Before

In previous chapters you’ve . . .

- Solved equations using multiplication or division
- Written fractions in simplest form
- Solved equations with rational numbers

Now

In Chapter 6 you’ll study . . .

- Finding ratios and unit rates
- Writing and solving proportions
- Identifying similar and congruent figures
- Finding unknown side lengths of similar figures
- Finding probabilities

Why?

So you can solve real-world problems about . . .

- nature, p. 273
- lacrosse, p. 278
- gasoline, p. 283
- money, p. 292
- carpentry, p. 303
- websites, p. 311
- art classes, p. 316
Architecture An architectural model shows on a small scale what an actual building will look like. In this chapter, you will use proportions to solve problems about scale models.

What do you think? The small photograph shows a scale model of Fallingwater, a building designed by the American architect Frank Lloyd Wright. One of the balconies on the actual building is 32 times longer than the same balcony on the scale model. If the scale model balcony is $13\frac{1}{2}$ inches long, how long (in feet) is the actual balcony?
Chapter Prerequisite Skills

PREREQUISITE SKILLS QUIZ

Preparing for Success To prepare for success in this chapter, test your knowledge of these concepts and skills. You may want to look at the pages referred to in blue for additional review.

1. Vocabulary How can you tell whether two fractions are equivalent?

Solve the equation. (p. 97)

2. \(-45 = 5x\)
3. \(4y = -32\)
4. \(-12 = \frac{n}{4}\)
5. \(\frac{x}{6} = 7\)

Write the fraction in simplest form. (p. 182)

6. \(\frac{24}{40}\)
7. \(\frac{42}{144}\)
8. \(\frac{14x^2y^3}{35xy^5}\)
9. \(\frac{a^3b^2c^2}{bc^2}\)

Solve the equation. (p. 247)

10. \(\frac{2}{9}x + \frac{5}{9} = 3\)
11. \(-\frac{3}{8}x - 5 = 7\)
12. \(\frac{1}{4}x - \frac{3}{8} = 9\)
13. \(\frac{3}{5}x - \frac{2}{5} = 2\)

NOTETAKING STRATEGIES

Comparing and Contrasting When you learn related vocabulary words or ideas, it may be helpful to make a table comparing and contrasting the ideas.

<table>
<thead>
<tr>
<th>Definition</th>
<th>LCM of two or more numbers: The least of the multiples that the numbers have in common</th>
<th>GCF of two or more numbers: The greatest of the factors that the numbers have in common</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>Multiples of 10: 10, 20, 30, 40, 50, 60</td>
<td>Factors of 10: 1, 2, 5, 10</td>
</tr>
<tr>
<td></td>
<td>Multiples of 12: 12, 24, 36, 48, 60</td>
<td>Factors of 12: 1, 2, 3, 4, 6, 12</td>
</tr>
<tr>
<td></td>
<td>LCM = 60</td>
<td>GCF = 2</td>
</tr>
</tbody>
</table>

In Lesson 6.4, you can make a table comparing and contrasting similar and congruent figures.
Ratios and Rates

**BEFORE**
You wrote equivalent fractions.

**Now**
You’ll find ratios and unit rates.

**WHY?**
So you can see if you’ll have enough for a guitar, as in Ex. 38.

**Archery** An archer shoots 60 arrows at a target, with 44 arrows hitting the scoring area and 16 missing the scoring area. How can you evaluate the archer’s performance? You can compare the archer’s number of hits to the archer’s number of misses using a ratio. A ratio uses division to compare two quantities.

**Writing Ratios**
You can write the ratio of two quantities, \(a\) and \(b\), where \(b\) is not equal to 0, in three ways.

\[
\begin{align*}
\text{a to } b & \quad a : b & \quad \frac{a}{b}
\end{align*}
\]

Each ratio is read “the ratio of \(a\) to \(b\).” You should write the ratio in simplest form, as shown in Example 1 below.

**Example 1 Writing Ratios**

Use the archery information given above. Write the ratio in three ways.

- a. The number of hits to the number of misses
- b. The number of hits to the number of shots

**Solution**

a. \[
\frac{\text{Number of hits}}{\text{Number of misses}} = \frac{44}{16} = \frac{11}{4}
\]

b. \[
\frac{\text{Number of hits}}{\text{Number of shots}} = \frac{44}{60} = \frac{11}{15}
\]

Three ways to write the ratio are \(\frac{11}{4}\), 11 to 4, and 11 : 4.

Three ways to write the ratio are \(\frac{11}{15}\), 11 to 15, and 11 : 15.

**Checkpoint**

1. Using the archery information above, compare the number of misses to the number of shots using a ratio. Write the ratio in three ways.
Comparing Ratios  To compare two ratios, you can write both ratios as fractions or as decimals. Two ratios are called equivalent ratios when they have the same value.

Example 2  Comparing and Ordering Ratios

Biology  The ratio comparing the length of a bird’s wings to the average width of the bird’s wings is the bird’s aspect ratio. Order the birds in the table from the greatest aspect ratio to the least.

<table>
<thead>
<tr>
<th>Bird</th>
<th>Wing length (cm)</th>
<th>Average wing width (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>White-tailed eagle</td>
<td>209</td>
<td>30</td>
</tr>
<tr>
<td>European jay</td>
<td>47</td>
<td>12</td>
</tr>
<tr>
<td>Black-headed gull</td>
<td>83</td>
<td>8</td>
</tr>
</tbody>
</table>

Solution

Write each ratio as a fraction. Then use a calculator to write each fraction as a decimal. Round to the nearest hundredth and compare the decimals.

White-tailed eagle: \[
\frac{\text{Wing length}}{\text{Wing width}} = \frac{209}{30} \approx 6.97
\]

European jay: \[
\frac{\text{Wing length}}{\text{Wing width}} = \frac{47}{12} \approx 3.92
\]

Black-headed gull: \[
\frac{\text{Wing length}}{\text{Wing width}} = \frac{83}{8} \approx 10.38
\]

Answer  The gull’s aspect ratio of 10.38 is the greatest. The eagle’s aspect ratio of 6.97 is the next greatest. The jay’s aspect ratio of 3.92 is the least.

In the Real World

Biology  Birds with high aspect ratios are better suited for gliding over long distances, while birds with low aspect ratios have adapted for rapid takeoffs and maneuverability. Which bird in Example 2 is best suited for gliding over long distances?

Rates  A rate is a ratio of two quantities measured in different units. A unit rate is a rate that has a denominator of 1 when expressed in fraction form. Unit rates are often expressed using the word per, which means “for every.”

Example 3  Finding a Unit Rate

Party  You host a party for 12 people. The food and drinks for the party cost $66. What is the cost per person?

Solution

First, write a rate comparing the total cost of the party to the number of people at the party. Then rewrite the rate so the denominator is 1.

\[
\frac{66}{12 \text{ people}} = \frac{66 \div 12}{12 \text{ people} \div 12} \quad \text{Divide numerator and denominator by 12.}
\]

\[
= \frac{5.50}{1 \text{ person}} \quad \text{Simplify.}
\]

Answer  The cost of food and drinks is $5.50 per person.
Example 4  Writing an Equivalent Rate

Jet  A jet flies 540 miles per hour. Write its rate in miles per minute.

Solution

To convert from miles per hour to miles per minute, multiply the rate by a conversion factor. There are 60 minutes in 1 hour, so \( \frac{1 \text{ h}}{60 \text{ min}} = 1 \).

\[
\frac{540 \text{ mi}}{1 \text{ h}} \cdot \frac{1 \text{ h}}{60 \text{ min}} = \frac{9 \cdot 60 \text{ mi}}{1 \text{ h}} \cdot \frac{1 \text{ h}}{60 \text{ min}} \quad \text{Multiply rate by conversion factor.}
\]

\[
= \frac{9 \text{ mi}}{1 \text{ min}} \quad \text{Divide out common factor and unit.}
\]

Answer  The jet travels at a rate of 9 miles per minute.

Example 5  Using Equivalent Rates

Robots  Engineers designed a miniature robot that can crawl through pipes and vents that humans can't access. The robot travels 1 inch in 3 seconds. How many feet does the robot travel in 4 minutes?

Solution

1. Express the robot's rate in inches per minute.

\[
\frac{1 \text{ in.}}{3 \text{ sec}} = \frac{1 \text{ in.}}{3 \text{ sec}} \cdot \frac{20 \text{ sec}}{1 \text{ min}} \quad \text{Multiply by conversion factor.}
\]

\[
= \frac{20 \text{ in.}}{1 \text{ min}} \quad \text{Divide out common factor and unit.}
\]

\[
= \frac{20 \text{ in.}}{1 \text{ min}} \quad \text{Simplify.}
\]

2. Find the distance (in feet) that the robot can travel in 4 minutes.

\[
\text{Distance} = \text{Rate} \cdot \text{Time}
\]

\[
= \frac{20 \text{ in.}}{1 \text{ min}} \cdot 4 \text{ min}
\]

\[
= 80 \text{ in.}
\]

\[
= \frac{20 \text{ in.}}{1 \text{ min}} \cdot \frac{1 \text{ ft}}{12 \text{ in.}}
\]

\[
= \frac{6 \frac{2}{3} \text{ ft}}{1}
\]

Answer  The robot travels 6\( \frac{2}{3} \) feet in 4 minutes.

Checkpoint

Write the equivalent rate.

2. \( \frac{5 \text{ cm}}{1 \text{ min}} = \frac{? \text{ m}}{1 \text{ h}} \)  
3. \( \frac{5 \text{ m}}{1 \text{ sec}} = \frac{? \text{ m}}{1 \text{ h}} \)  
4. \( \frac{3 \text{ lb}}{\text{ $1$}} = \frac{? \text{ oz}}{\text{ $1$}} \)
Guided Practice

**Vocabulary Check**
1. What is a unit rate? Give an example.
2. Write the ratio 8 to 5 in two other ways.

**Skill Check**
Tell whether the ratio is in simplest form. If not, write it in simplest form.
Then write the ratio in two other ways.
3. 8 to 6  
4. 7 to 26  
5. 39 : 13  
6. 120 : 64

Order the ratios from least to greatest.
7. 2 to 9, 1 : 7, \( \frac{7}{28} \), 2 to 6, \( \frac{3}{10} \)  
8. 1 to 3, \( \frac{2}{8} \), 5 : 18, 7 to 20, \( \frac{9}{25} \)

**Guided Problem Solving**
9. **Roses** Three decorators purchased bouquets of roses. Decorator A paid $120 for 5 bouquets that contained 25 roses each. Decorator B paid $204 for 20 bouquets that contained 12 roses each. Decorator C paid $180 for 40 bouquets that contained 6 roses each. Which decorator paid the least amount per rose?
   1. Find the total number of roses each decorator bought.
   2. Find the price per rose for each decorator.
   3. Compare the unit prices to determine which decorator paid the least per rose.

Practice and Problem Solving

Tell whether the ratio is in simplest form. If not, write it in simplest form.
Then write the ratio in two other ways.
10. 9 to 12  
11. 4 : 5  
12. \( \frac{15}{3} \)  
13. \( \frac{50}{6} \)
14. 63 : 18  
15. 24 : 8  
16. 64 to 3  
17. 28 to 10

Order the ratios from least to greatest.
18. \( \frac{4}{2} \), 11 to 2, 22 : 3, \( \frac{30}{4} \), 36 : 5  
19. \( \frac{15}{4} \), 19 to 5, \( \frac{53}{15} \), 4 : 1, 18 to 6
20. \( \frac{7}{11} \), 8 : 12, 6 : 10, \( \frac{1}{2} \), 7 : 4
21. \( \frac{22}{4} \), 65 : 12, 9 : 2, \( \frac{100}{19} \), 5 : 1

Find the unit rate.
22. \( \frac{140 \text{ words}}{4 \text{ min}} \)  
23. \( \frac{\$161}{7 \text{ shares}} \)  
24. \( \frac{80 \text{ oz}}{2.5 \text{ servings}} \)  
25. \( \frac{70 \text{ mi}}{5 \text{ h}} \)
26. \( \frac{\$320}{4 \text{ people}} \)  
27. \( \frac{26 \text{ points}}{3 \text{ quarters}} \)  
28. \( \frac{24 \text{ muffins}}{\$15} \)  
29. \( \frac{25 \text{ wins}}{40 \text{ games}} \)
Write the equivalent rate.

30. \( \frac{15 \text{ mi}}{1 \text{ h}} = ? \frac{\text{mi}}{1 \text{ h}} \)  
31. \( \frac{300 \text{ mi}}{20 \text{ sec}} = ? \frac{\text{mi}}{1 \text{ min}} \)  
32. \( \frac{390 \text{ m}}{1 \text{ min}} = ? \frac{\text{m}}{1 \text{ h}} \)  
33. \( \frac{33,000 \text{ dollars}}{1 \text{ year}} = ? \frac{\text{dollars}}{1 \text{ month}} \)  
34. \( \frac{43 \text{ dollars}}{1 \text{ day}} = ? \frac{\text{dollars}}{1 \text{ week}} \)  
35. \( \frac{45 \text{ min}}{2 \text{ mi}} = ? \frac{\text{h}}{1 \text{ mi}} \)

36. **Nature** As a tadpole, the paradoxical frog is 24 centimeters long. As an adult, the frog is 6 centimeters long.
   a. Write the ratio of the tadpole’s length to the adult frog’s length.
   b. Something is called **paradoxical** if it seems impossible. What is paradoxical about the frog? Explain using your answer to part (a).

37. **Estimate** A store sells 16 cookies for $11.88. Estimate the cost per cookie. Explain how you made your estimate.

38. **Guitar** You want to save all the money you earn to buy a guitar that costs $400. You earn $9 per hour and plan to work 15 hours each week for the next 3 weeks. Will you earn enough money in that time to buy the guitar?

39. **Extended Problem Solving** Your family used two full tanks of gasoline on a road trip. Your car drives about 25 miles per gallon, and the tank holds 12 gallons of gasoline.
   a. Find the approximate number of gallons of gasoline used on the trip.
   b. Find the approximate number of miles you drove on the trip.
   c. **Calculate** Assume gasoline costs $1.50 per gallon. How much did you spend per mile on gasoline?
   d. **Apply** You have $20 to spend on gasoline for another trip. The trip is 350 miles. You spend the same amount per mile on gasoline as on the first trip. Do you have enough money for gasoline? Explain.

40. **Drinks** A restaurant sells drinks in 3 sizes of cups: small, medium, and large. The small cup costs $.89 and holds 9 ounces. The medium cup costs $1.29 and holds 12 ounces. The large cup costs $1.59 and holds 15 ounces. Which size cup costs the least per ounce?

41. **Aquarium** An aquarium has twice as many angelfish as goldfish. The aquarium contains only angelfish and goldfish. Write a ratio for the number of goldfish to the total number of fish.

42. **Geometry** For each rectangle below, the measure of the longer side is the length, and the measure of the shorter side is the width.

   ![Rectangles A, B, and C]

   a. Which rectangle has the greatest ratio of length to width?
   b. For which rectangle is the ratio of length to width closest to 1:1?
   c. **Critical Thinking** The ratio of another rectangle’s length to its width is 1:1. What type of rectangle is it?
Find the ratio of the area of the shaded region to the area of the unshaded region. The figures are composed of squares and triangles.

43. \[ \text{ratio: } \frac{s}{2s} \]

44. \[ \text{ratio: } \frac{s}{2s} \]

45. **Running** In 2002, Khalid Khannouchi set the world record for a marathon when he ran 26.2 miles in 2 hours, 5 minutes, and 38 seconds. Round your answers to the nearest tenth.
   a. Find Khannouchi's rate in miles per hour.
   b. Find Khannouchi's rate in miles per minute.
   c. Find Khannouchi's rate in feet per minute.

46. **Earth Science** Due to the movement of Earth's landmasses, Los Angeles and other portions of coastal Southern California are moving northwest toward San Francisco at an average rate of 46 millimeters per year.
   a. How many meters per year does Los Angeles move?
   b. How many meters per century does Los Angeles move?
   c. In 2000, San Francisco was 554,000 meters from Los Angeles. In about how many years will Los Angeles be where San Francisco was in 2000?

47. **Challenge** If you travel 55 miles per hour, how many minutes will it take you to travel 1 mile?

**Mixed Review**

Find the product. *(Lesson 5.4)*

48. \( \frac{3}{8} \cdot \left( \frac{6}{15} \right) \)

49. \( \frac{-6}{21} \cdot \left( \frac{14}{54} \right) \)

50. \( -2 \frac{3}{4} \cdot \left( -3 \frac{5}{9} \right) \)

51. **Stamps** You have 15 stamps from Canada in your stamp collection. These stamps make up \( \frac{3}{11} \) of your entire collection. The rest of the stamps are from the U.S. How many stamps are in your collection? How many stamps from the U.S. do you have? *(Lesson 5.6)*

**Solve the inequality.** *(Lesson 5.7)*

52. \( \frac{2}{3}x + 9 \leq \frac{17}{2} \)

53. \( \frac{1}{5}y + 14 \leq \frac{13}{5} \)

54. \( -\frac{5}{9}x + 1 > -\frac{22}{27} \)

**Standardized Test Practice**

55. **Multiple Choice** Which of the following ratios is greater than 5 : 12?

56. **Short Response** One afternoon, you read 24 pages of a novel in 30 minutes. Another afternoon, you read 33 pages in 45 minutes. How can you decide whether you read at the same rate or at different rates on the two afternoons? On which afternoon did you read at a faster rate? Explain.
Writing and Solving Proportions

Vocabulary
proportion, p. 275

BEFORE
You wrote and compared ratios.

Now
You'll write and solve proportions.

WHY?
So you can find the salinity of saltwater, as in Ex. 31.

Elephants  Each day, an elephant eats 5 pounds of food for every 100 pounds of its body weight. How much does a 9300 pound elephant eat per day?
In Example 3, you’ll see how to use a proportion to answer this question.

Reading Algebra

The proportion \( \frac{2}{3} = \frac{8}{12} \) is read “2 is to 3 as 8 is to 12.”

**Proportions**

**Words** A proportion is an equation that states that two ratios are equivalent.

**Numbers** \( \frac{2}{3} = \frac{8}{12} \)

**Algebra** \( \frac{a}{b} = \frac{c}{d} \), where \( b \neq 0 \) and \( d \neq 0 \)

**Equivalent Ratios** If one of the numbers in a proportion is unknown, you can solve the proportion to find the unknown number. One way to solve a proportion is to use mental math to find an equivalent ratio.

**Example 1**  Solving a Proportion Using Equivalent Ratios

Solve the proportion \( \frac{5}{6} = \frac{x}{18} \).

1. Compare denominators.

   \[
   \frac{5}{6} \times 3 \rightarrow \frac{x}{18}
   \]

2. Find \( x \).

   \[
   \frac{5}{6} \times 3 \rightarrow \frac{x}{18}
   \]

Answer  Because \( 5 \times 3 = 15 \), \( x = 15 \).

**Checkpoint**

Use equivalent ratios to solve the proportion.

1. \( \frac{2}{7} = \frac{x}{21} \)  2. \( \frac{3}{8} = \frac{x}{32} \)  3. \( \frac{x}{2} = \frac{20}{10} \)  4. \( \frac{x}{48} = \frac{6}{12} \)
Using Algebra You can use the same methods you used to solve equations to solve proportions that have a variable in the numerator.

Example 2 Solving a Proportion Using Algebra

Solve the proportion \( \frac{x}{12} = \frac{2}{8} \). Check your answer.

\[
\frac{x}{12} = \frac{2}{8} \\
12 \cdot \frac{x}{12} = 12 \cdot \frac{2}{8} \\
x = \frac{24}{8} \\
x = 3
\]

\( \checkmark \) Check \( \frac{x}{12} = \frac{2}{8} \)

\[
\frac{3}{12} = \frac{2}{8} \\
\frac{1}{4} = \frac{1}{4} \checkmark
\]

\( \checkmark \) Checkpoint

Use algebra to solve the proportion.

5. \( \frac{2}{5} = \frac{x}{25} \)  
6. \( \frac{3}{10} = \frac{x}{100} \)  
7. \( \frac{x}{9} = \frac{42}{54} \)  
8. \( \frac{x}{4} = \frac{13}{2} \)

Setting up a Proportion There are different ways to set up a proportion. Consider the following problem.

Yesterday you rode your bike 18 miles in 2.5 hours. Today you plan to ride for 3.5 hours. If you ride at the same rate as yesterday, how far will you ride?

The tables below show two ways of arranging the information from the problem. In each table, \( x \) represents the number of miles that you can ride in 3.5 hours. The proportions follow from the tables.

<table>
<thead>
<tr>
<th>Today</th>
<th>Yesterday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles</td>
<td>( x )</td>
</tr>
<tr>
<td>Hours</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Proportion: \( \frac{x}{3.5} = \frac{18}{2.5} \)

<table>
<thead>
<tr>
<th>Miles</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Today</td>
<td>( x )</td>
</tr>
<tr>
<td>Yesterday</td>
<td>18</td>
</tr>
</tbody>
</table>

Proportion: \( \frac{x}{18} = \frac{3.5}{2.5} \)

When writing a proportion, make sure you use comparable ratios. For example, you cannot write a proportion to compare \( \frac{\text{miles}}{\text{hours}} \) and \( \frac{\text{hours}}{\text{miles}} \).
Example 3  Writing and Solving a Proportion

Use the information on page 275 to write and solve a proportion to determine how much food an elephant that weighs 9300 pounds eats per day.

Solution

First, write a proportion involving two ratios that compare the weight of the food with the weight of the elephant.

\[
\frac{5}{100} = \frac{x}{9300} \quad \text{Weight of food, Weight of elephant}
\]

Then, solve the proportion.

\[
9300 \cdot \frac{5}{100} = 9300 \cdot \frac{x}{9300} \quad \text{Multiply each side by 9300.}
\]

\[
\frac{46500}{100} = x \quad \text{Simplify.}
\]

\[
465 = x \quad \text{Divide.}
\]

Answer  A 9300 pound elephant eats about 465 pounds of food per day.

Checkpoint

9. Use the information given on page 275 to determine how much food a 12,500 pound elephant eats per day.

Guided Practice

Vocabulary Check
1. Give an example of a proportion that uses the numbers 2, 3, 4, and 6.
2. Explain how to use equivalent ratios to solve the proportion \( \frac{3}{2} = \frac{x}{12} \).

Skill Check
Solve the proportion.

3. \( \frac{1}{2} = \frac{x}{50} \)
4. \( \frac{3}{4} = \frac{y}{24} \)
5. \( \frac{a}{9} = \frac{21}{27} \)
6. \( \frac{b}{5} = \frac{28}{35} \)

7. Error Analysis Describe and correct the error in writing a proportion to find the cost of 30 pencils if 12 pencils cost $2.00.

8. Pizza You know that 3 pizzas are enough to feed 12 people. Write and solve a proportion to find the number of pizzas that will feed 28 people.
Use equivalent ratios to solve the proportion.

9. \( \frac{5}{6} = \frac{x}{30} \)  
10. \( \frac{6}{7} = \frac{y}{49} \)  
11. \( \frac{a}{12} = \frac{33}{36} \)  
12. \( \frac{b}{14} = \frac{27}{42} \)

13. \( \frac{14}{3} = \frac{a}{15} \)  
14. \( \frac{11}{9} = \frac{y}{81} \)  
15. \( \frac{x}{5} = \frac{200}{25} \)  
16. \( \frac{b}{15} = \frac{26}{30} \)

Use algebra to solve the proportion.

17. \( \frac{x}{8} = \frac{35}{56} \)  
18. \( \frac{y}{4} = \frac{42}{28} \)  
19. \( \frac{a}{32} = \frac{9}{16} \)  
20. \( \frac{b}{45} = \frac{8}{9} \)

21. \( \frac{25}{60} = \frac{c}{12} \)  
22. \( \frac{39}{54} = \frac{d}{18} \)  
23. \( \frac{17}{26} = \frac{w}{52} \)  
24. \( \frac{3}{7} = \frac{z}{63} \)

25. **School Supplies** At a store, 5 erasers cost $2.50. How many erasers can you buy for $7.50?

26. **Driving** You are driving 2760 miles across the country. During the first 3 days of your trip, you drive 1380 miles. If you continue to drive at the same rate each day, how many days will it take you to complete the trip?

27. **Lacrosse** Last season, a lacrosse player scored 41 points in 15 games. So far this season, the player has scored 24 points in 9 games.
   a. Does the player have a greater number of goals per game this season compared with last season?
   b. Suppose the player plays in as many games this season as last season and continues to score at this season's rate. Write and solve a proportion to find the number of goals the player will score this season.

28. **Exchange Rates** In 2003, the exchange rate between the United States and Canada was about 3 Canadian dollars to 2 U.S. dollars. Cindy had 78 U.S. dollars to exchange when she visited Canada. How many Canadian dollars could she get in exchange?

29. **Writing** Write a proportion without using any variables. In how many different ways can you rearrange the four numbers so that ratios are still equivalent? Explain your answer.

30. **Population Density** A region’s population density is the number of people per square mile. The tiny country of Monaco has the highest population density in the world, with 33,000 people living in an area of 0.75 square mile. The state of New York has a population of about 19,000,000 people living in an area of about 47,000 square miles. Use a calculator to complete the following. Round your answers to the nearest whole number.
   a. Write and solve a proportion to find how many people would live in Monaco if Monaco had the population density of New York.
   b. Write and solve a proportion to find how many people would live in New York if New York had the population density of Monaco.
31. **Saltwater** The salinity of saltwater is the ratio of the mass of the salt in the water to the mass of the salt and fresh water mixed together.
   a. A sample of saltwater is made by mixing 3 grams of salt with 75 grams of water. Find the salinity of the sample.
   b. A sample of saltwater has a salinity of 3:45. The sample has a mass of 30 kilograms. How much salt is in the sample?

32. **Jewel Cases** Store A sells 10 CD jewel cases for $9. Store B sells 15 CD jewel cases for $12. How much money will you save if you buy 30 CD jewel cases at the store with the lower unit price?

33. **Knitting** You are knitting an afghan with red, green, and blue stripes. There are equal numbers of red and blue stripes. There are twice as many green stripes as there are red stripes. The afghan has 20 stripes.
   a. Find the ratio of the number of red stripes to the total number of stripes on the afghan.
   b. How many red stripes are there on the afghan?

34. **Election** In an election, the winning candidate received 3 votes for every vote the opponent received. Altogether, 1000 votes were cast. How many votes did the winner receive?

35. **Critical Thinking** In the proportion \(\frac{10}{x} = \frac{y}{6}\), how does the value of \(y\) change as the value of \(x\) increases?

36. **Challenge** A painter is making a specific shade of green that requires 3 parts of yellow paint for every 4 parts of blue paint. To make the mixture, the painter uses 9 ounces of yellow paint and 2 tubes of blue paint. How many ounces are in each tube of blue paint?

---

**Mixed Review**

Simplify the expression. *(Lesson 4.5)*

37. \(\frac{8m^3 \cdot 9m^4}{3m^2}\)  
38. \(\frac{7n^3 \cdot n^2}{n^4}\)  
39. \(\frac{5a^2 \cdot 2a^2}{10a^4}\)  
40. \(\frac{2x^4 \cdot x^3}{6x^5}\)

Find the quotient. *(Lesson 5.5)*

41. \(-\frac{\frac{3}{20}}{\frac{4}{5}}\)  
42. \(\frac{\frac{15}{16}}{\frac{-\frac{5}{8}}{}}\)  
43. \(\frac{\frac{11}{42}}{\frac{4}{7}}\)  
44. \(\frac{\frac{25}{36}}{\frac{8}{9}}\)

Solve the inequality. *(Lesson 5.7)*

45. \(\frac{3}{8} - \frac{4}{5} > \frac{9}{10}\)  
46. \(\frac{1}{3} < \frac{6}{7}y - \frac{7}{15}\)  
47. \(\frac{1}{3} \geq \frac{7}{12}x - \frac{11}{15}\)  
48. \(-\frac{2}{5}x + \frac{6}{5} \geq \frac{1}{10}\)

---

**Standardized Test Practice**

49. **Multiple Choice** Solve the proportion \(\frac{48}{28} = \frac{x}{63}\).
   A. \(\frac{51}{3}\)  
   B. 96  
   C. 108  
   D. 432

50. **Short Response** A 12 ounce box of pasta costs $0.99. A 2 pound box costs $2.09. Which box has the lower price per ounce? You buy 6 pounds of pasta. How much money do you save if you buy pasta in the box with the lower price per ounce? Explain.
Solving Proportions Using Cross Products

**Vocabulary**
cross product, p. 280

**BEFORE**
You solved simple proportions.

**Now**
You’ll solve proportions using cross products.

**WHY?**
So you can find the mass of gold in a ring, as in Ex. 35.

Every pair of ratios has two *cross products*. A **cross product** of two ratios is the product of the numerator of one ratio and the denominator of the other ratio.

**Ratios:**

\[
\frac{3}{5} \quad \frac{6}{10} \\
\frac{2}{3} \quad \frac{6}{11}
\]

**Cross products:**

\[3 \cdot 10 \quad 5 \cdot 6 \quad 2 \cdot 11 \quad 3 \cdot 6\]

Notice that for the ratios \(\frac{3}{5}\) and \(\frac{6}{10}\), the ratios are equal and their cross products are also equal. For the ratios \(\frac{2}{3}\) and \(\frac{6}{11}\), the ratios are not equal, and neither are their cross products.

You can use cross products to tell whether two ratios form a proportion. If the cross products are equal, then the ratios form a proportion.

**Example 1**

**Determining if Ratios Form a Proportion**

Tell whether the ratios form a proportion.

a. \(\frac{9}{51} \div \frac{6}{34}\)

b. \(\frac{12}{20} \div \frac{32}{50}\)

**Solution**

a. \(\frac{9}{51} \div \frac{6}{34}\) Write proportion.

\[9 \cdot 34 \div 51 \cdot 6 \quad \text{Form cross products.}\]

\[306 = 306 \quad \text{Multiply.}\]

**Answer** The ratios form a proportion.

b. \(\frac{12}{20} \div \frac{32}{50}\) Write proportion.

\[12 \cdot 50 \div 20 \cdot 32 \quad \text{Form cross products.}\]

\[600 \neq 640 \quad \text{Multiply.}\]

**Answer** The ratios do not form a proportion.
**Checkpoint**

Tell whether the ratios form a proportion.

1. \( \frac{6}{14} : \frac{3}{7} \)
2. \( \frac{14}{35} : \frac{8}{20} \)
3. \( \frac{6}{11} : \frac{9}{16} \)
4. \( \frac{15}{24} : \frac{10}{16} \)

You can use the multiplication property of equality to demonstrate an important property about the cross products of a proportion.

\[
\frac{a}{b} = \frac{c}{d} \quad \text{Given}
\]

\[
\frac{\frac{a}{b}}{\frac{1}{d}} = \frac{\frac{c}{d}}{\frac{1}{1}} \quad \text{Multiply each side by } bd.
\]

\[
\frac{ad}{bd} = \frac{cb}{bd} \quad \text{Divide out common factors.}
\]

\[
ad = cb \quad \text{Simplify.}
\]

This result proves the following property.

**Cross Products Property**

**Words** The cross products of a proportion are equal.

**Numbers** Given that \( \frac{2}{5} = \frac{6}{15} \), you know that \( 2 \cdot 15 = 5 \cdot 6 \).

**Algebra** If \( \frac{a}{b} = \frac{c}{d} \), where \( b \neq 0 \) and \( d \neq 0 \), then \( ad = bc \).

You can use the cross products property to solve proportions.

**Example 2**

**Writing and Solving a Proportion**

**Hair Growth** Human hair grows about 0.7 centimeter in 2 weeks. How long does hair take to grow 14 centimeters?

**Solution**

\[
\frac{0.7}{2} = \frac{14}{x} \quad \text{Length of hair grown}
\]

\[
0.7 \cdot x = 2 \cdot 14 \quad \text{Cross products property}
\]

\[
0.7x = 28 \quad \text{Multiply.}
\]

\[
\frac{0.7x}{0.7} = \frac{28}{0.7} \quad \text{Divide each side by } 0.7.
\]

\[
x = 40 \quad \text{Simplify.}
\]

**Answer** Hair takes about 40 weeks to grow 14 centimeters.

**Checkpoint**

Use the cross products property to solve the proportion.

5. \( \frac{18}{42} = \frac{3}{r} \)
6. \( \frac{16}{p} = \frac{10}{45} \)
7. \( \frac{9}{b} = \frac{1.5}{7} \)
8. \( \frac{0.4}{6} = \frac{18}{z} \)
Note Worthy

The main ideas from Lessons 6.2 and 6.3 are summarized at the right. You may want to include a summary like this one in your notes.

### Summary

#### Methods for Solving a Proportion

To solve the proportion \( \frac{5}{12} = \frac{x}{36} \), use one of the following:

**Equivalent ratios**

\[
\frac{5}{12} \times 3 \rightarrow \frac{x}{36} \quad \frac{5}{12} \times 3 \rightarrow \frac{15}{36}
\]

**Algebra**

\[
36 \cdot \frac{5}{12} = 36 \cdot \frac{x}{36}
\]

Multiply each side by 36.

15 = x

Simplify.

**Cross products**

\[
5 \cdot 36 = 12x
\]

Cross products property

15 = x

Divide each side by 12.

### Exercises

**6.3 Exercises**

More Practice, p. 808

**Guided Practice**

**Vocabulary Check**

1. Find the cross products of the proportion \( \frac{3}{4} = \frac{9}{12} \).

2. Explain how to use cross products to determine if two ratios are equal.

**Skill Check**

Tell whether the ratios form a proportion.

3. \( \frac{5}{8} \frac{10}{16} \)

4. \( \frac{9}{32} \frac{3}{8} \)

5. \( \frac{40}{125} \frac{8}{25} \)

6. \( \frac{6}{9} \frac{12}{16} \)

Use the cross products property to solve the proportion.

7. \( \frac{24}{36} = \frac{2}{x} \)

8. \( \frac{60}{15} = \frac{12}{y} \)

9. \( \frac{0.8}{a} = \frac{3.2}{8} \)

10. \( \frac{1.6}{b} = \frac{8}{25} \)

**Guided Problem Solving**

11. **Long Distance**

You made a 12 minute phone call using a calling card. The call cost $0.66. There is $1.21 left on your calling card. The cost per minute of long distance calls is constant. How many more minutes can you talk long distance using your calling card?

   1. Write a ratio of the form \( \frac{\text{Cost of phone call}}{\text{Minutes of phone call}} \) for the phone call.

   2. Let \( m \) represent the number of minutes you can talk for $1.21. Write a ratio of the same form as the one in Step 1.

   3. Use the two ratios to write a proportion. Solve the proportion.
Tell whether the ratios form a proportion.

12. \( \frac{12}{30} : \frac{18}{45} \)
13. \( \frac{42}{20} : \frac{63}{60} \)
14. \( \frac{8}{25} : \frac{42}{6} \)
15. \( \frac{45}{35} : \frac{9}{21} \)

16. \( \frac{40}{210} : \frac{60}{630} \)
17. \( \frac{588}{105} : \frac{84}{20} \)
18. \( \frac{70}{147} : \frac{50}{105} \)
19. \( \frac{75}{15} : \frac{40}{8} \)

Solve the proportion.

20. \( \frac{16}{36} = \frac{4}{d} \)
21. \( \frac{3}{21} = \frac{c}{35} \)
22. \( \frac{30}{w} = \frac{24}{12} \)
23. \( \frac{35}{z} = \frac{7}{5} \)

24. \( \frac{144}{40} = \frac{x}{5} \)
25. \( \frac{9}{105} = \frac{15}{y} \)
26. \( \frac{r}{12} = \frac{20}{8} \)
27. \( \frac{s}{21} = \frac{16}{12} \)

28. \( \frac{7}{m} = \frac{0.6}{3} \)
29. \( \frac{26}{p} = \frac{13}{0.4} \)
30. \( \frac{51}{3.4} = \frac{n}{4} \)
31. \( \frac{1.4}{1.05} = \frac{4}{r} \)

32. **Drink Mix** A store is selling powdered drink mix in two different sizes. You can buy 10 ounces for $2.25, or you can buy 35 ounces for $7. Are the two rates equivalent? Explain.

33. **Gasoline** You paid $5 at a gas station for 3 gallons of gasoline.
   a. How much would 12 gallons of gasoline cost?
   b. How much gasoline can you buy for $30?

34. **Biking** You travel 24 miles in 2 hours while biking.
   a. At this rate, how far can you bike in 5 hours?
   b. At this rate, how long will it take to bike 30 miles? Write your answer in hours and minutes.

35. **Gold** Jewelers often mix gold with other metals. A **karat** is a unit of measure that compares the mass of the gold in an object with the mass of the object. Karats are expressed as a number that is understood to be the numerator of a ratio whose denominator is 24. For example, 24 karat gold means an object is pure gold, and 18 karat gold means that \( \frac{18}{24} \), or \( \frac{3}{4} \), of the object’s mass is gold.
   a. A 15 karat gold ring has a mass of 200 grams. How much gold is in the ring?
   b. An 18 karat gold bracelet contains 27 grams of gold. What is the mass of the bracelet?

36. **Writing** Describe three ways you could solve the proportion \( \frac{6}{10} = \frac{x}{40} \).

Find the value of \( x \).

37. \( \frac{36}{54} = \frac{18}{x + 5} \)
38. \( \frac{39}{x + 7} = \frac{21}{7} \)
39. \( \frac{15 - x}{45} = \frac{15}{75} \)
40. \( \frac{28}{16} = \frac{x - 8}{20} \)

41. **Critical Thinking** Use the cross products property to show that if \( \frac{a}{b} = \frac{c}{d} \), then \( \frac{d}{c} = \frac{b}{a} \).
42. **Extended Problem Solving** The tables show ingredients needed to make colored glass. Use the numbers in the *Parts* columns to form ratios comparing the masses of ingredients. For example, if you use 65 grams of sand to make yellow glass, you need 3 grams of chalk.

<table>
<thead>
<tr>
<th>Red Glass</th>
<th>Yellow Glass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ingredients</td>
<td>Parts</td>
</tr>
<tr>
<td>Sand</td>
<td>50</td>
</tr>
<tr>
<td>Red lead</td>
<td>100</td>
</tr>
<tr>
<td>Copper oxide</td>
<td>3</td>
</tr>
<tr>
<td>Ferric oxide</td>
<td>3</td>
</tr>
</tbody>
</table>

**a. Calculate** A piece of red glass contains 60 grams of sand. How much ferric oxide does it contain?

**b. Calculate** A piece of yellow glass contains 31 kilograms of soda ash. How much chalk does it contain?

**c. Compare** Which has more sand: red glass with 200 grams of red lead, or yellow glass with 4 grams of wood charcoal? Explain.

43. **Critical Thinking** The ratio $\frac{a}{b}$ is equivalent to $\frac{3}{4}$. The ratio $\frac{b}{c}$ is equivalent to $\frac{4}{5}$. What is the ratio $\frac{a}{c}$ equivalent to? Explain.

44. **Challenge** For a half circle like the one shown below, if you know lengths $a$ and $b$, then length $x$ can be found by solving $\frac{a}{x} = \frac{x}{b}$.

- **a.** Let $a = 9$ and $b = 4$. What is the value of $x$?
- **b.** Let $a = 18$ and $b = 8$. What is the value of $x$?
- **c.** In terms of $x$, what does $ab$ equal?

**Mixed Review**

**Use a ruler to draw a segment with the given length.** (*p. 787*)

- 45. 1.2 centimeters
- 46. 3.5 centimeters
- 47. 0.2 centimeters

**Write the number in scientific notation.** (*Lesson 4.7*)

- 48. 34,000,000,000
- 49. 5,001,000
- 50. 0.000000000672

**Write the ratio in simplest form.** (*Lesson 6.1*)

- 51. 8 to 18
- 52. 6 : 22
- 53. $\frac{14}{24}$

**Standardized Test Practice**

**54. Multiple Choice** Solve the proportion $\frac{90}{y} = \frac{27}{12}$.

- A. 3.6
- B. 40
- C. 90
- D. 1080

**55. Multiple Choice** Last week you saved $24. At this rate, how many weeks will it take you to save $600?

- F. 24 days
- G. 14,400 weeks
- H. 25 weeks
- I. 15 weeks
Basic Geometry Concepts

**Points, Lines, and Planes**

<table>
<thead>
<tr>
<th>Word</th>
<th>Notation</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>point</td>
<td>point $A$</td>
<td><img src="point.png" alt="" /></td>
</tr>
<tr>
<td>line</td>
<td>$\overrightarrow{BC}$</td>
<td><img src="line.png" alt="" /></td>
</tr>
<tr>
<td>plane</td>
<td>$M$</td>
<td><img src="plane.png" alt="" /></td>
</tr>
</tbody>
</table>

**Example** Use the diagram to name three points, two lines, and a plane.

Three points are point $J$, point $K$, and point $L$.
Two lines are $\overrightarrow{KL}$ and $\overrightarrow{JP}$.
The plane is plane $P$.

---

**Segments, Rays, and Angles**

<table>
<thead>
<tr>
<th>Word</th>
<th>Notation</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>line segment, or segment length of a line segment</td>
<td>$\overline{AB} \quad AB$</td>
<td><img src="segment.png" alt="" /></td>
</tr>
<tr>
<td>ray</td>
<td>$\overrightarrow{CD}$</td>
<td><img src="ray.png" alt="" /></td>
</tr>
<tr>
<td>angle measure of an angle</td>
<td>$\angle EFG$ or $\angle F$ $m\angle EFG$ or $m\angle F$</td>
<td><img src="angle.png" alt="" /></td>
</tr>
</tbody>
</table>

**Example** Use the diagram to name two segments and their lengths, two rays, and an angle and its measure.

A segment is $\overline{MP}$, and $MP = 4$ centimeters.
Another segment is $\overline{NP}$, and $NP = 6$ centimeters.
Two rays are $\overrightarrow{PM}$ and $\overrightarrow{PN}$.
An angle is $\angle P$, and $m\angle P = 80^\circ$. 

Continued
**Triangles, Quadrilaterals, and Congruent Parts**

<table>
<thead>
<tr>
<th>Word</th>
<th>Notation</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>triangle</td>
<td>$\triangle ABC$</td>
<td>![Triangle Diagram]</td>
</tr>
<tr>
<td>A <strong>quadrilateral</strong> is made of four segments that intersect only at their endpoints.</td>
<td>quadrilateral $PQRS$</td>
<td>![Quadrilateral Diagram]</td>
</tr>
</tbody>
</table>

**Congruent segments** have equal lengths, and **congruent angles** have equal measures. Congruent sides of a figure are marked using tick marks, and congruent angles of a figure are marked using arcs. The symbol for congruence is $\cong$.

**Example** Identify the angles, sides, congruent angles, and congruent sides of the triangle.

The angles of the triangle are $\angle X$, $\angle Y$, and $\angle Z$.

The sides of the triangle are $\overline{XY}$, $\overline{YZ}$, and $\overline{XZ}$.

Congruent angles: $\angle X \cong \angle Z$

Congruent sides: $\overline{XY} \cong \overline{YZ}$

---

**Checkpoint**

**In Exercises 1–6, use figure 1.**
1. Name three points.
2. Name two lines.
3. Name two planes.
4. Name two rays.
5. Name a segment.
6. Name an angle and give its measure.

**In Exercises 7–10, use figure 2.**
7. Name the quadrilateral.
8. Name the sides.
9. Name the angles.
10. Identify the congruent angles and congruent sides.
Two figures are similar if they have the same shape but not necessarily the same size. For example, when a figure is enlarged, the enlarged figure is similar to the original figure.

**Investigate**

**Compare corresponding parts of a figure and its enlargement.**

1. Draw \( \triangle ABC \) so that \( m\angle A = 90^\circ \), \( AB = 1.5 \) cm, and \( AC = 2 \) cm. Find \( BC \) to the nearest 0.1 cm.

2. Draw \( \triangle DEF \) so that \( m\angle D = 90^\circ \), \( DE = 2 \cdot AB \), and \( DF = 2 \cdot AC \). Find the side lengths of \( \triangle DEF \).

3. Use a protractor to find the measures of the angles of both triangles to the nearest degree.

**Draw Conclusions**

1. Copy and complete the tables.

<table>
<thead>
<tr>
<th>Side of ( \triangle ABC )</th>
<th>Corresponding side of ( \triangle DEF )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( AB = 1.5 ) cm</td>
<td>( DE = ? )</td>
</tr>
<tr>
<td>( AC = 2 ) cm</td>
<td>( DF = ? )</td>
</tr>
<tr>
<td>( BC = ? )</td>
<td>( EF = ? )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle of ( \triangle ABC )</th>
<th>Corresponding angle of ( \triangle DEF )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m\angle A = 90^\circ )</td>
<td>( m\angle D = 90^\circ )</td>
</tr>
<tr>
<td>( m\angle B = ? )</td>
<td>( m\angle E = ? )</td>
</tr>
<tr>
<td>( m\angle C = ? )</td>
<td>( m\angle F = ? )</td>
</tr>
</tbody>
</table>

2. For each pair of corresponding sides, find the ratio of the length of a side of \( \triangle ABC \) to the length of the corresponding side of \( \triangle DEF \). What do you notice about these ratios?

3. What do you notice about the measures of the corresponding angles?

4. **Conjecture** Use your answers to Exercises 2 and 3 to write two conjectures about similar figures.
Two figures are similar figures if they have the same shape but not necessarily the same size. The symbol \( \sim \) indicates that two figures are similar. When working with similar figures, you should identify the corresponding parts of the figures. Corresponding parts of figures are sides or angles that have the same relative position.

\[ \triangle ABC \sim \triangle DEF \]

\[ \triangle XYZ \text{ is not similar to } \triangle UVW \]

**Properties of Similar Figures**

1. Corresponding angles of similar figures are congruent.
   \[ \angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F \]

2. The ratios of the lengths of corresponding sides of similar figures are equal.
   \[ \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{2} \]

**Example 1**

Identifying Corresponding Parts of Similar Figures

Given \( \triangle LMN \sim \triangle PQR \), name the corresponding angles and the corresponding sides.

**Solution**

**Corresponding angles:** \( \angle L \) and \( \angle P \), \( \angle M \) and \( \angle Q \), \( \angle N \) and \( \angle R \)

**Corresponding sides:** \( \overline{LM} \) and \( \overline{PQ} \), \( \overline{MN} \) and \( \overline{QR} \), \( \overline{LN} \) and \( \overline{PR} \)
**Checkpoint**

1. Given \(ABCD \sim WXYZ\), name the corresponding angles and the corresponding sides.

**Example 2**

**Finding the Ratio of Corresponding Side Lengths**

Given \(ABCD \sim JKL\), find the ratio of the lengths of corresponding sides of \(ABCD\) to \(JKL\).

Write a ratio comparing the lengths of a pair of corresponding sides. Then substitute the lengths of the sides and simplify.

\[
\frac{AB}{JK} = \frac{8}{12} = \frac{2}{3}
\]

**Answer** The ratio of the lengths of the corresponding sides is \(\frac{2}{3}\).

**Example 3**

**Checking for Similarity**

**Soccer** A soccer field is a rectangle that is 70 yards long and 40 yards wide. The penalty area of the soccer field is a rectangle that is 35 yards long and 14 yards wide. Is the penalty area similar to the field?

**Solution**

Because all rectangles have four right angles, the corresponding angles are congruent. To decide if the rectangles are similar, determine whether the ratios of the lengths of corresponding sides are equal.

\[
\frac{\text{Length of field}}{\text{Length of penalty area}} = \frac{\text{Width of field}}{\text{Width of penalty area}}
\]

\[
\frac{70}{35} = \frac{40}{14}
\]

\[
70 \cdot 14 = 35 \cdot 40
\]

\[
980 \neq 1400
\]

**Answer** The ratios of the lengths of corresponding sides are not equal, so the penalty area is not similar to the field.

**Checkpoint**

2. Inside the penalty area of the soccer field in Example 3 is a smaller rectangle known as the goal area. The goal area of a soccer field is 19 yards long and 6 yards wide. Is the goal area similar to the penalty area? Explain.
**Congruent Figures** Two figures are **congruent** if they have the same shape *and* the same size. If two figures are congruent, then the corresponding angles are congruent and the corresponding sides are congruent. Congruent figures are also similar.

In the diagram, \( \triangle JKL \cong \triangle PQR \) because:

1. \( \angle J \cong \angle P, \angle K \cong \angle Q, \) and \( \angle L \cong \angle R. \)
2. \( JK \cong PQ, KL \cong QR, \) and \( JL \cong PR. \)

**Example 4**

**Finding Measures of Congruent Figures**

Given \( ABCD \cong WXYZ, \) find the indicated measure.

a. \( WZ \)

b. \( m\angle W \)

**Solution**

Because the quadrilaterals are congruent, the corresponding angles are congruent and the corresponding sides are congruent.

a. \( WZ \cong AD. \) So, \( WZ = AD = 12 \text{ m}. \)

b. \( \angle W \cong \angle A. \) So, \( m\angle W = m\angle A = 105^\circ. \)

### Guided Practice

**Vocabulary Check**

1. What do you know about the corresponding angles and corresponding sides of two figures that are congruent?

2. Given \( \triangle JKL \sim \triangle PQR, \) identify all pairs of corresponding sides and corresponding angles.

**Skill Check**

In Exercises 3–6, \( \triangle ABC \sim \triangle DEF. \)

3. Identify all corresponding sides and corresponding angles.

4. Find the ratio of the lengths of corresponding sides.

5. Find \( m\angle B. \)

6. **Error Analysis** Describe and correct the error in writing another similarity statement for the triangles.
**Practice and Problem Solving**

Name the corresponding angles and the corresponding sides.

7. $\triangle ABC \sim \triangle DEF$

8. $JKLM \cong XWYZ$

The figures are similar. Find the ratio of the lengths of corresponding sides of figure A to figure B.

9.

10.

11.

12.

Given $RSTU \cong ABCD$, find the indicated measure.

13. $m\angle R$

14. $m\angle B$

15. $AB$

**Critical Thinking** Copy and complete the statement using always, sometimes, or never. Explain your answer.

16. Congruent figures are ? similar. 17. Similar figures are ? congruent.

18. Two squares are ? similar. 19. Two rectangles are ? congruent.

20. **Screens** The table shows the heights and widths of various rectangular viewing screens. Use the table to complete the following.

a. Are the two computer screens similar? Explain.

b. Is the television screen similar to either computer screen? Explain.

c. **Compare** Compare the height to width ratios of the high definition TV and movie screen.

<table>
<thead>
<tr>
<th>Item</th>
<th>Height</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>Television</td>
<td>18 in.</td>
<td>24 in.</td>
</tr>
<tr>
<td>Computer 1</td>
<td>9 in.</td>
<td>12 in.</td>
</tr>
<tr>
<td>Computer 2</td>
<td>12 in.</td>
<td>15 in.</td>
</tr>
<tr>
<td>High definition TV</td>
<td>48 in.</td>
<td>27 in.</td>
</tr>
<tr>
<td>Movie screen</td>
<td>32 ft</td>
<td>18 ft</td>
</tr>
</tbody>
</table>
21. **Money**  It is illegal to reproduce a genuine U.S. bill except according to the following rule: Every side length of the reproduction must be less than \(\frac{3}{4}\) times, or greater than \(1\frac{1}{2}\) times, the corresponding side length of a genuine bill. Genuine bills are 6.14 inches long and 2.61 inches wide.

a. Is it legal to make a reproduction that is 9.41 inches long and 4.00 inches wide? Explain.

b. Is it legal to make a reproduction that is 4.00 inches long and 1.70 inches wide? Explain.

c. If a reproduction is 2 feet long and 9 inches wide, is it similar to a genuine U.S. bill?

22. **Extended Problem Solving**  Use the similar rectangles to complete the following.

![Diagram of rectangles A, B, and C with dimensions 1x2, 2x4, and 3x6 respectively.]

a. Copy and complete the table.

<table>
<thead>
<tr>
<th>Figures</th>
<th>Ratio of side lengths</th>
<th>Ratio of areas</th>
</tr>
</thead>
<tbody>
<tr>
<td>A to B</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>A to C</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>B to C</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

b. **Writing** Explain how the ratio of the areas of similar rectangles is related to the ratio of the lengths of corresponding sides.

c. **Predict** Rectangle D is similar to rectangle A. The ratio of a side length of rectangle D to a corresponding side length of rectangle A is 10 : 1. Predict the area of rectangle D. Explain your thinking.

23. **Challenge**  Draw \(\triangle ABC\) and \(\triangle DEF\) so that \(\triangle ABC\) is congruent to both \(\triangle DEF\) and \(\triangle DFE\).

**Mixed Review**

Write the fraction in simplest form. *(Lesson 4.3)*

24. \(\frac{48}{64}\)

25. \(\frac{90}{108}\)

26. \(\frac{7ab^5}{21a^2}\)

27. \(\frac{24x}{60x^2y^3}\)

28. A bird flies 266 miles in 19 hours. Find the bird’s speed in miles per hour. *(Lesson 6.1)*

29. You can hike 6.5 miles in 2 hours. At this rate, how long will it take you to hike 19.5 miles? *(Lesson 6.3)*

**Standardized Test Practice**

30. **Multiple Choice** If quadrilateral \(ABCD \cong\) quadrilateral \(GHEF\), which angle is congruent to \(\angle C\)?

A. \(\angle E\)  B. \(\angle F\)  C. \(\angle G\)  D. \(\angle H\)

31. **Short Response**  A tablecloth is spread over a 5 foot by 3 foot rectangular table. The tablecloth extends 1 foot beyond the table’s surface on each side. Is the tablecloth similar to the surface of the table? Explain.
**Cactus**  A man who is 6 feet tall is standing near a saguaro cactus. The length of the man’s shadow is 2 feet. The cactus casts a shadow 5 feet long. How tall is the cactus?

In Example 2, you will see how to use similar triangles to measure the cactus’s height indirectly.

---

**Example 1**  *Finding an Unknown Side Length in Similar Figures*

Given $ABCD \sim EFGH$, find $EH$.

**Solution**

Use the ratios of the lengths of corresponding sides to write a proportion involving the unknown length, $EH$.

\[
\frac{BC}{FG} = \frac{AD}{EH}
\]

Write proportion involving $EH$.

\[
\frac{12}{30} = \frac{16}{x}
\]

Substitute.

\[
12x = 30 \cdot 16
\]

Cross products property

\[
12x = 480
\]

Multiply.

\[
x = 40
\]

Divide each side by 12.

**Answer**  The length of $EH$ is 40 inches.

---

**Checkpoint**

1. Given $\triangle STU \sim \triangle DEF$, find $DF$.
2. Given $JKLM \sim PQRS$, find $PQ$.
**Indirect Measurement** You can use similar figures to find lengths that are difficult to measure directly.

**Example 2** Using Indirect Measurement

Use indirect measurement to find the height of the cactus described on page 293. The cactus and the man are perpendicular to the ground. The sun's rays strike the cactus and the man at the same angle, forming two similar triangles.

**Solution**

Write and solve a proportion to find the height $h$ (in feet) of the cactus.

$$\frac{\text{Height of cactus}}{\text{Height of man}} = \frac{\text{Length of cactus's shadow}}{\text{Length of man's shadow}}$$

$$\frac{h}{6} = \frac{5}{2} \quad \text{Substitute.}$$

$$2h = 6 \cdot 5 \quad \text{Cross products property}$$

$$2h = 30 \quad \text{Multiply.}$$

$$h = 15 \quad \text{Divide each side by 2.}$$

**Answer** The cactus has a height of 15 feet.

**Checkpoint**

3. A cactus is 5 feet tall and casts a shadow that is 1.5 feet long. How tall is a nearby cactus that casts a shadow that is 8 feet long?

**Example 3** Using Algebra and Similar Triangles

Given $\triangle ABC \sim \triangle DEC$, find $BE$.

To find $BE$, write and solve a proportion.

$$\frac{AB}{DE} = \frac{BC}{EC}$$

$$\frac{20}{15} = \frac{x + 36}{36} \quad \text{Write proportion.}$$

$$20 \cdot 36 = 15(x + 36) \quad \text{Use the fact that } BC = BE + EC.$$  

$$720 = 15x + 540 \quad \text{Cross products property}$$

$$180 = 15x \quad \text{Multiply.}$$

$$12 = x \quad \text{Subtract 540 from each side.}$$

**Answer** The length of $BE$ is 12 inches.
Guided Practice

Vocabulary Check
1. \( \triangle EFGH \sim \triangle JKL \). Which side of \( EFGH \) corresponds with \( \overline{JM} \)?
2. Describe how similar triangles are useful for indirect measurement.

In Exercises 3 and 4, \( \triangle ABC \sim \triangle DEF \).

Skill Check
3. Find \( EF \).
4. Find \( FD \).

Guided Problem Solving
5. Palm Tree A man who is 74 inches tall stands beside a palm tree. The length of the man's shadow is 26 inches. The palm tree's shadow is 80 inches long. How tall is the palm tree?

1) The rays of the sun create similar triangles for the man and the palm tree. Write a proportion using the triangles.

2) Find the height of the palm tree to the nearest inch.

Practice and Problem Solving

6. Given \( \triangle ABC \sim \triangle LMN \), find \( LN \).

7. Given \( LMNP \sim QRST \), find \( RS \).

8. Given \( ABCD \sim KLMJ \), find \( KL \).

9. Given \( ABCD \sim RSTU \), find \( UR \).

Lesson 6.5 Similarity and Measurement
Find the length of \( \overline{DE} \).

10. \( \triangle ABC \sim \triangle ADE \)

11. \( \triangle ABC \sim \triangle AGF \)

12. **Bryce Canyon** Bryce Canyon National Park in Utah is known for its unusual rock formations. One rock casts a shadow 21 feet long. A girl standing near this rock is 5 feet 3 inches tall and casts a shadow 7 feet long.

   a. **Convert** Write the girl’s height in inches.

   b. Write and solve a proportion to find the height of the rock in feet and inches.

13. **Poster** You are enlarging a photograph to make a poster. The poster will be similar to the original photograph. The photograph is 6 inches tall and 4 inches wide. The poster will be 2.5 feet wide. How tall will the poster be? Find the poster’s perimeter.

14. **Surveying** You can use indirect measurement to find the distance across a river by following these steps.

   1. Start at a point \( A \) directly across the river from a landmark, such as a tree, at point \( B \).

   2. Walk 30 feet along the river to point \( C \) and place a stake.

   3. Walk 20 feet farther along the river to point \( D \).

   4. Turn and walk directly away from the river. Stop at point \( E \), where the stake you planted lines up with the landmark. \( \triangle ABC \sim \triangle DEC \).

Suppose you walk 24 feet away from the river along \( \overline{DE} \) before the stake lines up with the landmark. Write and solve a proportion to find the distance \( \overline{AB} \) across the river.
15. In the figure, \( \triangle ABC \), \( \triangle ADE \), and \( \triangle AFG \) are all similar.
   a. Find \( DE \) and \( FG \).
   b. Find \( AE \) and \( AG \).

16. **Critical Thinking** Given \( \triangle ABC \sim \triangle DEF \), tell whether the given information is enough to find the specified measurements. Explain your thinking.
   a. You know \( AB \), \( BC \), \( CA \), and \( DE \). You want to find \( EF \) and \( FD \).
   b. You know \( AB \), \( BC \), and \( FD \). You want to find \( CA \).
   c. You know \( m \angle B \). You want to find \( m \angle E \).

17. **Challenge** A rectangular box has a length of 3 inches, a width of 2 inches, and a height of 4 inches. Find the dimensions of three similar boxes: one that has a length of 6 inches, one that has a width of 6 inches, and one that has a height of 6 inches.

**Mixed Review**

Solve the equation. *(Lesson 5.6)*

18. \( \frac{5}{7}x = 16 \)  
19. \( -\frac{4}{9}x = 12 \)  
20. \( \frac{3}{4}x + \frac{5}{8} = 1 \)  
21. \( \frac{1}{2}x + 6 = \frac{4}{5} \)

Order the ratios from least to greatest. *(Lesson 6.1)*

22. \( 5 : 3, 6 : 4, \frac{11}{3}, 3 \) to \( 1, 33 : 100 \)  
23. \( 15 \) to \( 9, 35 : 25, \frac{44}{33}, \frac{22}{20}, 8 : 3 \)

**Standardized Test Practice**

In Exercises 24 and 25, \( \triangle RST \sim \triangle VUT \).

**24. Multiple Choice** Find \( UV \).
   
   A. 3 yards  
   B. 4 yards  
   C. 4.5 yards  
   D. 5 yards

**25. Multiple Choice** Find \( ST \).
   
   F. 8 yards  
   G. 10 yards  
   H. 12 yards  
   I. 15 yards

**Brain Game**

Putting the Pieces Together

Arrange the four congruent triangles to create a larger similar triangle.
Arrange the four congruent rectangles to create a larger similar rectangle.
Sketch your answers.

Can you arrange the four congruent quadrilaterals in the bottom row to create a larger similar quadrilateral?
1. **Activities** The table gives the number of hours Chad spends doing various activities on a school day. Write the following ratios in simplest form.

   a. The hours he is at school or traveling to and from school to the hours in a day

   b. The hours he is asleep to the hours in a day

   c. The hours he is awake to the hours he is asleep

<table>
<thead>
<tr>
<th>Daily activities</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>School</td>
<td>7 hours</td>
</tr>
<tr>
<td>Relaxing or doing homework at home</td>
<td>5 hours</td>
</tr>
<tr>
<td>Football practice</td>
<td>2 hours</td>
</tr>
<tr>
<td>Traveling to and from school</td>
<td>1 hour</td>
</tr>
<tr>
<td>Sleeping</td>
<td>9 hours</td>
</tr>
</tbody>
</table>

2. **Car** A car travels at a speed of 44 feet per second. What is this speed in miles per hour?

   **Solve the proportion.**
   
   3. \( \frac{w}{7} = \frac{36}{42} \)
   
   4. \( \frac{x}{10} = \frac{35}{50} \)
   
   5. \( \frac{3}{4} = \frac{y}{52} \)
   
   6. \( \frac{7}{12} = \frac{z}{105} \)
   
   7. \( \frac{5}{8} = \frac{25}{a} \)
   
   8. \( \frac{b}{8} = \frac{60}{75} \)
   
   9. \( \frac{0.4}{c} = \frac{1.2}{21} \)
   
   10. \( \frac{5.2}{3} = \frac{78}{d} \)

   **Tell whether the ratios form a proportion.**
   
   11. \( \frac{25}{36}, \frac{5}{6} \)
   
   12. \( \frac{8}{9}, \frac{36}{32} \)

15. **American Flag** The blue portion of the American flag is known as the union. Using the measurements in the diagram, determine if the rectangle enclosing the union is similar to the rectangle enclosing the entire flag.

16. Given \( \triangle ABC \sim \triangle FGH \), find \( FG \).

17. Given \( PQRS \sim JKLM \), find \( KL \).
6.6 Making a Scale Drawing

**Goal**
Make a scale drawing of an object.

**Materials**
- metric ruler

A scale drawing is a drawing that is similar to the object it represents. You are making a scale drawing of a rectangular stage so that you can plan the arrangement of props for a school play. The stage is 12 meters long and 9 meters wide. In your drawing, 1 centimeter represents 3 meters on the stage. What dimensions should you use for the stage in the drawing?

**Investigate**

**Use proportions to make a scale drawing.**

1. Use a proportion to find the length \( l \) (in centimeters) of the stage in the drawing.

\[
\frac{1 \text{ cm}}{3 \text{ m}} = \frac{l}{12 \text{ m}}
\]

\[
1 \cdot 12 = 3 \cdot l
\]

\[
4 = l
\]

2. Use a proportion to find the width \( w \) (in centimeters) of the stage in the drawing.

\[
\frac{1 \text{ cm}}{3 \text{ m}} = \frac{w}{9 \text{ m}}
\]

\[
1 \cdot 9 = 3 \cdot w
\]

\[
3 = w
\]

3. Use a ruler to draw a 4 cm by 3 cm rectangle. This rectangle represents the stage.

**Draw Conclusions**

1. The table shows the dimensions of rectangular pieces of furniture to be placed on the stage. Find the length and width you should use for each piece of furniture in the scale drawing.

<table>
<thead>
<tr>
<th>Item</th>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sofa</td>
<td>1.8 m</td>
<td>0.9 m</td>
</tr>
<tr>
<td>Table</td>
<td>1.2 m</td>
<td>0.9 m</td>
</tr>
<tr>
<td>Upright piano</td>
<td>1.5 m</td>
<td>0.6 m</td>
</tr>
</tbody>
</table>

2. Make a scale drawing of the stage. Include scale drawings of the sofa, the table, and the piano so that the sofa is near the back of the stage, the table is stage left near the front, and the piano is stage right near the front.
Scale Drawings

The map shows a portion of Teotihuacan, a large city built over 2000 years ago. The ruins of the city still exist in central Mexico.

The map is an example of a scale drawing. A **scale drawing** is a two-dimensional drawing that is similar to the object it represents. A **scale model** is a three-dimensional model that is similar to the object it represents.

The **scale** of a scale drawing or scale model gives the relationship between the drawing or model's dimensions and the actual dimensions. For example, in the map shown, the scale $1 \text{ cm} : 200 \text{ m}$ means that 1 centimeter in the scale drawing represents an actual distance of 200 meters.

**Example 1** Using a Scale Drawing

Teotihuacan On the map, the center of the Pyramid of the Sun is 4 centimeters from the center of the Pyramid of the Moon. What is the actual distance from the center of the Pyramid of the Sun to the center of the Pyramid of the Moon?

**Solution**

Let $x$ represent the actual distance (in meters) between the two pyramids. The ratio of the map distance between the two pyramids to the actual distance $x$ is equal to the scale of the map. Write and solve a proportion using this relationship.

\[
\frac{1 \text{ cm}}{200 \text{ m}} = \frac{4 \text{ cm}}{x \text{ m}} \leftarrow \text{Map distance} \quad \frac{4 \text{ cm}}{x \text{ m}} \leftarrow \text{Actual distance}
\]

\[
1x = 200 \cdot 4 \quad \text{Cross products property}
\]

\[
x = 800 \quad \text{Multiply.}
\]

**Answer** The actual distance is 800 meters.
Checkpoint

1. On the map on page 300, the Pyramid of the Sun has a length and a width of 1.1 centimeters. Find the actual dimensions of the Pyramid of the Sun.

Example 2  Finding the Scale of a Drawing

Floral Carpet  Every few years, the Grand Place in Brussels, Belgium, is decorated with a large floral carpet made of begonias. Before making the carpet, designers make detailed scale drawings. Suppose the floral carpet is to be 40 meters wide. A designer creates a scale drawing of the carpet that is 20 centimeters wide. Find the drawing’s scale.

Solution

Write a ratio using corresponding side lengths of the scale drawing and the actual carpet. Then simplify the ratio so that the numerator is 1.

\[
\frac{20 \text{ cm}}{40 \text{ m}} = \frac{20 \text{ cm}}{2 \text{ m}}
\]

Simplify.

Answer  The drawing’s scale is 1 cm : 2 m.

The scale of a scale drawing or scale model can be written without units if the measurements have the same unit. To write the scale from Example 2 without units, write 2 meters as 200 centimeters, as shown.

<table>
<thead>
<tr>
<th>Scale with units</th>
<th>Scale without units</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cm : 2 m</td>
<td>1 cm : 2 m</td>
</tr>
<tr>
<td>1 cm : 200 cm</td>
<td>1 : 200</td>
</tr>
</tbody>
</table>

Example 3  Finding a Dimension of a Scale Model

Space Shuttle  A model of a space shuttle has a scale of 1 : 52. The space shuttle has a wingspan of 78 feet. Find the model’s wingspan.

Solution

Write a proportion using the scale.

\[
\frac{1}{52} = \frac{x}{78}
\]

Cross products property

\[78 = 52x\]

Divide each side by 52.

Answer  The wingspan of the model is 1.5 ft.

Checkpoint

2. The length of a space shuttle is a 122 feet. Find the length of the scale model in Example 3 to the nearest tenth of a foot.

Lesson 6.6  Scale Drawings
Guided Practice

**Vocabulary Check**
1. What is a scale drawing?
2. Write the scale 1 inch : 1 foot without units.

**Skill Check**
A map has a scale of 1 inch : 40 miles. Use the given map distance to find the actual distance.

3. 5 inches  
4. 12 inches  
5. 32 inches  
6. 1 foot

Write the scale without units.

7. 1 in. : 28 yd  
8. 1 in. : 4 ft  
9. 1 cm : 12 m  
10. 1 mm : 2 m

**Guided Problem Solving**
11. **Bedroom** Shown below is a scale drawing of a student’s bedroom. Use the scale drawing to determine the dimensions of the student’s desk.

   1. The student’s bed is 2 meters long. In the scale drawing, the bed is 2.5 centimeters long. Find the scale of the drawing.
   2. Write the scale from Step 1 without units.
   3. In the drawing, the desk is 1.5 centimeters long and 0.5 centimeter wide. Write and solve a proportion to find the dimensions of the student’s actual desk.

Practice and Problem Solving

A map has a scale of 1 centimeter : 5 kilometers. Use the given map distance to find the actual distance.

12. 6 cm  
13. 11 cm  
14. 26 cm  
15. 37 cm

16. 0.6 cm  
17. 1.5 cm  
18. 20 cm  
19. 9 cm

A map has a scale of 1 inch : 3 kilometers. Use the given actual distance to find the distance on the map.

20. 18 km  
21. 90 km  
22. 76 km  
23. 14 km

24. 0.9 km  
25. 1.5 km  
26. 0.3 km  
27. 0.5 km

Write the scale without units.

28. 1 in. : 10 yd  
29. 1 in. : 20 ft  
30. 1 cm : 1 m  
31. 1 mm : 36 cm

32. 1 cm : 3 km  
33. 1 cm : 5 km  
34. 1 cm : 2 cm  
35. 1 mm : 34 cm
36. **Architecture** In a scale drawing, a wall is 8 centimeters long. The actual wall is 20 meters long. Find the scale of the drawing.

37. **Interior Design** A sofa is 8 feet long. In a scale drawing, the sofa is 3 inches long. Find the scale of the drawing.

38. **Basketball Court** A scale drawing of a basketball court has a scale of 1 inch : 9 feet.
   a. The basketball court is 94 feet by 50 feet. Find the dimensions of the court in the drawing.
   b. The free throw line is 15 feet from the backboard. How far is the free throw line from the backboard in the drawing?

39. **Carpentry** A carpenter is building a house from an architect’s blueprint. The blueprint has a scale of 1 : 42.
   a. Find the actual length of a wall that is 3 inches long in the blueprint.
   b. A door on the blueprint is 2 inches high. Find the height of the actual door.
   c. A window on the house is drawn as a rectangle that is \( \frac{1}{2} \) inch by \( \frac{3}{4} \) inch. Find the actual dimensions of the window.

40. **Model Roller Coaster** You are building a model of the Viper roller coaster in California using a scale of 1 : 47. The model is 4 feet high. How many feet high is the Viper?

41. **Banner** You want to make a banner that says WELCOME HOME. You want the letters to be 2 feet high. You make a sketch in which the letters are 2 inches high. The entire phrase in your sketch is 20 inches long. What length of paper should you buy?

42. **Lincoln** A mask of Abraham Lincoln’s head was made when he was alive. The mask has a height of \( 9\frac{3}{4} \) inches.
   a. The profile of Lincoln’s head on a penny has a height of \( \frac{11}{32} \) inch.
      Write the scale of the penny to the mask without units.
   b. The carving of Lincoln’s face on Mount Rushmore is 60 feet high.
      Write the scale of the mask to the carving without units.
   c. Write the scale of the penny to the carving without units.
   d. Lincoln’s nose on the penny is about \( \frac{1}{16} \) inch long. Find the length of Lincoln’s nose on the carving and on the mask. Round your answers to the nearest inch.

43. **Critical Thinking** The ratio of the length of an object to its width is 3 : 2. A scale drawing of the object has a scale of 1 inch : 3 feet. In the scale drawing, what is the ratio of the object’s length to its width?

44. **Critical Thinking** Write a scale for a scale model whose dimensions are 20 times the dimensions of the actual object. Explain your reasoning.
45. **Ant** At the right is a scale drawing of a carpenter ant. The scale of the drawing is 1 cm : 2.5 mm. Find the actual length of the ant’s head, thorax, and abdomen. Round your answers to the nearest hundredth of a millimeter.

46. **Challenge** You made a scale model of Earth and the moon. The Earth’s diameter is about $1.3 \times 10^4$ kilometers. In the model, Earth’s diameter is 50 centimeters. The moon’s diameter is about $3.5 \times 10^2$ kilometers. Find the diameter of the moon in your model. Round your answer to the nearest tenth of a centimeter.

**Mixed Review**

Find the sum or difference. *(Lesson 5.2)*

47. $9 \frac{8}{11} + 7 \frac{6}{11}$
48. $7 \frac{5}{6} - 1 \frac{5}{6}$
49. $-6 \frac{7}{12} - 8 \frac{11}{12}$

50. **Baseball** A square tarp is spread over a 90 foot by 90 foot baseball infield. The tarp extends 15 feet beyond each edge of the infield. Is the tarp similar to the infield? *(Lesson 6.4)*

**Standardized Test Practice**

51. **Extended Response** An architect builds a scale model of a house. The model has a scale of 1 inch : 1 yard.
   
   a. The scale model is 1 foot high. How high is the actual house?
   
   b. The house’s deck is a 15 foot by 12 foot rectangle. Explain how to find the area of the deck in the model.

---

**Brain Game**

Sierpinski’s carpet is a pattern that involves repeatedly dividing a square into 9 smaller squares of equal size and removing the center square. The first and second stages of this pattern are shown below.

**Rolling out the Carpet**

How many new white squares are in the third stage?

Write a ratio that compares the side lengths of the white square in the first stage to one small white square in the second stage. Explain how you found this ratio.
6.7 Performing an Experiment

Goal
Use an experiment to estimate the likelihood that an event will occur.

Materials
- paper cup

Investigate

Perform an experiment to find the position in which a tossed paper cup will land most often.

1. Toss a small paper cup 30 times. Note whether the cup lands on its side, lands rim side up, or lands bottom side up.

2. For each toss, record the result in the Tally column of a frequency table like the one shown below. After 30 tosses, record the frequencies.

<table>
<thead>
<tr>
<th>Position of cup</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>On its side</td>
<td></td>
<td>?</td>
</tr>
<tr>
<td>Rim side up</td>
<td></td>
<td>?</td>
</tr>
<tr>
<td>Bottom side up</td>
<td></td>
<td>?</td>
</tr>
</tbody>
</table>

Draw Conclusions

1. **Analyze** For what fraction of the tosses did the cup land on its side? rim side up? bottom side up?

2. **Critical Thinking** In which position do you think the paper cup is most likely to land on your next toss? Explain your choice.

3. **Predict** Find the ratio of the number of times the cup landed on its side to the total number of times the cup was tossed. Use the ratio to predict the number of times the cup will land on its side if it is tossed 1000 times.

4. **Compare** Cut the paper cup so that it is only half as tall. Repeat the experiment. Then compare your results with the results of the first experiment.

5. **Predict** Use the results from Exercise 4 to predict the number of times the cut cup will land on its side if it is tossed 1000 times.
Probability and Odds

You wrote ratios.

You’ll find probabilities.

So you can describe the accuracy of a weather forecast, as in Ex. 6.

You are rolling a number cube and want to know how likely you are to roll a certain number. Each time you roll the number cube there are six possible results.

Rolling a number cube is an example of an experiment. The possible results of an experiment are outcomes. When you roll a number cube, there are 6 possible outcomes: rolling a 1, 2, 3, 4, 5, or 6. An event is an outcome or a collection of outcomes, such as rolling a 1 or rolling an odd number. Once you specify an event, the outcomes for that event are called favorable outcomes. The favorable outcomes for rolling an odd number are rolling a 1, rolling a 3, and rolling a 5.

The probability that an event occurs is a measure of the likelihood that the event will occur.

Probability of an Event

The probability of an event when all the outcomes are equally likely is:

\[ P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}} \]

Example 1 Finding a Probability

Suppose you roll a number cube. What is the probability that you roll an even number?

Solution

Rolls of 2, 4, and 6 are even, so there are 3 favorable outcomes. There are 6 possible outcomes.

\[ P(\text{rolling an even number}) = \frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}} = \frac{3}{6} = \frac{1}{2} \]

Answer The probability that you roll an even number is \( \frac{1}{2} \).
1. Suppose you roll a number cube. What is the probability that you roll a number greater than 1?

Experimental Probability The probability found in Example 1 is an example of a theoretical probability. A theoretical probability is based on knowing all of the equally likely outcomes of an experiment. A probability that is based on repeated trials of an experiment is called an experimental probability. Each trial in which the event occurs is a success.

Experimental Probability
The experimental probability of an event is:

\[ P(\text{event}) = \frac{\text{Number of successes}}{\text{Number of trials}} \]

Example 2 Finding Experimental Probability

Miniature Golf A miniature golf course offers a free game to golfers who make a hole-in-one on the last hole. Last week, 44 out of 256 golfers made a hole-in-one on the last hole. Find the experimental probability that a golfer makes a hole-in-one on the last hole.

Solution

\[ P(\text{hole-in-one}) = \frac{44}{256} = \frac{11}{64} \]

Answer The experimental probability that a golfer makes a hole-in-one on the last hole is \( \frac{11}{64} \), or about 0.17.

Checkpoint

2. You interviewed 45 randomly chosen students for the newspaper. Of the students you interviewed, 15 play sports. Find the experimental probability that the next randomly chosen student will play sports.

Interpreting Probabilities Probabilities can range from 0 to 1. The closer the probability of an event is to 1, the more likely the event will occur.

You can use probabilities to make predictions about uncertain occurrences.
**Example 3** Using Probability to Make a Prediction

**Basketball** Today, you attempted 50 free throws and made 32 of them. Use experimental probability to predict how many free throws you will make tomorrow if you attempt 75 free throws.

**Solution**

1. Find the experimental probability that you make a free throw.
   
   \[ P(\text{make a free throw}) = \frac{32}{50} = 0.64 \]

2. Multiply the experimental probability by the number of free throws you will attempt tomorrow.
   
   \[ 0.64 \cdot 75 = 48 \]

**Answer** If you continue to make free throws at the same rate, you will make 48 free throws in 75 attempts tomorrow.

**Checkpoint**

3. Use the information in Example 3 to predict how many free throws you would make if you attempt 150 free throws.

**Odds** When all outcomes are equally likely, the ratio of the number of favorable outcomes to the number of unfavorable outcomes is called the **odds in favor** of an event. The ratio of the number of unfavorable outcomes to the number of favorable outcomes is called the **odds against** an event.

\[
\text{Odds in favor} = \frac{\text{Number of favorable outcomes}}{\text{Number of unfavorable outcomes}}
\]

\[
\text{Odds against} = \frac{\text{Number of unfavorable outcomes}}{\text{Number of favorable outcomes}}
\]

**Example 4** Finding the Odds

Suppose you randomly choose a number between 1 and 20.

a. What are the odds in favor of choosing a prime number?

b. What are the odds against choosing a prime number?

**Solution**

a. There are 8 favorable outcomes (2, 3, 5, 7, 11, 13, 17, and 19) and \(20 - 8 = 12\) unfavorable outcomes.

   \[ \text{Odds in favor} = \frac{\text{Number of favorable outcomes}}{\text{Number of unfavorable outcomes}} = \frac{8}{12} = \frac{2}{3} \]

   The odds are \(\frac{2}{3}\) or 2 to 3, that you choose a prime number.

b. The odds against choosing a prime number are \(\frac{3}{2}\), or 3 to 2.
Guided Practice

**Vocabulary Check**
1. Copy and complete: When a coin is flipped 20 times and lands heads up 11 times, the probability that the coin lands heads up is $\frac{11}{20}$.

2. The odds in favor of event A are 2 to 1. The odds against event B are 2 to 1. Which event is more likely to occur, event A or event B?

**Skill Check**
In Exercises 3–5, suppose you roll a number cube. Find the probability of the event.

3. A prime number
4. A multiple of 2
5. A number less than 5

**Guided Problem Solving**
6. **Forecast** Over the course of a month, you keep track of how many times the next day’s weather forecast is accurate. The forecast is correct 22 times in a month of 30 days. Predict how many days over the course of a year the forecast will be correct.

1) Find the experimental probability that the forecast is correct.

2) Multiply your answer from Step 1 by 365 to predict how many days the forecast will be correct over the course of a year.

Practice and Problem Solving

In Exercises 7–10, use the spinner to find the probability. The spinner is divided into equal parts.

7. What is the probability that the spinner stops on a multiple of 3?

8. What are the odds in favor of stopping on a multiple of 4?

9. What are the odds against stopping on a 1 or a 2?

10. If you spin the spinner 100 times, how many times do you expect it to stop on 8?

11. Each letter in the word THEORETICAL is written on a separate slip of paper and placed in a hat. A letter is chosen at random from the hat.

   a. What is the probability that the letter chosen is an E?

   b. What is the probability that the letter chosen is a vowel?

   c. What are the odds in favor of choosing a consonant?

   d. **Critical Thinking** Find a word for which the probability that you choose an R when you randomly choose a letter from the word is $\frac{2}{5}$.
12. **Experiment** Use a coin to complete the following.

   a. What is the theoretical probability that the coin lands heads up when tossed?

   b. Flip the coin 20 times. Record whether it lands heads up or tails up for each flip. Then find the experimental probability that the coin lands heads up when tossed.

   c. **Writing** Compare the theoretical probability with the experimental probability. What do you think would happen if you tossed the coin 100 times? Explain.

13. **Flowers** You plant 30 African violet seeds and 9 of them sprout. Use an experimental probability to predict how many African violet seeds will sprout if you plant 20 more seeds.

14. **Extended Problem Solving** In normal English texts, letters appear with regular frequency. The table gives the probability that a letter, chosen at random from a page of English text, will be a certain letter.

<table>
<thead>
<tr>
<th>Letter</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>0.131</td>
</tr>
<tr>
<td>t</td>
<td>0.104</td>
</tr>
<tr>
<td>a</td>
<td>0.081</td>
</tr>
<tr>
<td>s</td>
<td>0.061</td>
</tr>
<tr>
<td>x</td>
<td>0.002</td>
</tr>
<tr>
<td>z</td>
<td>0.001</td>
</tr>
</tbody>
</table>

   a. **Predict** A page contains 300 letters. Predict how many e’s are on the page and how many a’s.

   b. **Predict** An essay contains 1000 letters. How many z’s would you predict are in the essay? How many x’s?

   c. **Compare** The three sentences in part (b) contain 68 letters. Find the experimental probability that a letter randomly chosen from these sentences is a t and the probability that it is an s. How do these probabilities compare with the probabilities given in the table?

15. **Critical Thinking** If you know the probability of an event, explain how to find the odds in favor of and the odds against the event.

16. **Writing** Describe a situation where you would use experimental probability and a situation where you would use theoretical probability.

In Exercises 17 and 18, use geometric probability to solve the problem.

If a point in a region is chosen at random, the geometric probability that the point is located in a specified part of the region is given by

\[
P(\text{point in part}) = \frac{\text{Area of specified part}}{\text{Area of entire region}}
\]

17. **Ring** You lost a ring in a rectangular field that is 110 yards by 65 yards. You search a rectangular section of the field that is 25 yards by 32 yards. What is the probability that the ring is in the section you search?

18. **Treasure Chest** A treasure chest is buried somewhere in a rectangular field. The field is 100 feet by 60 feet. You search 25 square feet of the field. What is the chance that the chest is in the region you search?
19. **Websites** Many websites have ads whose appearance is based on probability. Advertisers pay the website based on the number of times the ad appears.

   a. The probability that an ad appears when a particular website is loaded is 0.2. The website gets 2000 hits a day. About how many times does the ad appear?
   
   b. The probability that an ad appears when a particular website is loaded is 0.05. About how many hits must the website have for the ad to appear 1000 times?

20. **Critical Thinking** An event has *even odds* when the odds in favor of (or against) the event are $1:1$. What is the probability of an event with even odds?

21. **Critical Thinking** The probability of rolling a 1 on a number cube is $\frac{1}{6}$.
   
   What is the probability of *not* rolling a 1? In general, if the probability of an event is $\frac{1}{n}$, what is the probability that the event does *not* occur? Explain your thinking.

22. **Challenge** A train runs every 15 minutes. You arrive at the train station without consulting the train’s time schedule. What is the probability that you will wait more than 10 minutes for the train?

---

**Mixed Review**

**Algebra Basics** Evaluate the expression when $x = 3$, $y = -3$, and $z = 4$. *(Lesson 1.7)*

23. $2xy$  
24. $5yz$  
25. $7xyz$  
26. $6xz$

27. $\frac{5}{9}$, $-\frac{3}{5}$, $\frac{13}{5}$, $-1.5$, $-2.7$  
28. $-0.625$, $-\frac{3}{8}$, $\frac{5}{8}$, $\frac{21}{8}$, $-1.6$

29. Given $ABCD \sim EFGH$, find $EH$. *(Lesson 6.5)*
30. Given $\triangle LMN \sim \triangle PQR$, find $LN$. *(Lesson 6.5)*

---

**Standardized Test Practice**

31. **Multiple Choice** You have a bag filled with 12 red marbles, 9 blue marbles, and 14 green marbles. You randomly select a marble from the bag. What is the probability that you select a blue marble?
   
   A. $\frac{1}{9}$  
   B. $\frac{1}{35}$  
   C. $\frac{9}{35}$  
   D. $\frac{9}{26}$

32. **Short Response** A traffic light is green for 17 seconds, yellow for 3 seconds, and red for 20 seconds. Suppose your approach to the light is not affected by traffic or other factors. What is the probability that the light will be green when you first see the light? Explain.
6.7 Generating Random Numbers

**Goal** Use a calculator to generate random integers.

**Example**

Use a calculator to solve the following problem.

On average, how many people do you need to assemble in a group for two members of the group to have the same birth month?

Although months have different numbers of days, you can make a good prediction by assuming that the 12 months of the year are equally likely to be a person’s birth month. Assign the integers 1 through 12 to the months of the year and use a calculator’s random number generator to generate numbers as described below.

To generate a random integer from 1 through 12, use the random integer generator on a calculator. Note that your calculator may generate a different number than the one in the display below.

**Keystrokes**

![Keystroke Example]

To tell the calculator that you want random integers from 1 to 12, you should enter 1 and 12 after RANDI(.)

1. **Tech Help**
   - To tell the calculator that you want random integers from 1 to 12, you should enter 1 and 12 after RANDI(.)

2. **2nd [.] 12 =**

Continue to generate random numbers until one of the numbers repeats. A repeated number means that two members of the group have the same birth month. Record how many random numbers you generate until a number repeats.

Perform the experiment described in Steps 1 and 2 a total of 10 times. Keep a tally of your results in a table like the one below.

<table>
<thead>
<tr>
<th>Trial</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of people in group</td>
<td>3</td>
<td>6</td>
<td>7</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

**Draw Conclusions**

1. **Analyze** The number of random numbers generated in each trial represents the size of the group you need to assemble for two people to have the same birth month. What is the average (mean) size of such a group?

2. **Left-handed** About 10% of people are left-handed. Choose a number from 1 to 10 to represent a left-handed person. Then use your calculator to perform an experiment to determine how many people you need to assemble before 2 of them are left-handed. Perform the experiment 10 times and find the average (mean) of your results.
The Counting Principle

**Vocabulary**
- tree diagram, p. 313
- counting principle, p. 314

**Before**
You counted outcomes to find probabilities.

**Now**
You’ll use the counting principle to find probabilities.

**Why?**
So you can count possible NBA finals matchups, as in Ex. 9.

**Eyeglasses**
You are buying new eyeglasses and must choose the frame material and shape. The frame material can be plastic or metal. The frame shape can be rectangular, oval, cat’s eye, or round. How many different frames are possible?

One way to count the number of possibilities is to use a [tree diagram](#). A tree diagram uses branching to list choices.

**Example 1**
**Making a Tree Diagram**

To count the number of possible choices for frames, as described above, make a tree diagram.

<table>
<thead>
<tr>
<th>List the frame materials.</th>
<th>List the frame shapes for each frame material.</th>
<th>List the possibilities.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>plastic</strong></td>
<td><em>rectangular</em></td>
<td><em>plastic rectangular</em></td>
</tr>
<tr>
<td></td>
<td><em>oval</em></td>
<td><em>plastic oval</em></td>
</tr>
<tr>
<td></td>
<td><em>cat’s eye</em></td>
<td><em>plastic cat’s eye</em></td>
</tr>
<tr>
<td></td>
<td><em>round</em></td>
<td><em>plastic round</em></td>
</tr>
<tr>
<td><strong>metal</strong></td>
<td><em>rectangular</em></td>
<td><em>metal rectangular</em></td>
</tr>
<tr>
<td></td>
<td><em>oval</em></td>
<td><em>metal oval</em></td>
</tr>
<tr>
<td></td>
<td><em>cat’s eye</em></td>
<td><em>metal cat’s eye</em></td>
</tr>
<tr>
<td></td>
<td><em>round</em></td>
<td><em>metal round</em></td>
</tr>
</tbody>
</table>

**Answer**
Eight different frames are possible.

**Checkpoint**

1. Suppose each of the different eyeglasses in Example 1 also comes in two colors, black and red. Copy the tree diagram above and add the new choices. How many possible choices for frames are there?
**Counting Principle** A quick way to count the number of possibilities displayed in a tree diagram is to use the counting principle. The counting principle uses multiplication to find the number of possible ways two or more events can occur.

**The Counting Principle**

If one event can occur in \( m \) ways, and for each of these ways a second event can occur in \( n \) ways, then the number of ways that the two events can occur together is \( m \cdot n \).

The counting principle can be extended to three or more events.

**Example 2** Using the Counting Principle

You roll a blue and a red number cube. Use the counting principle to find the number of different outcomes that are possible.

<table>
<thead>
<tr>
<th>Number of outcomes for the red cube</th>
<th>Number of outcomes for the blue cube</th>
<th>Total number of possible outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 1</td>
<td>1, 2</td>
<td>6 ( \cdot ) 6 = 36</td>
</tr>
</tbody>
</table>

**Answer** There are 36 different possible outcomes.

**Checkpoint**

2. How many different outcomes are possible when you flip a coin and roll a number cube?

**Example 3** Finding a Probability

**Combination Lock** A combination lock has 40 numbers on its dial. To open the lock, you must turn the dial right to the first number, left to the second number, then right to the third number. You randomly choose three numbers on the lock. What is the probability that you choose the correct combination?

**Solution**

First find the number of different combinations.

\[ 40 \cdot 40 \cdot 40 = 64,000 \]

Use the counting principle.

Then find the probability that you choose the correct combination.

\[ P(\text{correct combination}) = \frac{1}{64,000} \]

There is only one correct combination.

**Answer** The probability that you choose the correct combination is \( \frac{1}{64,000} \).
Guided Practice

Vocabulary Check
1. Draw a tree diagram to show the possible outcomes when you flip two coins.
2. Explain how to use the counting principle to determine how many outcomes are possible if you roll 3 number cubes.

Skill Check
In Exercises 3–5, use the counting principle to determine the number of possible outfits that can be made using 1 of each type of item from the articles of clothing listed.

3. 4 shirts and 3 pairs of pants
4. 5 shirts, 3 pairs of pants, and 5 pairs of socks
5. 8 shirts, 4 pairs of pants, 4 pairs of socks, and 2 belts

Guided Problem Solving
6. Coins You flip three coins. What is the probability that all three coins show heads or all three coins show tails?
   1) Make a tree diagram to show all the different possible outcomes.
   2) How many different possible outcomes are there?
   3) How many favorable outcomes are there?
   4) Find the probability that all three coins show heads or all three coins show tails.

Practice and Problem Solving

7. Computers You are ordering a computer and must choose a multimedia drive, a hard drive, and a monitor. Using the choices listed in the tables, make a tree diagram of the different computers you can order. How many different computers can you order?

<table>
<thead>
<tr>
<th>Multimedia Drive</th>
<th>HARD Drive</th>
<th>Type of Monitor</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD-ROM</td>
<td>40 GB</td>
<td>CRT</td>
</tr>
<tr>
<td>DVD-ROM</td>
<td>60 GB</td>
<td>Flat panel</td>
</tr>
<tr>
<td>CD-RW</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. Music Store A music store manager wants to arrange the store's merchandise into different sections. The manager wants to put CDs, tapes, and singles in different sections. Each section will be divided into different genres: rock, R & B, rap, classical, international music, and country. How many different divisions will the store have?
9. **Playoffs** In the National Basketball Association (NBA), there are 15 teams in the Eastern Conference and 14 teams in the Western Conference. One team from each conference advances to the finals.
   
a. How many different team matchups could there be in the finals?

b. Eight teams from the Eastern Conference and eight teams from the Western Conference make the NBA playoffs. How many different matchups of the playoff teams could meet in the finals?

c. **Writing** Can you use the result from part (b) to determine the probability that two teams in the NBA playoffs meet in the finals? Explain why or why not.

10. **Critical Thinking** A student claims that because he has 3 sweaters and 3 pairs of pants, he has 6 different possible combinations of sweaters and pants. Describe and correct the student’s error.

In Exercises 11–14, use the counting principle to find the total number of possible outcomes. Then determine the probability of the specified event. Each spinner is divided into equal parts.

11. You spin spinner A two times. Find the probability that the spinner stops on 1, then 2.

12. You spin spinner B two times. Find the probability that the spinner stops on 1, then 2.

13. You spin each spinner once. Find the probability that both spinners stop on the same number.

14. You spin spinners A and B two times each. Find the probability that the spinner stops on the same number in all four spins.

15. **Password** Your computer password has 4 capital letters followed by 4 digits. Your friend randomly chooses 4 capital letters and 4 digits. Use a calculator to find the probability that your friend chooses your password.

16. **Art Classes** You want to take two art classes after school. You may take only one painting and one sculpture class per week. The table gives the days that the classes are offered. The classes are offered at 3:00 each day, so you cannot take both classes on the same day.

   a. Make a tree diagram of all the possible schedules for the two classes you could take. Be sure to eliminate the possibilities where you have both classes on the same day.

   b. How many different possible schedules for the two classes are there?

   c. You sign up for the two classes and are randomly assigned a schedule. What is the probability that your sculpture class is on Tuesday?

   d. **Writing** Explain why, for a situation like this one, it is better to make a tree diagram than to use the counting principle.
17. **Multiple Choice Tests** A multiple choice test contains four answer choices (A, B, C, and D) for each question. You guess randomly on two questions on the test.
   a. Make a tree diagram of the possible answers for the two questions.
   b. What is the probability that you answer both questions correctly?
   c. **Analyze** Suppose the answer to both questions is A. Use the tree diagram to count how many outcomes there are in which you answer at least one of the two questions correctly. What is the probability that you answer at least one question correctly?

18. **Challenge** You roll an 8-sided number octahedron, and your friend rolls a 4-sided number pyramid.
   a. How many different possible outcomes are there for the pairs of numbers rolled?
   b. Find the probability that you and your friend roll the same number.
   c. Find the odds in favor of rolling a number greater than the number your friend rolls.

19. **Phone Numbers** You remember part of your friend’s 7-digit phone number, but you cannot remember the rest.
   a. Your friend’s number begins with 79 and ends with five other digits. How many different phone numbers can begin with 79?
   b. You remember that the next digit after 79 is 8. How many possible phone numbers can begin with 798?
   c. How many digits of a 7-digit phone number do you have to know before the probability that you randomly guess the number correctly on the first try is $\frac{1}{100}$?

---

### Mixed Review

**Find the quotient.** *(Lesson 5.5)*

20. $\frac{-4}{9} \div \frac{2}{3}$
21. $\frac{5}{6} \div \left( -\frac{2}{3} \right)$
22. $\frac{11}{24} \div \frac{3}{8}$
23. $\frac{25}{63} \div \frac{8}{9}$

**Solve the proportion.** *(Lesson 6.3)*

24. $\frac{y}{20} = \frac{15}{4}$
25. $\frac{15}{6} = \frac{p}{8}$
26. $\frac{5}{g} = \frac{2}{14}$
27. $\frac{8}{9} = \frac{12}{b}$

28. **Cash** You have four $1 bills, two $5 bills, and one $20 bill in your wallet. You choose one of the bills at random. Find the odds in favor of and the odds against choosing a bill greater than $1$. *(Lesson 6.7)*

---

### Standardized Test Practice

29. **Multiple Choice** A car comes in two different styles. Each style comes in four colors. How many different versions of the car are available?
   - A. 2
   - B. 4
   - C. 6
   - D. 8

30. **Multiple Choice** A hot dog stand sells 2 sizes of hot dog, 6 kinds of soda, and 3 sizes of soda. How many different combinations of a hot dog and soda could you order?
   - F. 12
   - G. 24
   - H. 36
   - I. 11

---

Lesson 6.8  The Counting Principle 317
Chapter Review

Vocabulary Review

- ratio, p. 269
- equivalent ratios, p. 270
- proportion, p. 275
- cross product, p. 280
- similar figures, p. 288
- corresponding parts, p. 288
- congruent figures, p. 290
- scale drawing, p. 300
- scale model, p. 300
- scale, p. 300
- outcomes, p. 306
- event, p. 306
- favorable outcomes, p. 306
- probability, p. 306
- theoretical probability, p. 307
- experimental probability, p. 307
- odds against, p. 308
- tree diagram, p. 313
- counting principle, p. 314
- odds in favor, p. 308

1. Explain how to tell if two ratios are equivalent.
2. What is a proportion?
3. How are similar figures different from congruent figures?
4. What is the difference between an outcome and a favorable outcome?
5. Describe the difference between the probability that an event occurs and the odds in favor of an event.
6. What is the counting principle?

6.1 Ratios and Rates

Goal

Find and interpret a unit rate.

Example

You worked 15 hours and earned $195. How much did you earn per hour?

\[
\frac{195}{15 \text{ hours}} = \frac{195 \div 15}{15 \text{ hours} \div 15} \quad \text{Divide numerator and denominator by 15.}
\]

\[
= \frac{13}{1 \text{ hour}} \quad \text{Simplify.}
\]

Answer

You earned $13 per hour.

Find the unit rate.

7. 330 miles \text{ \ } 6 \text{ hours}
8. 60 minutes \text{ \ } 4 \text{ games}
9. 5 laps \text{ \ } 20 \text{ minutes}
10. 24 ounces \text{ \ } 4 \text{ servings}

11. Tickets

The drama club pays $144.50 to buy 17 movie tickets. What is the cost per ticket?

12. Cereal

One brand of cereal contains 18 ounces and costs $3.20. Another brand contains 1 pound and costs $3.00. Which brand has a lower price per ounce?
6.2 Writing and Solving Proportions

**Goal** Solve proportions using algebra.

**Example** Solve the proportion \( \frac{x}{9} = \frac{15}{27} \).

\[
\frac{x}{9} = \frac{15}{27}
\]

Write original proportion.

\[
9 \cdot \frac{x}{9} = 9 \cdot \frac{15}{27}
\]

Multiply each side by 9.

\[
x = \frac{15}{3}
\]

Simplify.

\[
x = 5
\]

Divide.

**Use algebra to solve the proportion.**

13. \( \frac{10}{25} = \frac{a}{5} \)

14. \( \frac{18}{24} = \frac{x}{4} \)

15. \( \frac{35}{6} = \frac{y}{42} \)

16. \( \frac{b}{54} = \frac{8}{9} \)

17. **Lions** Lions sleep 5 out of every 6 hours. Write and solve a proportion to find how many hours per day lions sleep.

6.3 Solving Proportions Using Cross Products

**Goal** Solve proportions using the cross products property.

**Example** Solve the proportion \( \frac{8}{20} = \frac{38}{y} \) using the cross products property.

\[
\frac{8}{20} = \frac{38}{y}
\]

Write original proportion.

\[
8 \cdot y = 20 \cdot 38
\]

Cross products property

\[
y = \frac{760}{8}
\]

Multiply.

\[
8y = 760
\]

Divide each side by 8.

\[
y = 95
\]

Simplify.

**Use the cross products property to solve the proportion.**

18. \( \frac{6}{2.7} = \frac{40}{z} \)

19. \( \frac{6}{17} = \frac{3}{x} \)

20. \( \frac{17}{c} = \frac{34}{46} \)

21. \( \frac{50}{a} = \frac{25}{7} \)

22. **Field Trip** Your school is going on a field trip. It takes 2 buses to carry 64 people. Write and solve a proportion to find the number of buses needed to carry 150 people.
6.4 Similar and Congruent Figures

**Example** Given $\triangle ABC \sim \triangle MNP$, find the ratio of the lengths of corresponding sides of $\triangle ABC$ to $\triangle MNP$.

Two corresponding sides are $AB$ and $MN$.

The ratio of the length of these sides is $\frac{AB}{MN}$.

Substitute the lengths of the sides and simplify.

$$\frac{AB}{MN} = \frac{40}{48} = \frac{5}{6}$$

**Answer** The ratio of the lengths of the corresponding sides is $\frac{5}{6}$.

**Goal** Find the ratio of the lengths of corresponding sides of figure A to figure B.

23. $GHJK \sim PQRS$

24. $KLMN \sim WXYZ$

6.5 Similarity and Measurement

**Example** Given $DEFG \sim JKLM$, find $KL$.

Write proportion involving $KL$.

$$\frac{EF}{KL} = \frac{FG}{LM}$$

Substitute.

$$\frac{6}{x} = \frac{14}{21}$$

Cross products property

$6 \cdot 21 = 14x$

$126 = 14x$

Multiply.

$9 = x$

Divide each side by 14.

**Answer** The length of $KL$ is 9 centimeters.

**Goal** Find unknown side lengths of similar figures.

**In Exercises 25 and 26, use the similar figures above.**

25. Find $JM$.

26. Find $DE$. 

320 Chapter 6 Ratio, Proportion, and Probability
6.6 Scale Drawings

**Goal**
Find distances using scales and scale drawings.

**Example**
The distance between two cities on a map is 7 centimeters. The map has a scale of 1 cm : 20 km.
Find the actual distance between the two cities.

\[
\frac{1 \text{ cm}}{20 \text{ km}} = \frac{7 \text{ cm}}{x \text{ km}}
\]
Write proportion.

\[
x = 20 \cdot 7
\]
Cross products property

\[
x = 140
\]
Multiply.

**Answer** The actual distance is 140 kilometers.

A scale drawing has a scale of 1 inch : 3 yards. Use the given distance from the drawing to find the actual distance.

27. 7 inches  28. 14 inches  29. 18 inches  30. 22 inches

6.7 Probability and Odds

**Goal**
Find the probability of an event.

**Example**
The spinner shown is divided into equal parts. Find the probability that the spinner stops on a 5.

\[
P(\text{stopping on a 5}) = \frac{\text{Number of favorable outcomes}}{\text{Number of outcomes}} = \frac{3}{8}
\]

**Answer** Find the probability that the spinner above stops on the number.

31. 4  32. 3  33. 2  34. 1

6.8 The Counting Principle

**Goal**
Use the counting principle to count possibilities.

**Example**
In a game, you are to choose a 2 letter code from the 26 capital letters of the alphabet. Find the number of possible codes.

Number of possibilities for first letter × Number of possibilities for second letter = Total number of possibilities

\[
26 \cdot 26 = 676
\]

**Answer** 35. Clothing You have 6 shirts and 3 pairs of pants. How many outfits are possible using one of each item?
Chapter Test

Order the ratios from least to greatest.

1. \(51 \text{ to } 25, \frac{5}{4}, 13 : 10, \frac{33}{20}, 17 \text{ to } 20\)

2. \(\frac{64}{25}, 9 \text{ to } 40, \frac{59}{20}, \frac{53}{25}, 37 : 20\)

Write the equivalent rate.

3. \(\frac{18 \text{ ft}}{1 \text{ sec}} = \frac{? \text{ ft}}{1 \text{ min}}\)

4. \(\frac{\$5.60}{1 \text{ lb}} = \frac{?}{1 \text{ oz}}\)

5. \(\frac{1296 \text{ cars}}{1 \text{ day}} = \frac{? \text{ cars}}{1 \text{ hour}}\)

6. \(\frac{8.5 \text{ km}}{1 \text{ h}} = \frac{? \text{ m}}{1 \text{ h}}\)

7. **Cashews** Cashews cost \$.40 per ounce. You have $6. Can you buy one pound of cashews? Explain.

Solve the proportion.

8. \(\frac{5}{12} = \frac{x}{36}\)

9. \(\frac{4}{7} = \frac{a}{35}\)

10. \(\frac{b}{54} = \frac{12}{18}\)

11. \(\frac{7}{8} = \frac{x}{12}\)

12. \(\frac{9}{t} = \frac{3}{8}\)

13. \(\frac{21}{t} = \frac{9}{p}\)

14. \(\frac{6}{14} = \frac{15}{c}\)

15. \(\frac{8}{w} = \frac{1.2}{3}\)

16. Given \(\triangle ABC \cong \triangle EFG\), name the corresponding angles and the corresponding side lengths. Then find the unknown side lengths.

17. **Football** The shadow of a goalpost on a football field is 20 feet long. A football player who is 6 feet tall stands next to the goalpost and casts a shadow 32 inches long. How tall is the goalpost?

A scale drawing has a scale of 1 inch : 10 feet. Use the given actual length to find the length of the object in the scale drawing.

18. 8 feet

19. 7 feet

20. 6 feet

21. 4 feet

22. **Marbles** A bag contains 8 blue marbles, 6 red marbles, 15 green marbles, and 16 orange marbles. A marble is chosen at random from the bag. What is the probability that the marble is red? What are the odds in favor of choosing a green marble?

23. **Wildlife** A wildlife preserve identifies each animal in the preserve with a one-digit number and a capital letter. How many animals can the preserve identify? How many animals can the preserve identify using a two-digit number and a capital letter?
Chapter Standardized Test

Test-Taking Strategy Work at a comfortable pace. Do not pay attention to how fast other students are working.

1. Which rate is equivalent to \(\frac{15 \text{ mi}}{1 \text{ h}}\)?
   - A. \(\frac{22 \text{ ft}}{1 \text{ min}}\)
   - B. \(\frac{1320 \text{ ft}}{1 \text{ min}}\)
   - C. \(\frac{79200 \text{ ft}}{1 \text{ min}}\)
   - D. \(\frac{4752000 \text{ ft}}{1 \text{ min}}\)

2. What is the solution of the proportion \(\frac{x}{5} = \frac{16}{20}\)?
   - F. 2
   - G. 4
   - H. 6
   - I. 8

3. What is the solution of the proportion \(\frac{2.4}{4} = \frac{3}{y}\)?
   - A. 0.2
   - B. 0.5
   - C. 2
   - D. 5

4. Which statement is not necessarily true?
   - F. Corresponding angles of similar figures are congruent.
   - G. Corresponding sides of similar figures are congruent.
   - H. Two squares with different side lengths are similar figures.
   - I. Corresponding sides of congruent figures are congruent.

5. A woman is standing next to a tree. She is 64 inches tall and casts a shadow 24 inches long. The tree’s shadow is 67.5 inches long. How tall is the tree?
   - A. 10 feet
   - B. 15 feet
   - C. 22.75 feet
   - D. 180 feet

6. A bag holds 8 slips of paper numbered 1 through 8. You randomly choose one slip of paper. What is the probability that the number on the slip of paper is greater than 5?
   - F. \(\frac{1}{8}\)
   - G. \(\frac{3}{8}\)
   - H. \(\frac{1}{2}\)
   - I. \(\frac{5}{8}\)

7. An identification system assigns each item a code using 3 capital letters. How many different codes are possible?
   - A. 78
   - B. 7800
   - C. 15,600
   - D. 17,576

8. **Short Response** You make a pattern by drawing three similar rectangles. The width of the smallest rectangle is \(\frac{4}{5}\) of the width of the medium-sized rectangle. The width of the medium-sized rectangle is \(\frac{4}{5}\) of the width of the largest rectangle. The largest rectangle is 12 inches long and 8 inches wide. Find the dimensions of the smallest rectangle. Explain your reasoning.

9. **Extended Response** You are making a scale drawing of a room using a scale of 1 inch : 4 feet.
   - a. The room is 14 feet by 18 feet. Find its dimensions in the drawing.
   - b. A sofa in the room has a length of 6 feet. Find the length of the sofa in the drawing.
   - c. You want to enlarge the scale drawing. How would you change the scale to double the dimensions of the drawing? Explain.
Making a **Business Decision**

**Goal**
Decide what the prices of ads in the school yearbook will be.

**Key Skill**
Solving proportions

**Materials**
• graph paper

To help pay for the cost of publishing a school yearbook, some yearbook staffs sell ads in the yearbook. Suppose you want to raise $1800 for your school’s yearbook. What prices should you set for the ads in order to reach your goal?

Here are the guidelines:

- You have 10 pages of ads to sell.
- The ad pages are divided into 9 sections.
- A single ad can cover 1, 2, 4, or 9 sections, as shown.

![Diagram showing ad sections]

1 section  |  2 sections  |  2 sections  |  4 sections  |  9 sections

A reasonable way to set the prices for ads of different sizes is to let the price of an ad be proportional to its area.

**Investigate**

1. In order to raise $1800, how much must you raise from each page of ads?

2. Let the amount you calculated in Step 1 be the cost of a 9-section (full-page) ad. Write and solve a proportion to calculate the cost of a 4-section ad.

3. Write and solve proportions to calculate the cost of a 2-section ad and a 1-section ad.

4. On graph paper, sketch a few different ways that an ad page can be filled with the ad sizes shown above.
Consider and Decide

Decide on the prices for the ads. Consider the following:

- When designing the layout of a page with ads, you may have some ads that include photos and text and others that include only text. You may want to charge more for ads that include photos.

- You may want to slightly discount the price of larger ads in order to encourage customers to buy larger ads. For example, you may want to make the price of a 4-section ad slightly less than 4 times the cost of a 1-section ad.

Present Your Results

Make a poster that shows the sizes of the yearbook ads and the prices that you chose. Explain how the prices you chose will help you meet the financial goal for the yearbook.

Project Extensions

Using Proportions: Suppose you decide also to allow 3-section and 6-section ads, as shown. What prices should you set for these ads if price is proportional to area and the price of a 9-section ad is $210? Explain your answers.

Research

Ad prices for newspapers and magazines are sometimes listed on rate cards. Use the Internet to find rate cards for several different newspapers or magazines. Are ad prices for sections of a page proportional to ad prices for a full page? Explain how you found your answer.

Career: Most magazines and newspapers are at least partially funded by advertising sales. Find out more about careers in selling advertising space and careers in designing print ads.

Explore: Find several yearbooks, magazines, or newspapers. How do these publications divide pages into sections for ads? Describe your findings.