Evaluate the expression.** (*Lessons 1.6, 1.7*)

34. \(\frac{8 - (-1)}{4 - 1}\)  
35. \(\frac{-3 - (-5)}{6 - 8}\)  
36. \(\frac{4 - 24}{9 - 5}\)  
37. \(\frac{-7 - 11}{-12 - (-3)}\)

**Identify the percent of change as an increase or decrease.** Then find the percent of change. (*Lesson 7.5*)


**Tell whether the ordered pair is a solution of the equation.** (*Lesson 8.2*)

42. \(y = -2x + 7; (8, -9)\)  
43. \(y = 10x - 4; (0, 10)\)  
44. \(5x + y = 15; (-6, 15)\)  
45. \(3x - 8y = 12; (-4, -3)\)

### Standardized Test Practice

**46. Multiple Choice** What is the \(x\)-intercept of the graph of \(y = 4x + 32\)?

A. \(-32\)  B. \(-8\)  C. \(8\)  D. \(32\)

**47. Multiple Choice** What is the \(y\)-intercept of the graph of \(5x + 2y = 30\)?

F. \(-15\)  G. \(-6\)  H. \(6\)  I. \(15\)

**48. Short Response** A car wash charges \$8 for a basic wash and \$12 for a deluxe wash that includes a wax. On a certain day, sales at the car wash total \$960. Write and graph an equation describing the possible numbers of basic and deluxe washes that could have been done. Give three possible combinations of basic and deluxe washes.
8.4 Investigating Slope

**Goal**
Use slope to describe the steepness of a ramp.

**Materials**
- 5 books
- 2 rulers

A ramp's steepness is described by its *slope*, the ratio of the vertical rise to the horizontal run.

\[
\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{1}{4}
\]

**Investigate**

Use slope to describe the steepness of a ramp.

1. **Make** a stack of books. Use one ruler as a ramp. Using the other ruler, measure and record the rise and the run of the ramp. Calculate and record the slope of the ramp.

2. **Create** ramps with the same rise but three different runs by moving the lower end of the ruler. Measure and record the rise and the run of each ramp. Calculate and record each slope.

3. **Create** ramps with the same run but three different rises. Keep the lower end of the ruler in one spot. Add or subtract books to change the rise. Record the rise, the run, and the slope of each ramp.

**Draw Conclusions**

1. **Writing** If one ramp is steeper than a second ramp, what is true about the slopes of the two ramps?

2. **Describe** What is the relationship between the rise and the run of a ramp when the slope is 1? Explain.

3. **Critical Thinking** What happens to the slope of a ramp when the rise increases and the run stays the same?
The Slope of a Line

Before
You graphed lines in a coordinate plane.

Now
You’ll find and interpret slopes of lines.

Why?
So you can compare animal speeds, as in Ex. 17.

Wakeboarding
How steep is a wakeboard ramp like the one shown? To find out, you can calculate the ramp’s slope. The slope of a line is the ratio of the line’s vertical change, called the rise, to its horizontal change, called the run.

Example 1
Finding Slope

A wakeboard ramp has a rise of 6 feet and a run of 10 feet. Find its slope.

\[
\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{6}{10} = \frac{3}{5}
\]

Answer
The wakeboard ramp has a slope of \(\frac{3}{5}\).

To determine the slope of a line in a coordinate plane, you can find the ratio of the vertical change between two points on the line and the horizontal change between the points.

Slope of a Line

Given two points on a nonvertical line, you can find the slope \(m\) of the line using this formula:

\[
m = \frac{\text{rise}}{\text{run}} = \frac{\text{difference of } y\text{-coordinates}}{\text{difference of } x\text{-coordinates}}
\]

Example
\[
m = \frac{4 - 1}{5 - 3} = \frac{3}{2}
\]
**Comparing Slopes** You can use the diagrams below to compare the slopes of different lines. Imagine that you are walking to the right.

- **Positive slope**
  - If the line rises, the slope is positive.

- **Negative slope**
  - If the line falls, the slope is negative.

- **Zero slope**
  - If the line is horizontal, the slope is zero.

- **Undefined slope**
  - If the line is vertical, the slope is undefined.

---

**Example 2** Finding Positive and Negative Slope

Find the slope of the line shown.

a. \( m = \frac{\text{rise}}{\text{run}} = \frac{\text{difference of } y\text{-coordinates}}{\text{difference of } x\text{-coordinates}} \)
   \[ m = \frac{5 - 2}{4 - 1} = \frac{3}{3} = 1 \]
   **Answer** The slope is 1.

b. \( m = \frac{\text{rise}}{\text{run}} = \frac{\text{difference of } y\text{-coordinates}}{\text{difference of } x\text{-coordinates}} \)
   \[ m = \frac{-3 - 1}{3 - 0} = \frac{-4}{3} \]
   **Answer** The slope is \( -\frac{4}{3} \).

---

**Checkpoint**

Find the slope of the line through the given points.

1. \((1, 2), (4, 7)\)  
2. \((-2, 5), (6, 1)\)  
3. \((0, 0), (3, -9)\)  
4. \((5, 0), (7, 8)\)
Example 3  Zero and Undefined Slope

Find the slope of the line shown.

a. \( m = \frac{\text{rise}}{\text{run}} = \frac{\text{difference of } y\text{-coordinates}}{\text{difference of } x\text{-coordinates}} \)
   \[ = \frac{3 - 3}{4 - 1} \]
   \[ = \frac{0}{3} = 0 \]

Answer The slope is 0.

b. \( m = \frac{\text{rise}}{\text{run}} = \frac{\text{difference of } y\text{-coordinates}}{\text{difference of } x\text{-coordinates}} \)
   \[ = \frac{3 - (-1)}{2 - 2} \]
   \[ = \frac{4}{0} \quad \text{Division by zero is undefined.} \]

Answer The slope is undefined.

Checkpoint

Find the slope of the line through the given points. Tell whether the slope is positive, negative, zero, or undefined.

5. (2, 3), (4, 5)  6. (6, 3), (6, -1)  7. (-7, 4), (5, 4)  8. (1, 5), (4, 1)

Example 4  Interpreting Slope as a Rate of Change

The graph shows the distance traveled by a wakeboarder as a function of time. The slope of the line gives the wakeboarder’s speed, which is the rate of change in distance traveled with respect to time. Find the wakeboarder’s speed.

Solution

Use the points (2, 52) and (7, 182) to find the slope of the line.

\[ m = \frac{\text{difference of } y\text{-coordinates}}{\text{difference of } x\text{-coordinates}} \]
\[ = \frac{182 \text{ ft} - 52 \text{ ft}}{7 \text{ sec} - 2 \text{ sec}} \]
\[ = \frac{130 \text{ ft}}{5 \text{ sec}} \]
\[ = 26 \text{ ft/sec} \]

Answer The wakeboarder’s speed is 26 feet per second.
Guided Practice

Vocabulary Check
1. Copy and complete: The vertical change between two points on a line is called the _slope_, and the horizontal change is called the _x_.

2. Why is the slope of a vertical line undefined?

Skill Check
3. Error Analysis Describe and correct the error in calculating the slope of the line through the points (5, 4) and (0, 2).

Tell whether the slope of the line is positive, negative, zero, or undefined. Then find the slope.

4. 

5. 

6. 

7. Writing A wakeboard ramp has a rise of 5 feet and a run of 12 feet. Find the slope of the ramp. Compare this slope with the slope of the ramp in Example 1.

Practice and Problem Solving

Tell whether the slope of the line is positive, negative, zero, or undefined. Then find the slope.

8. 

9. 

10. 

Find the coordinates of two points on the line with the given equation. Then use the coordinates to find the slope of the line.

11. \( y = 2x + 4 \)

12. \( y = -1 \)

13. \( y = \frac{3}{2}x - 5 \)

14. \( x + 2y = 6 \)

15. \( 4x - 3y = 12 \)

16. \( x = 3 \)
17. **Extended Problem Solving**  The graph shows the distance run by a cheetah as a function of time.

a. Find the slope of the line.

b. **Interpret**  What information about the cheetah can you obtain from the slope?

c. **Compare and Contrast**  A gazelle’s top speed is about 22 meters per second. Suppose you made a graph showing the distance run by a gazelle as a function of time. How would the graph for the gazelle compare with the graph for the cheetah? Explain your thinking.

**Sketch an example of the type of line described.**

18. A line with zero slope  
19. A line with undefined slope  
20. A line with positive slope  
21. A line with negative slope

**Find the slope of the line through the given points.**

22. (3, 3), (5, 7)  
23. (6, 1), (4, 3)  
24. (7, 3), (7, 2)  
25. (−3, −5), (6, −11)  
26. (4, 1), (12, 8)  
27. (5, −7), (0, −7)  
28. (−1, 0), (0, −5)  
29. (3, −2), (−8, −2)  
30. (−2, −6), (−2, 6)  
31. (−8, −8), (−2, −6)  
32. (65, 87), (82, 16)  
33. (−10, 10), (−10, 0)

34. **Writing**  Describe the difference between a line with zero slope and a line with undefined slope.

35. **Wheelchair Ramp**  You are building a wheelchair ramp that leads to a doorway 22 inches above the ground. The slope of the ramp must be $\frac{1}{12}$. Find the length of ground (in feet) that the ramp covers.

36. **Cinder Cones**  A cinder cone is a type of volcano. To describe the steepness of a cinder cone from one point on the cone to another, you can find the gradient between the two points.

   \[ \text{Gradient} = \frac{\text{Change in elevation (in feet)}}{\text{Horizontal change (in miles)}} \]

   The graph shows a cross section of a cinder cone. Use the information in the graph to find the gradient between the given points on the cinder cone. Include units in your answers.

a. $A$ and $B$  
b. $B$ and $C$  
c. $A$ and $C$
37. **Roads** The grade of a road is its slope written as a percent. A warning sign must be posted if a section of road has a grade of at least 8% and is more than 750 feet long.
   
   a. **Interpret and Apply** A road rises 63 feet over a horizontal distance of 840 feet. Should a warning sign be posted? Explain your thinking.
   
   b. **Critical Thinking** The grade of a section of road that stretches over a horizontal distance of 1000 feet is 9%. How many feet does the road rise over that distance?

38. **Horseback Riding** A riding instructor takes students on mountain trails. The instructor wants to avoid steep trails. On the steepest part of trail A, the path rises 15 feet over a horizontal distance of 50 feet. On the steepest part of trail B, the path rises 30 feet over a horizontal distance of 75 feet. Which trail should the instructor take? Explain.

39. **Logical Reasoning** Choose three different pairs of points on the given line, and find the slope of the line using each pair. What conclusion can you draw from your results?

40. **Challenge** Without graphing, choose a point \( P \) so that the slope of the line through \((-1, 1)\) and \( P \) is \( \frac{1}{9} \).

---

**Mixed Review**

**Solve the equation. Check your solution.** *(Lessons 2.5, 2.6)*

41. \( x + 7 = -5 \)  
42. \( x - 3 = 21 \)  
43. \(-3y = 33\)  
44. \( \frac{m}{5} = 10 \)

**Find the greatest common factor of the numbers.** *(Lesson 4.2)*

45. 15, 48  
46. 64, 56  
47. 105, 125  
48. 121, 132

**Find the intercepts of the equation’s graph. Then graph the equation.** *(Lesson 8.3)*

49. \( 2x - y = 2 \)  
50. \( 9x + 2y = 18 \)  
51. \( 3x + 4y = -24 \)

---

**Standardized Test Practice**

52. **Multiple Choice** What is the slope of the line that passes through the points \((-1, -14)\) and \((5, 4)\)?

   A. \(-3\)  
   B. \(-\frac{1}{3}\)  
   C. \(\frac{1}{3}\)  
   D. 3

53. **Multiple Choice** The slope of a line through the point \((0, 0)\) is 2. Which point is also on the line?

   F. \((-4, 2)\)  
   G. \((2, 4)\)  
   H. \((-2, 4)\)  
   I. \((2, -4)\)
Parallel, Perpendicular, and Skew Lines

**Parallel Lines**
Two lines are **parallel lines** if they lie in the same plane and do not intersect. The symbol \( \parallel \) is used to state that two lines are parallel. Triangles (\( \triangle \)) are used in a diagram to indicate that lines are parallel. In the diagram below, lines \( t \) and \( v \) are parallel.

**Example** Name one pair of parallel lines that lie in plane \( P \).
Because lines \( a \) and \( c \) are marked as being parallel, you know that \( a \parallel c \).

**Perpendicular Lines**
Two lines are **perpendicular lines** if they intersect to form a right angle. The symbol \( \perp \) is used to state that two lines are perpendicular. In the diagram, lines \( m \) and \( n \) are perpendicular.

**Example** Name two lines that are perpendicular to line \( f \).
Because lines \( g \) and \( j \) intersect line \( f \) at right angles, you know that \( g \perp f \) and \( j \perp f \).
**Skew Lines**

Two lines are **skew lines** if they do not lie in the same plane and do not intersect. In the diagram, lines \( r \) and \( s \) are skew lines.

**Example** Name two lines that are skew.

Lines \( u \) and \( w \) are skew. Note that lines \( u \) and \( v \) are not skew because they intersect.

**Checkpoint**

Tell whether the lines are **parallel or perpendicular**.

1. Lines \( a \) and \( b \)
2. Lines \( a \) and \( c \)
3. Lines \( d \) and \( b \)
4. Lines \( c \) and \( d \)

Tell whether the lines are skew. Explain.

5. Lines \( k \) and \( m \)
6. Lines \( k \) and \( j \)
7. Lines \( j \) and \( m \)

In Exercises 8–10, use the radio shown. The radio has the shape of a box with rectangular sides. Consider the antenna and each edge of the radio as part of a line.

8. Name three lines perpendicular to \( \overrightarrow{GE} \).
9. Name two lines parallel to \( \overrightarrow{AC} \).
10. Name two lines that are skew to \( \overrightarrow{CD} \).
Slope-Intercept Form

**Vocabulary**
slope-intercept form, p. 412

**BEFORE**
You used intercepts to graph linear equations.

**Now**
You’ll graph linear equations in slope-intercept form.

**WHY?**
So you can find how long it will take to knit a scarf, as in Ex. 9.

The graph of $y = 2x + 3$ is shown. You can see that the line’s $y$-intercept is 3, and the line’s slope $m$ is 2:

$$m = \frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$$

Notice that the slope is equal to the coefficient of $x$ in the equation $y = 2x + 3$. Also notice that the $y$-intercept is equal to the constant term in the equation. These results are always true for an equation written in **slope-intercept form**.

### Slope-Intercept Form

**Words** A linear equation of the form $y = mx + b$ is said to be in **slope-intercept form**. The slope is $m$ and the $y$-intercept is $b$.

**Algebra**

$$y = mx + b$$

**Numbers**

$$y = 2x + 3$$

---

**Example 1** **Identifying the Slope and $y$-Intercept**

Identify the slope and $y$-intercept of the line with the given equation.

**a.** $y = x - 4$

**b.** $3x + 5y = 10$

**Solution**

**a.** Write the equation $y = x - 4$ as $y = 1x + (−4)$.

**Answer** The line has a slope of 1 and a $y$-intercept of $−4$.

**b.** Write the equation $3x + 5y = 10$ in slope-intercept form by solving for $y$.

$$3x + 5y = 10 \quad \text{Write original equation.}$$

$$5y = −3x + 10 \quad \text{Subtract 3x from each side.}$$

$$y = −\frac{3}{5}x + 2 \quad \text{Multiply each side by } \frac{1}{5}.$$

**Answer** The line has a slope of $−\frac{3}{5}$ and a $y$-intercept of 2.

---

**Reading Algebra**

Recall that you wrote linear equations in function form in Lesson 8.2. In part (b) of Example 1, notice that writing $3x + 5y = 10$ in slope-intercept form is equivalent to writing the equation in function form.
Study Strategy

Reasonableness To check the line drawn in Example 2, substitute the coordinates of the second plotted point, (3, 2), into the equation \( y = -\frac{2}{3}x + 4 \). You should get a true statement.

Example 2

**Graphing an Equation in Slope-Intercept Form**

Graph the equation \( y = -\frac{2}{3}x + 4 \).

1. The \( y \)-intercept is 4, so plot the point (0, 4).
2. The slope is \( -\frac{2}{3} = -\frac{2}{3} \).
   Starting at (0, 4), plot another point by moving right 3 units and down 2 units.
3. Draw a line through the two points.

**Checkpoint**

Identify the slope and \( y \)-intercept of the line with the given equation. Use the slope and \( y \)-intercept to graph the equation.

1. \( y = -x + 1 \)
2. \( 3x - 2y = 6 \)
3. \( y = 4x \)

Real-Life Situations In a real-life problem involving a linear equation, the \( y \)-intercept is often an initial value, and the slope is a rate of change.

Example 3

**Using Slope and \( y \)-Intercept in Real Life**

Earth Science The temperature at Earth’s surface averages about 20°C. In the crust below the surface, the temperature rises by about 25°C per kilometer of depth.

a. Write an equation that approximates the temperature below Earth’s surface as a function of depth.

b. Underground bacteria exist that can survive temperatures of up to 110°C. Find the maximum depth at which these bacteria can live.

Solution

a. Let \( x \) be the depth (in kilometers) below Earth’s surface, and let \( y \) be the temperature (in degrees Celsius) at that depth. Write a verbal model. Then use the verbal model to write an equation.

\[
\text{Temperature below surface} = \text{Temperature at surface} + \text{Rate of change in temperature} \times \text{Depth below surface}
\]

\[
y = 20 + 25x
\]

b. Graph \( y = 20 + 25x \) on a graphing calculator. Trace along the graph until the cursor is on a point where \( y = 110 \). For this point, \( x \approx 3.6 \). So, the maximum depth at which the bacteria can live is about 3.6 kilometers.

Scientists use airtight enclosures like the one shown to preserve bacteria found in Earth’s crust.
Parallel and Perpendicular Lines There is an important relationship between the slopes of two nonvertical lines that are parallel or perpendicular.

Slopes of Parallel and Perpendicular Lines

Two nonvertical parallel lines have the same slope. For example, the parallel lines \( a \) and \( b \) below both have a slope of \( 2 \).

Two nonvertical perpendicular lines, such as lines \( a \) and \( c \) below, have slopes that are negative reciprocals of each other.

Example 4 Finding Slopes of Parallel and Perpendicular Lines

Find the slope of a line that has the given relationship to the line with equation \( 4x + 3y = -18 \).

a. Parallel to the line

b. Perpendicular to the line

Solution

a. First write the given equation in slope-intercept form.

\[
4x + 3y = -18 \quad \text{Write original equation.}
\]

\[
3y = -4x - 18 \quad \text{Subtract } 4x \text{ from each side.}
\]

\[
y = -\frac{4}{3}x - 6 \quad \text{Multiply each side by } \frac{4}{3}.
\]

The slope of the given line is \(-\frac{4}{3}\). Because parallel lines have the same slope, the slope of a parallel line is also \(-\frac{4}{3}\).

b. From part (a), the slope of the given line is \(-\frac{4}{3}\). The slope of a perpendicular line is the negative reciprocal of \(-\frac{4}{3}\), or \(\frac{3}{4}\).

Checkpoint

For the line with the given equation, find the slope of a parallel line and the slope of a perpendicular line.

4. \( y = -3x \)  
5. \( y = 4x + 10 \)  
6. \( 2x - 5y = 15 \)
Guided Practice

Vocabulary Check
1. Copy and complete: An equation of the form $y = mx + b$ is written in ___ form.

2. Without graphing, tell whether the lines with equations $y = 7x - 1$ and $y = 7x + 3$ are parallel, perpendicular, or neither. Explain.

Skill Check
Identify the slope and $y$-intercept of the line with the given equation. Use the slope and $y$-intercept to graph the equation.

3. $y = 2x$
4. $y = -3x + 4$
5. $x - 2y = 2$

For the line with the given equation, find the slope of a parallel line and the slope of a perpendicular line.

6. $y = x$
7. $y = -6x + 9$
8. $3x + 2y = 16$

Guided Problem Solving
9. **Knitting** You and a friend are knitting a scarf that will be 72 inches long. Your friend knits the first 24 inches and then gives you the scarf to finish. You expect to knit at a rate of 8 inches per day. After how many days will you finish the scarf?

   1. Use the verbal model to write an equation giving the length $y$ of the scarf (in inches) after you have been knitting for $x$ days.

   \[
   \text{Length of scarf} = \text{Length knitted by your friend} + \text{Knitting rate} \times \text{Knitting time}
   \]

   2. Identify the slope and $y$-intercept of the line with the equation from Step 1. Then graph the equation.

   3. Use the graph to estimate how long you will take to finish the scarf.

Practice and Problem Solving

Match the equation with its graph.

10. $y = x + 2$
11. $y = -x + 2$
12. $y = x - 2$

13. **Critical Thinking** Give the equations of three lines that are parallel to the line with equation $y = 3x + 2$. 
Identify the slope and y-intercept of the line with the given equation. Use the slope and y-intercept to graph the equation.

14. \( y = -2x + 3 \)  
15. \( y = \frac{1}{4}x + 1 \)  
16. \( y = -2 \)
17. \( 3x + y = -1 \)  
18. \( 2x - 3y = 0 \)  
19. \( 5x - 2y = -4 \)

20. **Robotics** In 2002, a robot explored a tunnel 210 feet long inside the Great Pyramid in Egypt. The robot could travel about 10 feet per minute. Write and graph an equation giving the distance \( y \) (in feet) that the robot could travel in \( x \) minutes. Use the graph to estimate how quickly the robot could reach the end of the tunnel.

21. **Paramotor** A paramotor is a parachute propelled by a fan-like motor. Suppose that \( x \) minutes after beginning a descent, a paramotorist has an altitude \( y \) (in feet) given by \( y = 2000 - 250x \).

   a. Graph the given equation on a graphing calculator. Use the trace feature to find how long it takes the paramotorist to reach the ground.

   b. **Interpret** Identify the slope and y-intercept of the graph. What real-life quantities do the slope and y-intercept represent?

For the line with the given equation, find the slope of a parallel line and the slope of a perpendicular line.

22. \( y = 8x + 5 \)  
23. \( y = -x - 9 \)  
24. \( y = -7x + 4 \)
25. \( 4x - 5y = 30 \)  
26. \( 11x + 6y = 18 \)  
27. \( x = 3y - 7 \)

Find the slope of a line parallel to the given line and the slope of a line perpendicular to the given line.

28. 

29. 

30. 

31. **Extended Problem Solving** Two farmers each harvest 50 acres of corn per day from their fields. The area of one farmer’s field is 1000 acres, and the area of the other farmer’s field is 600 acres.

   a. Write an equation giving the unharvested area \( y \) of the larger field (in acres) after \( x \) days.

   b. Write an equation giving the unharvested area \( y \) of the smaller field (in acres) after \( x \) days.

   c. Graph the equations from parts (a) and (b) in the same coordinate plane. Identify the slope and y-intercept of each graph.

   d. **Compare** What is the geometric relationship between the graphs from part (c)? How do you know?

   e. **Interpret and Apply** How long does it take to harvest the corn in the larger field? in the smaller field?
32. **Walk-a-thon** You are participating in a walk-a-thon. Donors can pledge a certain amount of money for each mile that you walk, or a fixed amount that doesn’t depend on how far you walk, or both. The table gives the amounts pledged by four donors on your street.

<table>
<thead>
<tr>
<th>Donor</th>
<th>Amount per mile</th>
<th>Fixed amount</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Janette</td>
<td>None</td>
<td>$35</td>
<td>?</td>
</tr>
<tr>
<td>Ben</td>
<td>$2</td>
<td>$20</td>
<td>?</td>
</tr>
<tr>
<td>Sall</td>
<td>$5</td>
<td>None</td>
<td>?</td>
</tr>
<tr>
<td>Mary</td>
<td>$3</td>
<td>$15</td>
<td>?</td>
</tr>
</tbody>
</table>

**a.** Copy the table. For each person, write an equation giving the amount of money $y$ the person will donate if you walk $x$ miles.

**b.** Write an equation giving the total amount of money $y$ you will raise from the donors on your street if you walk $x$ miles.

**c.** **Writing** Consider the equations from part (a) and the equation from part (b). Which equation has the graph with the greatest slope? Explain why this is so.

33. **Challenge** Complete the following steps to show that the slope of the line $y = mx + b$ is $m$.

**a.** Show that two points on the graph of $y = mx + b$ are $(0, b)$ and $(1, m + b)$. (*Hint: Find $y$ when $x = 0$ and when $x = 1$.*)

**b.** For the points $(0, b)$ and $(1, m + b)$, what is the difference of the second $y$-coordinate and the first $y$-coordinate? What is the difference of the second $x$-coordinate and the first $x$-coordinate?

**c.** Use your results from part (b) to write an expression for the slope of the line $y = mx + b$. Show that the slope is equal to $m$.

---

**Mixed Review**

Solve the equation. Check your solution. *(Lesson 3.2)*

34. $2(x - 4) = 16$  
36. $-6 + 5a + 13 = -8$  
35. $-20 = 4(7 - 3x)$  
37. $14c + 33 - 10c = 5$

Use the percent equation to answer the question. *(Lesson 7.4)*

38. What number is 20% of 50?  
39. What number is 125% of 80?  
40. 45 is 75% of what number?  
41. What percent of 140 is 56?

Find the slope of the line through the given points. *(Lesson 8.4)*

42. $(0, 0), (2, 8)$  
43. $(1, 5), (4, -1)$  
44. $(2, 6), (5, 4)$  
45. $(-3, 7), (1, 17)$

**Standardized Test Practice**

46. **Multiple Choice** Which equation’s graph has the greatest slope?
   
   A. $y = 3x$  
   B. $y = x + 12$  
   C. $y = 5x - 1$  
   D. $y = 8x + 4$

47. **Short Response** You buy a prepaid phone card that has 500 minutes of calling time. You use about 25 minutes of calling time per week. Write and graph an equation that approximates your remaining calling time $y$ (in minutes) after $x$ weeks.
Mid-Chapter Quiz

Represent the relation as a graph and as a mapping diagram. Then tell whether the relation is a function. Explain your reasoning.
1. (2, 1), (2, 2), (2, 3), (2, 4)  
2. (8, −1), (6, 0), (4, 0), (2, −1)

Graph the equation. Tell whether the equation is a function.
3. \( y = -x + 7 \)  
4. \( x = 5 \)  
5. \( y = -1 \)  
6. \( x + 4y = 32 \)

Find the intercepts of the equation’s graph. Then graph the equation.
7. \( 6x + 3y = 12 \)  
8. \( 4x - y = 8 \)  
9. \( y = 2x - 6 \)  
10. \( -5x + 2y = 10 \)

Find the slope of the line through the given points.
11. (1, 2), (2, 8)  
12. (0, 4), (4, 4)  
13. (−6, 10), (1, 2)  
14. (−1, 2), (−1, 6)

15. **Drama Club** The drama club pays a registration fee of $50 to take part in a festival of one-act plays and $40 for each play the club enters. Write and graph an equation giving the total cost \( y \) (in dollars) of entering \( x \) plays.

---

**Quarter Count**

The U.S. Mint began issuing special state quarters in 1999. Using a coordinate plane, follow the steps below to find out how many states had quarters issued each year. For each step after the first, start at the point in the plane where you ended in the previous step. All segments you draw should be 4 units long.

1. Start at (2, 4) and draw a segment that has a slope of 0 and an endpoint in Quadrant II.
2. Draw a segment on the line \( x = -2 \) with an endpoint on the \( x \)-axis.
3. Draw a segment on the line \( y = 0 \) that has a positive \( x \)-coordinate.
4. Draw a segment that has an undefined slope and an endpoint in Quadrant IV.
5. Draw a segment on the line \( y = -4 \) that has an endpoint in Quadrant III.
Bamboo  Bamboo is one of the fastest-growing plants on Earth. It can grow up to 4 feet in one day! In Example 4, you’ll see how to write a linear equation that describes the growth of a bamboo plant.

You can write a linear equation in slope-intercept form, \( y = mx + b \), if you know the slope \( m \) and the \( y \)-intercept \( b \) of the equation’s graph.

Example 1  Writing an Equation Given the Slope and \( y \)-Intercept

Write an equation of the line with a slope of 3 and a \( y \)-intercept of \(-7\).

\[
\begin{align*}
y &= mx + b \\
y &= 3x + (-7) \\
y &= 3x - 7
\end{align*}
\]

Write general slope-intercept equation.
Substitute 3 for \( m \) and \(-7 \) for \( b \).
Simplify.

Example 2  Writing an Equation of a Graph

Write an equation of the line shown.

1. Find the slope \( m \) using the labeled points.

\[
m = \frac{-3}{4 - 0} = \frac{-3}{4} = -\frac{3}{4}
\]

2. Find the \( y \)-intercept \( b \). The line crosses the \( y \)-axis at \((0, 3)\), so \( b = 3 \).

3. Write an equation of the form \( y = mx + b \).

\[
y = -\frac{3}{4}x + 3
\]

Checkpoint

1. Write an equation of the line with a slope of 1 and a \( y \)-intercept of 5.
2. Write an equation of the line through the points \((-2, 6)\) and \((0, -4)\).
Example 3  Writing Equations of Parallel or Perpendicular Lines

a. Write an equation of the line that is parallel to the line $y = 4x - 8$ and passes through the point $(0, 2)$.

b. Write an equation of the line that is perpendicular to the line $y = -5x + 1$ and passes through the point $(0, -9)$.

Solution

a. The slope of the given line is 4, so the slope of the parallel line is also 4. The parallel line passes through $(0, 2)$, so its $y$-intercept is 2.

Answer An equation of the line is $y = 4x + 2$.

b. Because the slope of the given line is $-5$, the slope of the perpendicular line is the negative reciprocal of $-5$, or $\frac{1}{5}$. The perpendicular line passes through $(0, -9)$, so its $y$-intercept is $-9$.

Answer An equation of the line is $y = \frac{1}{5}x + (-9)$, or $y = \frac{1}{5}x - 9$.

Example 4  Writing an Equation from a Table

The table shows a bamboo plant’s growth over 8 hours. Show that the table represents a linear function. Write an equation for the function.

<table>
<thead>
<tr>
<th>Time (h), x</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (in.), y</td>
<td>6</td>
<td>10</td>
<td>14</td>
<td>18</td>
<td>22</td>
</tr>
</tbody>
</table>

Solution

1) Make a scatter plot. The points lie on a nonvertical line, so the table represents a linear function.

2) Find the slope $m$ using any two points on the line, such as $(0, 6)$ and $(2, 10)$.

$$m = \frac{10 - 6}{2 - 0} = \frac{4}{2} = 2$$

3) Find the $y$-intercept $b$. The line intersects the $y$-axis at $(0, 6)$, so $b = 6$.

4) Write the equation $y = mx + b$.

$y = 2x + 6$

In the Real World

Bamboo  Bamboo is a rapidly renewable building material compared to trees such as oak. Bamboo takes about 5 years to grow, while oak takes about 120 years. How many bamboo forests can be grown and harvested in the time it takes to grow one oak forest?

Checkpoint

3. Which representation of a function more clearly shows whether or not the function is linear: a table of values or a graph? Explain.
**Best-Fitting Lines** In Example 4, the points in the scatter plot lie exactly on a line. Often, however, there is no single line that passes through all the points in a data set. In such cases, you can find the **best-fitting line**, which is the line that lies as close as possible to the data points.

The following example uses a graphical method to approximate the equation of a best-fitting line. In the activity on page 425, you’ll use a graphing calculator to find a better approximation of this line.

**Example 5**  
**Approximating a Best-Fitting Line**

**Medicine** The table shows the number of female physicians in the United States for the years 1992–1999.

<table>
<thead>
<tr>
<th>Years since 1992, ( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female physicians (in thousands), ( y )</td>
<td>110</td>
<td>117</td>
<td>125</td>
<td>140</td>
<td>148</td>
<td>158</td>
<td>168</td>
<td>177</td>
</tr>
</tbody>
</table>

a. Approximate the equation of the best-fitting line for the data.

b. Predict the number of female physicians in 2005.

**Solution**

a. First, make a scatter plot of the data pairs.

Next, draw the line that appears to best fit the data points. There should be about the same number of points above the line as below it. The line does not have to pass through any of the data points.

Finally, write an equation of the line. To find the slope, estimate the coordinates of two points on the line, such as (0, 108) and (7, 177).

\[
m = \frac{177 - 108}{7 - 0} = \frac{69}{7} \approx 9.86
\]

The line intersects the \( y \)-axis at (0, 108), so the \( y \)-intercept is 108.

**Answer** An approximate equation of the best-fitting line is \( y = 9.86x + 108 \).

b. Note that \( 2005 - 1992 = 13 \), so 2005 is 13 years after 1992. Calculate \( y \) when \( x = 13 \) using the equation from part (a).

\[
\begin{align*}
y &= 9.86x + 108 \\
y &= 9.86(13) + 108 \\
y &= 236
\end{align*}
\]

**Answer** In 2005, there will be about 236,000 female physicians in the United States.

---

**Watch Out**

In the table for Example 5, each year’s number \( y \) of female physicians is given in thousands. So in part (b), a \( y \)-value of 236 means that the number of female physicians in 2005 will be about 236,000, not 236.
Guided Practice

Vocabulary Check
1. Copy and complete: The line that lies as close as possible to the data points in a scatter plot is called the ___.

2. Describe the steps you would use to write an equation of the line through the points (−2, 3) and (0, 9).

Skill Check
Write an equation of the line through the given points.
3. (0, 8), (1, 9) 4. (−2, 13), (0, 1) 5. (0, −5), (3, −3)

6. Write an equation of the line that is perpendicular to the line $y = 2x − 11$ and passes through the point (0, −7).

Guided Problem Solving
7. Clams The table shows the dimensions of seven butter clams. What is the approximate length of a butter clam that is 85 millimeters wide?

<table>
<thead>
<tr>
<th>Width (mm), x</th>
<th>13</th>
<th>21</th>
<th>30</th>
<th>39</th>
<th>50</th>
<th>60</th>
<th>71</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (mm), y</td>
<td>17</td>
<td>28</td>
<td>40</td>
<td>52</td>
<td>62</td>
<td>77</td>
<td>91</td>
</tr>
</tbody>
</table>

1. Make a scatter plot of the data pairs. Draw the line that appears to best fit the data points.
2. Write an equation of your line.
3. Use your equation to predict, to the nearest millimeter, the length of a butter clam that is 85 millimeters wide.

Practice and Problem Solving

Write an equation of the line with the given slope and $y$-intercept.
8. slope = −3; $y$-intercept = 5 9. slope = 4; $y$-intercept = 10
10. slope = 13; $y$-intercept = −8 11. slope = −1; $y$-intercept = −20

Write an equation of the line.
12. 

13. 

14. 

Write an equation of the line through the given points.
15. (0, 9), (3, 15) 16. (0, −6), (8, −16) 17. (−2, −11), (0, −11)
Write an equation of the line that is parallel to the given line and passes through the given point.

18. \( y = 2x + 1; \ (0, \ 4) \)  
19. \( y = -x - 3; \ (0, \ 7) \)  
20. \( y = -8x + 9; \ (0, \ -2) \)

Write an equation of the line that is perpendicular to the given line and passes through the given point.

21. \( y = 3x + 4; \ (0, \ 6) \)  
22. \( y = \ x - 7; \ (0, \ -5) \)  
23. \( y = -\frac{1}{4}x + 3; \ (0, \ 1) \)

Show that the table represents a linear function. Then write an equation for the function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-5</td>
<td>-2</td>
<td>1</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

26. **Extended Problem Solving** Since 1912, scientists have created five maps of the glaciers on top of Mount Kilimanjaro in Africa. The maps indicate that the glaciers are shrinking, as shown by the table.

<table>
<thead>
<tr>
<th>Map number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year map was made</td>
<td>1912</td>
<td>1953</td>
<td>1976</td>
<td>1989</td>
<td>2000</td>
</tr>
<tr>
<td>Area of glaciers (km²)</td>
<td>12.1</td>
<td>6.7</td>
<td>4.2</td>
<td>3.3</td>
<td>2.2</td>
</tr>
</tbody>
</table>

a. **Graph** Let \( x \) be the number of years since 1912. Let \( y \) be the area of the glaciers (in square kilometers). Make a scatter plot of the data pairs \((x, y)\). Draw the line that appears to best fit the data points.

b. **Represent** Write an equation of your line.

c. **Predict** Estimate the year when the glaciers will disappear.

Two variables \( x \) and \( y \) show **direct variation** if \( y = kx \) for some nonzero number \( k \). In Exercises 27–30, write a direct variation equation that has the given ordered pair as a solution.

**Example** **Writing a Direct Variation Equation**

Write a direct variation equation that has \((4, \ 20)\) as a solution.

\[
y = kx \quad \text{Write general equation for direct variation.}
\]

\[
20 = k(4) \quad \text{Substitute 4 for} \ x \text{ and 20 for} \ y.
\]

\[
5 = k \quad \text{Divide each side by 4.}
\]

**Answer** A direct variation equation is \( y = 5x \).

27. \((5, \ 15)\)  
28. \((-3, \ 21)\)  
29. \((-8, \ -4)\)  
30. \((12, \ -16)\)

31. **Sales** Lisa and John work in different department stores. Lisa earns a salary of $18,000 per year plus a 2% commission on her sales. John receives no salary but earns a 6% commission on his sales. For each person, tell whether annual sales and annual earnings show direct variation. Justify your answers mathematically.
32. **Physics** The table below gives the length of a spring when different masses are suspended from it.

<table>
<thead>
<tr>
<th>Mass (g),</th>
<th>0</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (mm), y</td>
<td>80</td>
<td>110</td>
<td>140</td>
<td>170</td>
<td>200</td>
</tr>
</tbody>
</table>

a. Show that the table represents a linear function.

b. Write an equation for the function.

33. **Marathons** The table below shows the men’s winning times in the Boston Marathon for every tenth year from 1900 to 2000. In the table, \( x \) represents the number of years since 1900, and \( y \) represents the corresponding winning time (to the nearest minute).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>160</td>
<td>149</td>
<td>150</td>
<td>155</td>
<td>148</td>
<td>153</td>
<td>141</td>
<td>131</td>
<td>132</td>
<td>128</td>
<td>130</td>
</tr>
</tbody>
</table>

a. Make a scatter plot of the data pairs \((x, y)\). Draw the line that appears to best fit the data points.

b. Write an equation of your line.

c. **Predict** Use your equation to predict, to the nearest minute, the men’s winning time in the Boston Marathon for the year 2010.

d. **Writing** Do you think your equation will accurately predict winning times far into the future? Explain your reasoning.

34. **Challenge** Write an equation of the line through \((2, -1)\) and \((6, 5)\). Describe the method you used to determine the equation.

---

**Mixed Review**

Solve the equation. Check your solution. *(Lesson 3.3)*

35. \(8x - 5 = 5x + 7\)
36. \(-7y + 4 = -y + 22\)
37. \(4(m - 4) = 2m\)
38. \(6(1 - n) = -6n + 1\)

Write the fraction as a percent. *(Lesson 7.3)*

39. \(\frac{7}{10}\)
40. \(\frac{3}{8}\)
41. \(\frac{5}{2}\)
42. \(\frac{9}{5}\)

Identify the slope and \(y\)-intercept of the line with the given equation. Use the slope and \(y\)-intercept to graph the equation. *(Lesson 8.5)*

43. \(y = 3x - 2\)
44. \(y = -x + 5\)
45. \(3x + 2y = 0\)
46. \(x - 2y = -2\)

---

**Standardized Test Practice**

47. **Multiple Choice** What is an equation of the line through the points \((0, 8)\) and \((2, 0)\)?

   A. \(y = 4x + 2\)
   B. \(y = 4x + 8\)
   C. \(y = -4x + 2\)
   D. \(y = -4x + 8\)

48. **Multiple Choice** What is an equation of the line that is parallel to the line \(y = 5x + 3\) and passes through the point \((0, -1)\)?

   F. \(y = 5x - 1\)
   G. \(y = -5x - 1\)
   H. \(y = \frac{1}{5}x - 1\)
   I. \(y = -\frac{1}{5}x - 1\)

---

**424 Chapter 8 Linear Functions**
8.6 Finding Best-Fitting Lines

**Goal** Use a graphing calculator to find the best-fitting line for a scatter plot.

**Example**

Use a graphing calculator to make a scatter plot of the female physician data on page 421 and find the best-fitting line.

1. Press **LIST** and enter the data into two lists, L1 and L2. Use the arrow keys to navigate.

2. Press **2nd [PLOT]**, select Plot1, turn it from Off to On, and select the scatter plot icon. Then press **GRAPH**.

3. Press **2nd [STAT]**, select CALC, and select LinReg(ax+b). Then press **ENTER**.

4. The best-fitting line has slope \(a\) and y-intercept \(b\). Enter the equation of the line and graph it.

**Tech Help**

Press **ZOOM** and select ZoomStat to set a viewing window that will show all points in the scatter plot.

Press **TRACE** to move among the data points in the scatter plot or along the best-fitting line.

**Draw Conclusions**

1. **Predict** Use the best-fitting line from the example above to predict the number of female physicians in 2005.

2. **Dentistry** The table shows the average amount each person in the U.S. spent on dental services for the years 1992–1999. Use a graphing calculator to find the best-fitting line for the data.

<table>
<thead>
<tr>
<th>Years since 1992, (x)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount (dollars), (y)</td>
<td>140</td>
<td>148</td>
<td>156</td>
<td>166</td>
<td>173</td>
<td>184</td>
<td>193</td>
<td>202</td>
</tr>
</tbody>
</table>
Function Notation

**Science** In 2001, a scientific balloon like the one shown was launched near McMurdo Station in Antarctica. In Example 4, you’ll see how *function notation* can be used to describe the balloon’s altitude as a function of time.

When you use an equation to represent a function, it is often convenient to give the function a name, such as $f$ or $g$. For instance, the function $y = x + 2$ can be written in *function notation* as follows:

$$f(x) = x + 2$$

The symbol $f(x)$, which replaces $y$, is read “$f$ of $x$” and represents the value of the function $f$ at $x$. For instance, $f(3)$ is the value of $f$ when $x = 3$.

**Example 1** Working with Function Notation

Let $f(x) = -3x + 8$. Find $f(x)$ when $x = 5$, and find $x$ when $f(x) = -22$.

a. $f(x) = -3x + 8$ \hspace{1cm} Write function.

$\quad f(5) = -3(5) + 8$ \hspace{1cm} Substitute 5 for $x$.

$\qquad = -7$ \hspace{1cm} Simplify.

**Answer** When $x = 5$, $f(x) = -7$.

b. $f(x) = -3x + 8$ \hspace{1cm} Write function.

$\quad -22 = -3x + 8$ \hspace{1cm} Substitute $-22$ for $f(x)$.

$\quad -30 = -3x$ \hspace{1cm} Subtract 8 from each side.

$\quad 10 = x$ \hspace{1cm} Divide each side by $-3$.

**Answer** When $f(x) = -22$, $x = 10$.

**Checkpoint**

Let $g(x) = 4x - 5$. Find the indicated value.

1. $g(x)$ when $x = 2$ \hspace{1cm} 2. $g(-10)$ \hspace{1cm} 3. $x$ when $g(x) = 19$
**Graphing Functions** To graph a function written in function notation, you may find it helpful to first rewrite the function in terms of \( x \) and \( y \).

**Example 2**  
*Graphing a Function*

Graph the function \( f(x) = \frac{3}{4}x + 1 \).

1. Rewrite the function as \( y = \frac{3}{4}x + 1 \).
2. The \( y \)-intercept is 1, so plot the point \((0, 1)\).
3. The slope is \( \frac{3}{4} \). Starting at \((0, 1)\), plot another point by moving right 4 units and up 3 units.
4. Draw a line through the two points.

**Checkpoint**

Graph the function.

4. \( f(x) = 2x - 4 \)
5. \( g(x) = -\frac{3}{2}x + 3 \)
6. \( h(x) = -1 \)

If \( f(c) = d \) for a function \( f \), then you can conclude that the graph of \( f \) passes through the point \((c, d)\).

**Example 3**  
*Writing a Function*

Write a linear function \( g \) given that \( g(0) = 9 \) and \( g(3) = -6 \).

1. Find the slope \( m \) of the function’s graph. From the values of \( g(0) \) and \( g(3) \), you know that the graph of \( g \) passes through the points \((0, 9)\) and \((3, -6)\). Use these points to calculate the slope.
   \[ m = \frac{-6 - 9}{3 - 0} = \frac{-15}{3} = -5 \]
2. Find the \( y \)-intercept \( b \) of the function’s graph. The graph passes through \((0, 9)\), so \( b = 9 \).
3. Write an equation of the form \( g(x) = mx + b \).
   \[ g(x) = -5x + 9 \]

**Checkpoint**

Write a linear function that satisfies the given conditions.

7. \( f(0) = 1, f(2) = 9 \)
8. \( f(0) = -7, f(6) = 5 \)
9. \( g(-6) = 16, g(0) = -5 \)
10. \( r(-7) = 3, r(0) = 3 \)
Example 4  Using Function Notation in Real Life

After the balloon described on page 426 was launched, it rose at a rate of about 500 feet per minute to a final altitude of 120,000 feet.

a. Use function notation to write an equation giving the altitude of the balloon as a function of time.

b. How long did it take the balloon to reach its final altitude?

Solution

a. Let \( t \) be the elapsed time (in minutes) since the balloon was launched, and let \( a(t) \) be the altitude (in feet) at that time. Write a verbal model. Then use the verbal model to write an equation.

\[
\text{Altitude} = \text{Rate of climb} \cdot \text{Time since launch}
\]

\[
a(t) = 500t
\]

b. Find the value of \( t \) for which \( a(t) = 120,000 \).

\[
a(t) = 500t \quad \text{Write function for altitude.}
\]

\[
120,000 = 500t \quad \text{Substitute 120,000 for } a(t).
\]

\[
240 = t \quad \text{Divide each side by 500.}
\]

Answer  It took the balloon about 240 minutes (or about 4 hours) to reach its final altitude.

8.7  Exercises

More Practice, p. 810

Guided Practice

Vocabulary Check
1. Write the equation \( y = 4x - 3 \) using function notation.
2. Suppose \( f \) is a linear function with \( f(2) = 5 \) and \( f(6) = -1 \). Describe how you can find the slope of the graph of \( f \).

Skill Check

Let \( f(x) = 7x + 4 \). Find the indicated value.
3. \( f(x) \) when \( x = -8 \)  
4. \( f(3) \)  
5. \( x \) when \( f(x) = 67 \)

Graph the function.
6. \( f(x) = -x + 3 \)
7. \( g(x) = 3x - 5 \)
8. \( h(x) = 2x \)
9. Write a linear function \( f \) given that \( f(-4) = 12 \) and \( f(0) = 8 \).

10. **Cable TV**  The average monthly cost of basic cable TV was $9.73 in 1985 and has increased by about $1.35 each year since then. Let \( t \) be the number of years since 1985. Use function notation to write an equation giving the monthly cost of basic cable TV as a function of \( t \).
Let \( f(x) = -3x + 1 \) and \( g(x) = 10x - 4 \). Find the indicated value.

11. \( f(x) \) when \( x = -1 \)  
12. \( g(x) \) when \( x = 5 \)  
13. \( x \) when \( f(x) = -17 \)  
14. \( x \) when \( g(x) = 31 \)  
15. \( f(-20) \)  
16. \( f(4) + g(-3) \)

Match the function with its graph.

17. \( f(x) = 2x - 1 \)  
18. \( g(x) = x - 1 \)  
19. \( h(x) = 2x + 1 \)

A.  
B.  
C.  

Graph the function.

20. \( f(x) = -2x \)  
21. \( g(x) = 4x - 4 \)  
22. \( h(x) = -\frac{2}{3}x + 5 \)

23. Critical Thinking Write a linear function \( g \) whose graph passes through the origin and is parallel to the graph of \( f(x) = -8x - 2 \).

Write a linear function that satisfies the given conditions.

24. \( f(0) = 4, f(1) = 7 \)  
25. \( g(-2) = 10, g(0) = 0 \)  
26. \( h(0) = 13, h(3) = 1 \)  
27. \( r(-9) = -7, r(0) = -1 \)

28. Squid An arrow squid has a beak used for eating. Given the length \( b \) (in millimeters) of an arrow squid’s lower beak, you can approximate the squid’s mass (in grams) using the function \( m(b) = 236b - 513 \).

- a. The beak of an arrow squid washes ashore on a beach, where it is found and measured by a biologist. The lower beak has a length of 5 millimeters. Approximate the mass of the squid.
- b. To the nearest tenth of a millimeter, about how long is the lower beak of an arrow squid with a mass of 1100 grams?

29. Extended Problem Solving You make and sell birdhouses. Your fixed costs for your tools and workspace are $3000. The cost of wood and other materials needed to make a birdhouse is $10. You sell each birdhouse for $50. Let \( x \) represent the number of birdhouses you make and sell.

- a. Write a function for your total costs, \( c(x) \).
- b. Write a function for your income, \( i(x) \).
- c. Analyze Your profit is the difference of your income and total costs. Write a function for your profit, \( p(x) \).
- d. What is your profit when you make and sell 100 birdhouses?
- e. Interpret and Apply You are said to “break even” when your profit is $0. How many birdhouses do you need to make and sell in order to break even?
30. **Rivers**  Surveyors measured the speed of the current below a dam on the Columbia River in Washington. Based on their data, the speed (in feet per second) can be approximated by \( s(d) = -0.117d + 1.68 \), where \( d \) is the depth (in feet) below the river’s surface.

a. **Graph** the given function on a graphing calculator. Remember to replace \( d \) with \( x \) and \( s(d) \) with \( y \).

b. **Writing**  Describe what happens to the speed of the current as you go deeper below the river’s surface.

c. **Apply**  Approximate the speed of the current at a depth of 9 feet.

31. **Challenge**  The first four rectangles in a pattern are shown below.

![Rectangle Pattern](image)

a. For the \( n \)th rectangle in the pattern, what are the dimensions in terms of \( n \)?

b. Write a function for the area \( A(n) \) of the \( n \)th rectangle.

c. Write a function for the perimeter \( P(n) \) of the \( n \)th rectangle.

d. Find the area and the perimeter of the 50th rectangle in the pattern.

**Mixed Review**  

**Simplify.** *(Lesson 4.5)*

32. \( x^3 \cdot x^5 \)  
33. \( 2n^7 \cdot 5n^4 \)  
34. \( \frac{a^{12}}{a^8} \)  
35. \( \frac{30x^3}{12c^2} \)

**Write the percent as a fraction in simplest form.** *(Lesson 7.1)*

36. 40%  
37. 64%  
38. 99%  
39. 150%

**Write an equation of the line that is perpendicular to the given line and passes through the given point.** *(Lesson 8.6)*

40. \( y = 6x + 10; \) (0, -4)  
41. \( y = \frac{-5}{9}x - 1; \) (0, 3)

**Standardized Test Practice**

42. **Multiple Choice**  Let \( f(x) = -7x - 11 \). What is the value of \( f(-4) \)?

A. -39  
B. -22  
C. 0  
D. 17

43. **Multiple Choice**  Suppose \( g \) is a linear function with \( g(-3) = 28 \) and \( g(0) = 4 \). What is the slope of the graph of \( g \)?

F. -8  
G. \( -\frac{1}{8} \)  
H. \( \frac{1}{8} \)  
I. 8

44. **Short Response**  For Oregon counties with population \( p \), the function \( w(p) = 0.878p - 4764 \) approximates the amount of solid waste (in tons) that was disposed of during 1998. The population of Marion County, Oregon, was 271,750 in 1998. To the nearest thousand tons, about how much solid waste was disposed of in Marion County during 1998?
Systems of Linear Equations

**Before**

You graphed linear equations.

**Now**

You’ll graph and solve systems of linear equations.

**Why?**

So you can decide which of two printers to buy, as in Ex. 25.

**Internet** Some providers of high-speed Internet service offer a choice of two plans. With plan A, you buy the modem and pay a monthly fee for Internet service. With plan B, the modem is free, but you pay a higher monthly fee than for plan A.

When is plan A a better deal than plan B? In Example 4, you’ll see how to answer this question by solving a **system of linear equations**.

A **system of linear equations**, or simply a **linear system**, consists of two or more linear equations with the same variables. Below is an example.

\[
\begin{align*}
y &= 2x - 4 & \text{Equation 1} \\
y &= -3x + 1 & \text{Equation 2}
\end{align*}
\]

A **solution of a linear system** in two variables is an ordered pair that is a solution of each equation in the system. A linear system has a solution at each point where the graphs of the equations in the system intersect.

**Example 1**

**Solving a System of Linear Equations**

Solve the linear system: \[ \begin{align*}
y &= 2x - 4 \\
y &= -3x + 1
\end{align*} \]

**1.** Graph the equations.

**2.** Identify the apparent intersection point, \((1, -2)\).

**3.** Verify that \((1, -2)\) is the solution of the system by substituting 1 for \(x\) and \(-2\) for \(y\) in each equation.

**Equation 1**

\[
\begin{align*}
y &= 2x - 4 \\
-2 &= 2(1) - 4 \\
-2 &= -2 \checkmark
\end{align*}
\]

**Equation 2**

\[
\begin{align*}
y &= -3x + 1 \\
-2 &= -3(1) + 1 \\
-2 &= -2 \checkmark
\end{align*}
\]

**Answer** The solution is \((1, -2)\).
**Numbers of Solutions**  As you saw in Example 1, when the graphs of two linear equations have exactly one point of intersection, the related system has exactly one solution. It is also possible for a linear system to have no solution or infinitely many solutions.

**Example 2**  **Solving a Linear System with No Solution**

Solve the linear system:  
\[ y = -2x + 1 \]  \[ y = -2x + 5 \]

Graph the equations. The graphs appear to be parallel lines. You can confirm that the lines are parallel by observing from their equations that they have the same slope, \(-2\), but different \(y\)-intercepts, 1 and 5.

**Answer** Because parallel lines do not intersect, the linear system has no solution.

**Example 3**  **Solving a Linear System with Many Solutions**

Solve the linear system:  
\[ 2x - y = -3 \]  \[ -4x + 2y = 6 \]

Write each equation in slope-intercept form.

**Equation 1**  
\[ 2x - y = -3 \]
\[ -y = -2x - 3 \]
\[ y = 2x + 3 \]

**Equation 2**  
\[ -4x + 2y = 6 \]
\[ 2y = 4x + 6 \]
\[ y = 2x + 3 \]

The slope-intercept forms of equations 1 and 2 are identical, so the graphs of the equations are the same line (shown at the right).

**Answer** Because the graphs have infinitely many points of intersection, the system has infinitely many solutions. Any point on the line \(y = 2x + 3\) represents a solution.

**Checkpoint**

Solve the linear system by graphing.

1. \[ y = 4x + 2 \]
   \[ y = x + 2 \]
2. \[ x - y = -3 \]
   \[ -4x + 4y = 12 \]
3. \[-3x + y = -1 \]
   \[ y = 3x + 4 \]
4. **Critical Thinking**  If the graphs of two linear equations have different slopes, how many solutions does the related system have? Give a verbal and a graphical justification for your answer.
A company offers two plans for high-speed Internet service, as described on page 431.

**Plan A:** You pay $200 for the modem and $30 per month for service.

**Plan B:** The modem is free and you pay $40 per month for service.

**Example 4**  **Writing and Solving a Linear System**

a. After how many months are the total costs of the plans the same?

b. When is plan A a better deal? When is plan B a better deal?

**Solution**

a. Let $y$ be the cost of each plan after $x$ months. Write a linear system.

**Plan A:** $y = 200 + 30x$

**Plan B:** $y = 40x$

Use a graphing calculator to graph the equations. Trace along one of the graphs until the cursor is on the point of intersection. This point is (20, 800).

**Answer** The total costs of the plans are the same after 20 months, when each plan costs $800.

b. The graph for plan A lies below the graph for plan B when $x > 20$, so plan A costs less if you have service for more than 20 months. The graph for plan B lies below the graph for plan A when $x < 20$, so plan B costs less if you have service for less than 20 months.

---

**Guided Practice**

**Vocabulary Check**

1. What is a solution of a system of linear equations in two variables?

2. If the graphs of the two equations in a system are parallel lines, what can you say about the solution(s) of the system?

**Skill Check**  **Solve the linear system by graphing.**

3. $y = 3x - 8$

4. $x + y = 3$

5. $y = -4x + 1$

6. $y = 2x - 5$

7. $x - y = -5$

8. $y = 5 - 4x$

**Shoes** One wall of a shoe store is used to display court shoes and running shoes. There is enough room on the wall for 120 styles of shoes. Based on past sales, the store manager wants to display twice as many running shoes as court shoes. Write and solve a linear system to find the number of each type of shoe to display.
Tell whether the ordered pair is a solution of the linear system.

7. \((0, -2)\);
\[3x - 2y = 4\]
\[-2x - y = -2\]

8. \((4, 2)\);
\[y = -5x + 22\]
\[y = 8x - 30\]

9. \((-24, -10)\);
\[x - 4y = 16\]
\[-2x + 6y = -12\]

Use the graph to identify the solution of the related linear system.

10. \[\text{Graph A}\]
11. \[\text{Graph B}\]
12. \[\text{Graph C}\]

Solve the linear system by graphing.

13. \[y = -3x + 2\]
\[y = x - 2\]
14. \[y = 2x - 1\]
\[y = 4x - 5\]
15. \[2x + 4y = 8\]
\[3x + 6y = 12\]
16. \[2x + y = -8\]
\[-x + y = 4\]
17. \[y = 5x - 3\]
\[y = 5x + 2\]
18. \[x + y = -7\]
\[y = x + 3\]
19. \[x - 3y = -6\]
\[2x + 3y = -3\]
20. \[3x + 2y = 8\]
\[4y = 16 - 6x\]
21. \[4x + y = 5\]
\[3x + 5y = 25\]

22. **Vacation Rentals** A business rents in-line skates and bicycles to tourists on vacation. A pair of skates rents for $15 per day. A bicycle rents for $20 per day. On a certain day, the owner of the business has 25 rentals and takes in $450. Using the verbal model below, write and solve a system of equations to find the number of each item rented.

\[
\begin{array}{ccc}
\text{Pairs of} & + & \text{Number of} \\
\text{skates} & = & \text{Total} \\
\text{rentals} & & \\
\hline
\end{array}
\]

\[
\begin{array}{ccc}
\text{Rent per} & \times & \text{Pairs of} & + & \text{Rent per} \\
\text{pair of} & \times & \text{skates} & = & \text{Number of} \\
\text{skates} & & & = & \text{Total} \\
\text{rentals} & & & & \text{income} \\
\hline
\end{array}
\]

23. **Advertising** You own a business that advertises in a local newspaper and over the radio. A newspaper ad costs $600. A radio ad costs $300. You have a monthly advertising budget of $24,000 and want to run 50 ads each month. Write and solve a system of equations to find how many newspaper ads and radio ads you should run each month.

24. **Geometry** The graphs of the three equations below form a triangle. Find the coordinates of the triangle’s vertices.

\[
\begin{align*}
2x - y &= 4 \\
2x + 3y &= 12 \\
10x + 3y &= -12 \\
\end{align*}
\]
25. **Extended Problem Solving** You are trying to decide whether to buy an inkjet printer for $100 or a laser printer for $400. The operating costs are estimated to be $0.15 per page for the inkjet printer and $0.03 per page for the laser printer.

   a. Write a system of equations describing the total cost of buying and operating each printer.

   b. Use a graphing calculator to solve the system of equations. After how many pages are the total costs of the printers equal?

   c. **Interpret** When does the inkjet printer have the lower total cost? When does the laser printer have the lower total cost?

   d. **Apply** You plan to own the printer you buy for 3 years. Which printer offers the lower total cost if you print an average of 2 pages per day? If you print an average of 4 pages per day? Explain.

**Visual Thinking** In Exercises 26–28, find values of $m$ and $b$ for which the system below has the given number of solutions. Justify your answers.

$$y = 3x - 2$$
$$y = mx + b$$

26. Exactly one \hspace{1cm} 27. None \hspace{1cm} 28. Infinitely many

29. **Challenge** You are designing a reflecting pool for a park. The design specifications say that the area of the pool should be 450 square feet. You want the pool to be rectangular and have a length that is twice the width. Let $l$ be the pool’s length, and let $w$ be its width.

   a. Write a system of two equations for this situation. Each equation should be solved for $l$.

   b. Enter the equations from part (a) into a graphing calculator. Use the table feature to make a table of solutions for each equation. What ordered pair ($w, l$) has positive coordinates and satisfies both equations? What should the dimensions of the reflecting pool be?

**Mixed Review**

**Solve the Inequality. Graph your solution.** *(Lessons 3.4, 3.5)*

30. $x + 4 > 9$ \hspace{1cm} 31. $y - 5 \leq 2$ \hspace{1cm} 32. $-3t \geq 12$ \hspace{1cm} 33. $\frac{n}{2} < 6$

**Write the number in scientific notation.** *(Lesson 4.7)*

34. 1200 \hspace{1cm} 35. 309,000 \hspace{1cm} 36. 0.0005 \hspace{1cm} 37. 0.00000748

**Write a linear function that satisfies the given conditions.** *(Lesson 8.7)*

38. $f(0) = 8, f(3) = 10$ \hspace{1cm} 39. $h(-4) = -7, h(0) = -27$

**Standardized Test Practice**

40. **Multiple Choice** Which ordered pair is the solution of the linear system $y = 2x + 16$ and $y = -x + 1$?

   A. (0, 16) \hspace{1cm} B. (2, -1) \hspace{1cm} C. (-5, 6) \hspace{1cm} D. (-8, 9)

41. **Multiple Choice** Which ordered pair is not a solution of the linear system $x - 3y = -12$ and $-3x + 9y = 36$?

   F. (-3, 2) \hspace{1cm} G. (0, 4) \hspace{1cm} H. (3, 5) \hspace{1cm} I. (6, 6)
8.9

Graphs of Linear Inequalities

Vocabulary
linear inequality in two variables, p. 436
solution of a linear inequality in two variables, p. 436
graph of a linear inequality in two variables, p. 436
half-plane, p. 436

You solved inequalities in one variable.
You’ll graph inequalities in two variables.
So you can find how many kites to make from paper, as in Ex. 34.

Pottery How many bowls and vases can you make from a fixed amount of clay? In Example 4, you’ll see how a linear inequality can be used to answer this question.

A linear inequality in two variables, such as $2x - 3y < 6$, is the result of replacing the equal sign in a linear equation with $<$, $\leq$, $>$, or $\geq$.

An ordered pair $(x, y)$ is a solution of a linear inequality if substituting the values of $x$ and $y$ into the inequality produces a true statement.

Example 1 Checking Solutions of a Linear Inequality

Tell whether the ordered pair is a solution of $2x - 3y < 6$.

a. $(0, 1)$

Solution

a. Substitute 0 for $x$ and 1 for $y$.

$2x - 3y < 6$

$2(0) - 3(1) \leq 6$

$-3 < 6 \checkmark$

$(0, 1)$ is a solution.

b. Substitute 4 for $x$ and $-2$ for $y$.

$2x - 3y < 6$

$2(4) - 3(-2) \leq 6$

$14 < 6$

$(4, -2)$ is not a solution.

Graphs The graph of a linear inequality in two variables is the set of points in a coordinate plane that represent all the inequality’s solutions.

Reading Algebra

In the graph shown, a dashed boundary line is used to indicate that points on the line are not solutions of $2x - 3y < 6$. A solid boundary line would indicate that points on the line are solutions.

All solutions of $2x - 3y < 6$ lie on one side of the boundary line $2x - 3y = 6$.

The boundary line divides the plane into two half-planes. The shaded half-plane is the graph of $2x - 3y < 6$. 
Graphing Linear Inequalities

1. Find the equation of the boundary line by replacing the inequality symbol with =. Graph this equation. Use a dashed line for < or >. Use a solid line for ≤ or ≥.

2. Test a point in one of the half-planes to determine whether it is a solution of the inequality.

3. If the test point is a solution, shade the half-plane that contains the point. If not, shade the other half-plane.

Example 2

Graphing a Linear Inequality

Graph \( y \geq 2x + 4 \).

1. Draw the boundary line \( y = 2x + 4 \). The inequality symbol is ≥, so use a solid line.

2. Test the point (0, 0) in the inequality.
   
   \[
   \begin{align*}
   y & \geq 2x + 4 \\
   0 & \geq 2(0) + 4 \\
   0 & \not\geq 4
   \end{align*}
   \]

3. Because (0, 0) is not a solution, shade the half-plane that does not contain (0, 0).

Example 3

Graphing Inequalities with One Variable

Graph \( x < 3 \) and \( y \geq -2 \) in a coordinate plane.

a. Graph \( x = 3 \) using a dashed line. Use (0, 0) as a test point.
   
   \[
   \begin{align*}
   x & < 3 \\
   0 & < 3 \checkmark
   \end{align*}
   \]
   Shade the half-plane that contains (0, 0).

b. Graph \( y = -2 \) using a solid line. Use (0, 0) as a test point.
   
   \[
   \begin{align*}
   y & \geq -2 \\
   0 & \geq -2 \checkmark
   \end{align*}
   \]
   Shade the half-plane that contains (0, 0).

Checkpoint

Graph the inequality in a coordinate plane.

1. \( x + 2y > 6 \)   
2. \( x \geq -1 \)   
3. \( y < 3 \)
You have 100 pounds of clay to use for making bowls and vases. You need 5 pounds of clay for each bowl and 2 pounds for each vase.

a. Write an inequality describing the possible numbers of bowls and vases that you can make.

b. Graph the inequality from part (a).

c. Give three possible combinations of bowls and vases that you can make.

Solution

a. Let \( x \) be the number of bowls you make. Let \( y \) be the number of vases you make. Write a verbal model. Then use the verbal model to write an inequality.

\[
\text{Clay per bowl} \cdot \text{Number of bowls} + \text{Clay per vase} \cdot \text{Number of vases} \leq \text{Total amount of clay}
\]

\[5x + 2y \leq 100\]

b. To graph the inequality, first draw the boundary line \( 5x + 2y = 100 \). Use a solid line because the inequality symbol is \( \leq \).

Test the point \((0, 0)\) in the inequality.

\[
5(0) + 2(0) \leq 100
\]

\[0 \leq 100 \checkmark
\]

Because \((0, 0)\) is a solution, all solutions of \(5x + 2y \leq 100\) lie in the half-plane containing \((0, 0)\). Shade the portion of this half-plane that lies in the first quadrant, as the numbers of bowls and vases made must be nonnegative.

c. Choose three points on the graph with whole-number coordinates, such as \((5, 20)\), \((10, 10)\), and \((20, 0)\). You can make 5 bowls and 20 vases, or 10 bowls and 10 vases, or 20 bowls and no vases.

Checkpoint

4. It is recommended that you get at least 60 milligrams of vitamin C each day. One fluid ounce of orange juice contains about 15 milligrams of vitamin C, and one fluid ounce of grapefruit juice contains about 12 milligrams. Write and graph an inequality describing the possible amounts of orange juice and grapefruit juice you can drink to meet your daily requirement for vitamin C.
Guided Practice

Vocabulary Check
1. Copy and complete: The graph of a linear inequality in two variables is called a(n) \( \square \).

2. When graphing a linear inequality in two variables, explain how to determine which side of the boundary line to shade.

Skill Check
Tell whether the ordered pair is a solution of \( 4x + y > -1 \).
3. \((-2, 5)\)  4. \((0, 0)\)  5. \((4, -4)\)  6. \((-1, 3)\)

Graph the inequality in a coordinate plane.
7. \(y < 3x + 1\)  8. \(4x - 5y \leq 20\)  9. \(x > -2\)  10. \(y \geq 1\)

Guided Problem Solving
11. Movies You have a gift certificate for $40 to use at a movie theater. Matinees cost $5 and evening shows cost $8. What are some possible combinations of matinees and evening shows that you can see?
   
   1) Write an inequality for this situation.
   2) Graph the inequality from Step 1.
   3) Given three possible combinations of matinees and evening shows that you can see.

Practice and Problem Solving

Homework Help

Error Analysis Describe and correct the error in the graph of the given inequality.
12. \(y > x - 1\)  13. \(y \leq 2x + 2\)

14. Critical Thinking When graphing the inequality \(y \geq 2x\), can you use \((0, 0)\) as a test point to determine which side of the boundary line to shade? Explain.

15. Logical Reasoning Find an ordered pair that is a solution of \(y \leq x + 5\) but is not a solution of \(y < x + 5\).
Tell whether the ordered pair is a solution of the inequality.

16. \(y \geq -7x + 9; \ (1, \ 4)\)
17. \(y < 10x - 1; \ (-1, \ -11)\)
18. \(x \leq 6; \ (8, \ -9)\)
19. \(5x - 8y \geq 2; \ (0, \ -3)\)

Graph the inequality in a coordinate plane.

20. \(y < x + 4\)
21. \(y > -3x\)
22. \(y \geq \frac{2}{3}x - 5\)
23. \(y \leq -2x - 3\)
24. \(x + y \geq -2\)
25. \(-x + 2y \leq 6\)
26. \(3x - 2y > 2\)
27. \(4x + 3y < -12\)
28. \(y > -3\)
29. \(x \geq 1\)
30. \(x < -4\)
31. \(y \leq -1\)

32. **Entertainment** At a county fair, you buy 20 tickets that you can use for carnival rides and other attractions. Some rides require 1 ticket while others require 2 tickets.
   a. Write an inequality describing the possible numbers of 1-ticket rides and 2-ticket rides that you can go on.
   b. Graph the inequality from part (a).
   c. **Interpret and Apply** Give three possible combinations of 1-ticket rides and 2-ticket rides that you can go on.

33. **Video** A widescreen format for a movie or TV show is one in which the image’s height \(x\) and width \(y\) satisfy the inequality \(y > \frac{4}{3}x\).
   a. Graph the given inequality.
   b. **Writing** Suppose the height of a widescreen image is 18 inches. Describe the possible widths of the image.

34. **Extended Problem Solving** You have 48 square feet of paper to use for making kites. You want to make the two types of kites shown below. Assume that the amount of paper needed for each kite is the area of the kite.

![Kite A](image1)

![Kite B](image2)

a. **Calculate** Find the area of kite A and the area of kite B in square inches. Then convert the areas to square feet.

b. **Graph** Write and graph an inequality describing how many of kite A and kite B you can make.

c. **Analyze** What property is shared by points that represent solutions where you use up all your paper? What property is shared by points that represent solutions where you have paper left over?
**Challenge**  In Exercises 35–37, use the system of linear inequalities shown below. A solution of the system is an ordered pair that is a solution of each inequality.

\[
\begin{align*}
y &< x + 3 \\
y &\geq -2x - 3
\end{align*}
\]

35. Tell whether each ordered pair is a solution of the system.
   a. \((0, -4)\)  
   b. \((1, 3)\)  
   c. \((-2, 1)\)

36. Graph the inequalities in the system. Draw both graphs in the same coordinate plane, and use a different color for each graph.

37. **Writing**  Describe the region of the plane that contains the solutions of the system.

**Mixed Review**

Write the product using an exponent.  (Lesson 1.2)

38. \(8 \cdot 8 \cdot 8 \cdot 8 \cdot 8\)  
39. \((1.2)(1.2)(1.2)\)  
40. \(x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x\)

Write the expression using only positive exponents.  (Lesson 4.6)

41. \(5x^{-2}\)  
42. \(2a^{-3}b^8\)  
43. \(9m^{-5}n^{-4}\)

Solve the linear system by graphing.  (Lesson 8.8)

44. \(\begin{align*}y &= 2x - 3 \\
y &= x + 1\end{align*}\)  
45. \(\begin{align*}y &= -3x - 6 \\
y &= 2x + 4\end{align*}\)  
46. \(\begin{align*}x + y &= -2 \\
2x + y &= 0\end{align*}\)

**Standardized Test Practice**

47. **Multiple Choice**  Which ordered pair is not a solution of \(y \geq -9x + 1\)?
   A. \((1, -5)\)  
   B. \((2, 1)\)  
   C. \((-1, 10)\)  
   D. \((-3, -1)\)

48. **Multiple Choice**  Which inequality has no solutions in the first quadrant of a coordinate plane?
   F. \(x < 1\)  
   G. \(y \leq -2x + 6\)  
   H. \(x + y \leq 5\)  
   I. \(-3x - y > 6\)

**Brain Game**

A list of ordered pairs is shown below.

\((-3, -6)\)  \((4, -4)\)  \((-6, 9)\)  \((2, -3)\)  \((5, 2)\)

Only one of the ordered pairs is a solution of all of the following inequalities. Which ordered pair is it?

\[x < 8\]  \[y \geq -5\]  \[3x - y \leq 9\]  \[y > -2x - 4\]  \[x + y < 3\]
Chapter Review

Vocabulary Review

- relation, p. 385
- domain, p. 385
- range, p. 385
- input, p. 385
- output, p. 385
- function, p. 386
- vertical line test, p. 387
- equation in two variables, p. 391
- solution of an equation in two variables, p. 391
- graph of an equation in two variables, p. 392
- linear equation, p. 392
- function form, p. 393
- x-intercept, p. 398
- y-intercept, p. 398
- slope, p. 404
- rise, p. 404
- run, p. 404
- slope-intercept form, p. 412
- best-fitting line, p. 421
- function notation, p. 426
- system of linear equations, p. 431
- solution of a linear system, p. 431
- linear inequality in two variables, p. 436
- solution of a linear inequality in two variables, p. 436
- graph of a linear inequality in two variables, p. 436
- half-plane, p. 436

1. What is the difference between the domain and the range of a relation?

2. Write a linear equation in slope-intercept form. Identify the slope and y-intercept.

3. How are the rise and run between two points on a line related to the line's slope?

4. Write the equation \( y = -5x + 2 \) using function notation.

8.1 Relations and Functions

**Examples on pp. 385–387**

**Goal**

Use graphs and mapping diagrams to represent relations.

**Example**

Represent the relation \((-2, -2), (-1, 3), (0, 4), (3, 0)\) as a graph and as a mapping diagram.

- a. Graph the ordered pairs as points in a coordinate plane.

- b. List the inputs and the outputs in order. Draw arrows from the inputs to their outputs.

- Input: \(-2, -1, 0, 3\)
- Output: \(2, 3, 4, 0\)

**✓** Represent the relation as a graph and as a mapping diagram.

5. \((-5, 6), (-4, 3), (0, 0), (4, -3)\)

6. \((7, -2), (6, 5), (2, 3), (2, -8), (3, 0)\)
8.2 Linear Equations in Two Variables

Example Tell whether \((-4, -6)\) or \((2, 8)\) is a solution of \(-3x + y = 6\).

a. \[-3x + y = 6\] Write original equation.
\[-3(-4) + (-6) \neq 6\] Substitute \(-4\) for \(x\) and \(-6\) for \(y\).
\[6 = 6 \checkmark\] Simplify.
Answer \((-4, -6)\) is a solution of \(-3x + y = 6\).

b. \[-3x + y = 6\] Write original equation.
\[-3(2) + 8 \neq 6\] Substitute \(2\) for \(x\) and \(8\) for \(y\).
\[2 \neq 6\] Simplify.
Answer \((2, 8)\) is not a solution of \(-3x + y = 6\).

Tell whether the ordered pair is a solution of the equation.

7. \(y = -8x - 2; (-1, 6)\)
8. \(14x + 2y = -22; (-2, -3)\)

8.3 Using Intercepts

Example Find the intercepts of the graph of \(9x + 3y = 27\).

To find the \(x\)-intercept, let \(y = 0\) and solve for \(x\).
\[9x + 3y = 27\] Write original equation.
\[9x + 3(0) = 27\] Substitute \(0\) for \(y\).
\[9x = 27\] Simplify.
\[x = 3\] Divide each side by \(9\).

To find the \(y\)-intercept, let \(x = 0\) and solve for \(y\).
\[9x + 3y = 27\] Write original equation.
\[9(0) + 3y = 27\] Substitute \(0\) for \(x\).
\[3y = 27\] Simplify.
\[y = 9\] Divide each side by \(3\).
Answer The \(x\)-intercept is \(3\), and the \(y\)-intercept is \(9\).

Find the intercepts of the equation’s graph.
9. \(3x - 12y = 24\)
10. \(y = 2x - 10\)
11. \(20x + 4y = -20\)
8.4 The Slope of a Line

**Goal**
Find the slope of a line.

**Example**
Find the slope of the line through the points \((-3, 6)\) and \((-1, 2)\).

\[
m = \frac{\text{rise}}{\text{run}} = \frac{\text{difference of } y\text{-coordinates}}{\text{difference of } x\text{-coordinates}}
\]

\[
= \frac{2 - 6}{-1 - (-3)} = \frac{-4}{2} = -2
\]

**Example**
Find the slope of the line through the given points.

12. \((4, -7), (-2, -10)\)  
13. \((6, 9), (-3, 9)\)  
14. \((3, 4), (7, -12)\)

8.5 Slope-Intercept Form

**Goal**
Find the slope and \(y\)-intercept of a line.

**Example**
Identify the slope and \(y\)-intercept of the line \(24x + 4y = 80\).

\[
24x + 4y = 80 \quad \text{Write original equation.}
\]

\[
4y = -24x + 80 \quad \text{Subtract } 24x \text{ from each side.}
\]

\[
y = -6x + 20 \quad \text{Multiply each side by } \frac{1}{4}.
\]

**Answer**
The line has a slope of \(-6\) and a \(y\)-intercept of \(20\).

**Example**
Identify the slope and \(y\)-intercept of the line with the given equation.

15. \(y = -3x + 2\)  
16. \(2x + 3y = -6\)  
17. \(-36x + 9y = 18\)

8.6 Writing Linear Equations

**Goal**
Write an equation of a line parallel to a given line.

**Example**
Write an equation of the line that is parallel to the line \(y = -3x + 4\) and passes through \((0, 7)\).

Because the slope of the given line is \(-3\), the slope of the parallel line is also \(-3\). The parallel line passes through \((0, 7)\), so its \(y\)-intercept is \(7\).

**Answer**
An equation of the line is \(y = -3x + 7\).

**Example**
Write an equation of the line that is parallel to the given line and passes through the given point.

18. \(y = 3x - 8; (0, 2)\)  
19. \(y = -x; (0, -6)\)  
20. \(y = -9x + 1; (0, 5)\)
### 8.7 Function Notation

**Goal**

Use function notation.

**Example** Let \( f(x) = 4x - 5 \). Find \( f(x) \) when \( x = -3 \).

\[
\begin{align*}
f(x) &= 4x - 5 \\
f(-3) &= 4(-3) - 5 = -17
\end{align*}
\]

Write function.

Substitute \(-3\) for \( x \) and simplify.

**✓** Let \( g(x) = -2x + 6 \). Find the indicated value.

21. \( g(x) \) when \( x = 4 \)  
22. \( x \) when \( g(x) = 14 \)  
23. \( g(-2) \)

### 8.8 Systems of Linear Equations

**Goal**

Solve linear systems in two variables by graphing.

**Example** Solve the linear system: \( y = -x + 4 \)  
\( y = 2x + 1 \)

1. Graph the equations.
2. Identify the apparent intersection point, \((1, 3)\).
3. Verify that \((1, 3)\) is the solution of the system by substituting 1 for \( x \) and 3 for \( y \) in each equation.

**✓** Solve the linear system by graphing.

24. \( y = -2x - 12 \)  
25. \( y = -2x + 5 \)  
26. \( 2x + y = 1 \)  
27. \( 4x - 2y = 22 \)

### 8.9 Graphs of Linear Inequalities

**Goal**

Graph linear inequalities in two variables.

**Example** Graph \( y > x - 3 \).

1. Graph the boundary line \( y = x - 3 \).
   Use a dashed line.
2. Use \((0, 0)\) as a test point.
   \[
   y > x - 3 \\
   0 > 0 - 3 = -3 \checkmark
   \]
3. Shade the half-plane that contains \((0, 0)\).

**✓** Graph the inequality in a coordinate plane.

27. \( y \leq 2x + 3 \)  
28. \( y \geq -4 \)  
29. \( 3x + y > -6 \)
Chapter Test

Represent the relation as a graph and as a mapping diagram. Then tell whether the relation is a function. Explain your reasoning.

1. (0, −2), (0, −1), (0, 0), (0, 1), (0, 2)  
2. (3, 5), (6, 7) (9, 9), (8, 1)

Tell whether each ordered pair is a solution of the equation.

3. \( y = 7 − 2x \); (5, 1), (6, −5), (2, 3)  
4. \( y = −3x − 4 \); (−1, −1), (0, −4), (10, 34)

Find the intercepts of the equation’s graph. Then graph the equation.

5. \( x + y = 4 \)  
6. \( 4x − 3y = 24 \)  
7. \( y = \frac{5}{2}x − 10 \)  
8. \( y = 3x + 6 \)

Find the slope of the line through the given points.

9. (8, −3), (10, 7)  
10. (4, 2), (0, 3)  
11. (−2, 0), (−2, 5)  
12. (4, 7), (10, 7)

Identify the slope and \( y \)-intercept of the line with the given equation. Use the slope and \( y \)-intercept to graph the equation.

13. \( y = \frac{4}{3}x − 7 \)  
14. \( y = 5x + 1 \)  
15. \( −6x + y = −2 \)  
16. \( 6x − 5y = 10 \)

17. **Televisions** The table shows the number of televisions sold each month at a retail store.

   a. Make a scatter plot of the data pairs. Draw the line that appears to best fit the data points.

   b. Write an equation of your line in slope-intercept form.

   c. Predict the number of televisions sold at the store during the 7th month.

<table>
<thead>
<tr>
<th>Month, ( x )</th>
<th>Televisions, ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>360</td>
</tr>
<tr>
<td>2</td>
<td>375</td>
</tr>
<tr>
<td>3</td>
<td>380</td>
</tr>
<tr>
<td>4</td>
<td>389</td>
</tr>
<tr>
<td>5</td>
<td>402</td>
</tr>
</tbody>
</table>

Write a linear function that satisfies the given conditions.

18. \( f(0) = 3, f(4) = 9 \)  
19. \( g(0) = −6, g(15) = −9 \)  
20. \( h(−4) = −5, h(0) = 10 \)

Solve the linear system by graphing.

21. \( x + 5y = −10 \)  
22. \( 3x − y = −7 \)  
23. \( 2x − y = 5 \)  
\[ x + 5y = 5 \]
\[ −3x + y = 7 \]  
\[ x + 2y = −10 \]

Graph the inequality in a coordinate plane.

24. \( x < 7 \)  
25. \( y ≤ 3x − 5 \)  
26. \( x + 2y > 6 \)
1. What is the domain of the relation (8, 2), (6, 4), (4, 2), (2, 4)?
   A. 2, 4  B. 2, 4, 6, 8  
   C. 6, 8  D. 2, 4, 8

2. Which equation is not a function?
   F. $x + y = 5$  G. $2x - y = 3$
   H. $x = 4$  I. $y = -1$

3. What is the $x$-intercept of the graph of $y = \frac{1}{4}x - 6$?
   A. $-6$  B. $\frac{1}{4}$  C. 6  D. 24

4. What is the slope of the line through the points (2, 2) and (4, 6)?
   F. $\frac{1}{2}$  G. $\frac{3}{4}$  H. $\frac{4}{3}$  I. 2

5. What is the $y$-intercept of the graph of $-x + 4y = -24$?
   A. $-6$  B. 4  C. 6  D. 24

6. What is the slope of the line with equation $-x + 4y = -24$?
   F. $-6$  G. $\frac{1}{4}$  H. 6  I. 24

7. What is the slope of a line perpendicular to the line with equation $y = 4x - 5$?
   A. $-4$  B. $-\frac{1}{4}$  C. $\frac{1}{5}$  D. 5

8. Given that $f(x) = -2x + 1$, what is $f(5)$?
   F. $-11$  G. $-9$  H. $-2$  I. 2

9. How many solutions does the system of equations $4x + 2y = 6$ and $y = -2x + 6$ have?
   A. 0  B. 1  C. 2  D. Infinitely many

10. What is the solution of the system of equations $3x + y = -11$ and $y = 2x + 9$?
    F. $(1, -4)$  G. $(-1, 4)$
    H. $(4, -1)$  I. $(-4, 1)$

11. The graph of which inequality is shown?
    A. $y < \frac{3}{2}x - 1$
    B. $2x + \frac{1}{3}y \geq -2$
    C. $3x + 2y > -2$
    D. $3x + 2y < -2$

12. **Short Response** For which value of $a$ are the lines $y = ax + 6$ and $x + 2y = 4$ parallel? For which value of $a$ are the lines $y = ax + 6$ and $x + 2y = 4$ perpendicular? Explain.

13. **Extended Response** You have a $100 gift card to a store that rents movies and video games. The rental fee for a movie is $4, and the rental fee for a video game is $5.

   a. Write an equation describing the possible numbers of movies and video games you can rent.
   b. Use intercepts to graph the equation from part (a).
   c. Give three possible combinations of movies and video games you can rent.
Solving the Problem

Problems with No Solution or Many Solutions

Not all problems have a single solution. Instead, a problem may have no solution or many solutions.

**Problem**
At your class picnic, you volunteer to grill hot dogs. The hot dogs come in packages of 8, and the hot dog buns come in packages of 6. After everyone is finished eating, you have an open package of hot dogs that contains 2 hot dogs and an open package of hot dog buns that contains 4 buns. How many hot dogs were eaten?

1. **Write an equation.**
   Let \( x \) be the number of complete packages of hot dogs used. The total number \( d \) of hot dogs eaten is the sum of the 8\( x \) hot dogs in the complete packages used and the \( 8 - 2 = 6 \) hot dogs used in the last package.
   \[
   d = 8x + 6
   \]
   Let \( y \) be the number of complete packages of buns used. The total number \( b \) of buns eaten is the sum of the 6\( y \) buns in the complete packages used and the \( 6 - 4 = 2 \) buns used in the last package.
   \[
   b = 6y + 2
   \]
   Assume that the same number of buns and hot dogs were eaten. Then you can write an equation giving \( y \) as a function of \( x \).
   \[
   b = d
   \]
   \[
   6y + 2 = 8x + 6
   \]
   \[
   6y = 8x + 4
   \]
   \[
   y = \frac{8}{6}x + \frac{4}{6}
   \]
   \[
   y = \frac{4}{3}x + \frac{2}{3}
   \]

   The possible numbers of complete hot dog and bun packages used are given by the solutions \((x, y)\) of \( y = \frac{4}{3}x + \frac{2}{3} \) where \( x \) and \( y \) are whole numbers.

2. **Make a graph.**
   Graph \( y = \frac{4}{3}x + \frac{2}{3} \). Label the points on the graph that have whole-number coordinates.

3. **Solve the problem.**
   Make a table of values for \( d = 8x + 6 \) using the \( x \)-coordinates of the labeled points on the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>Substitution</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( d = 8(1) + 6 )</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>( d = 8(4) + 6 )</td>
<td>38</td>
</tr>
<tr>
<td>7</td>
<td>( d = 8(7) + 6 )</td>
<td>62</td>
</tr>
</tbody>
</table>

**Answer** Some possible numbers of hot dogs eaten are 14, 38, and 62.