Chapter 5

Rational Numbers and Equations

Before

In previous chapters you’ve . . .
• Performed operations with decimals
• Solved equations and inequalities with integers
• Simplified fractions
• Compared fractions using the LCD

Now

In Chapter 5 you’ll study . . .
• Identifying rational numbers
• Writing fractions as decimals and decimals as fractions
• Performing operations with fractions and mixed numbers
• Solving equations and inequalities with rational numbers

Why?

So you can solve real-world problems about . . .
• recycling, p. 223
• shot put, p. 228
• horses, p. 235
• sewing, p. 240
• Panama Canal, p. 250
• pets, p. 256
Bison  To monitor bison populations, biologists make yearly counts of the adult bison and calves in a herd. In this chapter, you will use fractions to work with quantities like animal populations.

What do you think? Suppose that in one year there were 6 calves and 32 adult bison in a herd of bison. The next year there were 9 calves and 36 adult bison in the herd. In which year was the fraction of calves in the herd greater?
Chapter Prerequisite Skills

PREREQUISITE SKILLS QUIZ

Preparing for Success  To prepare for success in this chapter, test your knowledge of these concepts and skills. You may want to look at the pages referred to in blue for additional review.

1. **Vocabulary**  Describe how to find the least common denominator of two fractions.

Solve the equation. Check your solution.  (p. 97)

2. $4q = 48$  
3. $-9p = 81$  
4. $\frac{n}{4} = 8$  
5. $\frac{m}{-5} = 3$

Solve the inequality. Graph your solution.  (p. 144)

6. $3s > -27$  
7. $-7r > 49$  
8. $\frac{x}{-4} \leq -6$  
9. $\frac{y}{3} < -12$

Use the LCD to determine which fraction is greater.  (p. 187)

10. $\frac{3}{4} \cdot \frac{7}{9}$  
11. $\frac{2}{7} \cdot \frac{3}{5}$  
12. $\frac{7}{8} \cdot \frac{5}{6}$  
13. $\frac{13}{15} \cdot \frac{11}{18}$

Simplify.  (p. 194)

14. $\frac{18x^2}{24x}$  
15. $\frac{16a^3}{22a}$  
16. $\frac{15z^3}{63z}$  
17. $\frac{27m^4}{45m^2}$

NOTETAKING STRATEGIES

**Note Worthy**

You will find a notetaking strategy at the beginning of each chapter. Look for additional notetaking and study strategies throughout the chapter.

**Using Your Homework.** When you are doing your homework and come to an exercise you don’t understand, write a question for your teacher. Ask the question the next time you have class.

Write the prime factorization of 324.

$$324 = 81 \cdot 4$$

$$81 = 9 \cdot 9$$

$$4 = 2 \cdot 2$$

The prime factorization is $3^4 \cdot 2^2$.

Does it matter in what order I write the factors? Ask in class tomorrow.

As you do your homework in Chapter 5, write down questions you have about performing operations on fractions and mixed numbers.
Rational Numbers

**Before**  
You wrote decimals and fractions.  

**Now**  
You’ll write fractions as decimals and vice versa.  

**Why?**  
So you can assess a recycling plan, as in Ex. 45.

A **rational number** is a number that can be written as a quotient of two integers. Whole numbers and integers are part of the set of rational numbers, as shown in the Venn diagram.

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**Example 1**  
**Identifying Rational Numbers**

Show that the number is rational by writing it as a quotient of two integers.

a. 7  

b. −10  

c. \(\frac{5}{4}\)  

d. \(-\frac{3}{2}\)

**Solution**

a. Write the integer 7 as \(\frac{7}{1}\).

b. Write the integer −10 as \(-\frac{10}{1}\) or \(-\frac{10}{-1}\). These fractions are equivalent.

c. Write the mixed number \(\frac{5}{4}\) as the improper fraction \(\frac{23}{4}\).

d. Think of \(-\frac{3}{2}\) as the opposite of \(\frac{3}{2}\). First write \(\frac{3}{2}\) as \(\frac{7}{2}\). Then you can write \(-\frac{3}{2}\) as \(-\frac{7}{2}\). To write \(-\frac{7}{2}\) as a quotient of two integers, you can assign the negative sign to either the numerator or the denominator. You can write \(-\frac{7}{2}\) or \(-\frac{7}{2}\).

**Terminating and Repeating Decimals**

If you take a rational number in the form \(\frac{a}{b}\) and carry out the division of \(a\) by \(b\), the quotient will be either a **terminating decimal** or a **repeating decimal**. In a **terminating decimal**, the division ends because you obtain a final remainder of zero. In a **repeating decimal**, a digit or block of digits in the quotient repeats without end. Example 2 on page 220 shows how to write both a terminating decimal and a repeating decimal.
Example 2

**Writing Fractions as Decimals**

a. Write $\frac{3}{8}$ as a decimal.

<table>
<thead>
<tr>
<th>24</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>56</td>
<td>40</td>
</tr>
<tr>
<td>40</td>
<td>0</td>
</tr>
</tbody>
</table>

$$0.375$$

b. Write $\frac{5}{11}$ as a decimal.

<table>
<thead>
<tr>
<th>44</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>50</td>
</tr>
<tr>
<td>44</td>
<td>60</td>
</tr>
<tr>
<td>55</td>
<td>0</td>
</tr>
</tbody>
</table>

$$0.4545\ldots = 0.\overline{45}$$

**Answer** The remainder is 0, so the decimal is a terminating decimal: $\frac{3}{8} = 0.375$.

**Answer** Use a bar to show the repeating digits in the repeating decimal: $\frac{5}{11} = 0.\overline{45}$.

**Checkpoint**

Write the fraction or mixed number as a decimal.

1. $\frac{3}{10}$
2. $\frac{2}{3}$
3. $1\frac{9}{20}$
4. $\frac{29}{80}$

Example 3

**Using Decimals to Compare Fractions**

**Biology** Of the 50 mammal species found in Canyonlands National Park, 20 species belong to the order Rodentia. Of the 54 mammal species found in Badlands National Park, 24 belong to Rodentia. In which park is the fraction of mammal species belonging to Rodentia greater?

**Solution**

1. Write a fraction for each park. Then write each fraction as a decimal by dividing the numerator by the denominator.

   - **Canyonlands National Park**
     - Rodentia species : Mammal species = $20:50$
     - Write fraction.
     - Divide.
     - $0.4$

   - **Badlands National Park**
     - Rodentia species : Mammal species = $24:54$
     - Write fraction.
     - Divide.
     - $0.444\ldots = 0.\overline{4}$

2. Compare the decimals. By writing 0.4 as 0.400, you can see that $0.444\ldots$ is greater than 0.400. So $0.\overline{4} > 0.4$, and $\frac{24}{54} > \frac{20}{50}$.

**Answer** The fraction in Badlands National Park is greater.

**In the Real World**

**Biology** The yellow-bellied marmot belongs to the order Rodentia. Yellow-bellied marmots typically live at elevations from 6500 feet to 13,500 feet. Find the difference of these two elevations.
Writing Decimals as Fractions

To write a terminating decimal as a fraction or a mixed number, use the place of the last digit to determine the denominator of the fraction, as shown in Example 4. Example 5 shows a method for writing a repeating decimal as a fraction.

**Example 4**

**Writing Terminating Decimals as Fractions**

- **a.** \(0.7 = \frac{7}{10}\)  
  7 is in tenths’ place, so denominator is 10.
- **b.** \(-3.05 = -3\frac{5}{100}\)  
  5 is in hundredths’ place, so denominator is 100.

\[= -3\frac{1}{20}\]

Simplify fraction.

**Example 5**

**Writing a Repeating Decimal as a Fraction**

To write 0.93 as a fraction, let \(x = 0.93\).

1. Because 0.93 has 2 repeating digits, multiply each side of \(x = 0.93\) by \(10^2\), or 100. Then \(100x = 93.93\).

\[
\begin{align*}
100x &= 93.93 \\
\quad -\quad (x &= 0.93) \\
99x &= 93
\end{align*}
\]

2. Subtract \(x\) from 100x.

\[
\frac{99x}{99} = \frac{93}{99}
\]

3. Solve for \(x\) and simplify.

\[
x = \frac{31}{33}
\]

**Answer** The decimal 0.93 is equivalent to the fraction \(\frac{31}{33}\).

**Checkpoint**

5. **Critical Thinking** Compare writing 0.3 as a fraction with writing \(0.\overline{3}\) as a fraction.

**Example 6**

**Ordering Rational Numbers**

Order the numbers \(-\frac{5}{4}, -0.2, 4.31, -3, \frac{5}{2}, -\frac{13}{3}\) from least to greatest.

Graph the numbers on a number line. You may want to write improper fractions as mixed numbers.

\[
\begin{align*}
-\frac{13}{3} &= -4\frac{1}{3} \\
-\frac{5}{4} &= -1\frac{1}{4} \\
\frac{5}{2} &= 2\frac{1}{2}
\end{align*}
\]

Read the numbers from left to right: \(-\frac{13}{3}, -3, -\frac{5}{4}, -0.2, \frac{5}{2}, 4.31\).
Guided Practice

**Vocabulary Check**
Tell whether the number is a terminating decimal or a repeating decimal.

1. 0.667  
2. 0.4747...  
3. 35.35  
4. 2.413  
5. How can you tell whether a number is a rational number?

**Skill Check**
Show that the number is rational by writing it as a quotient of two integers.

6. 15  
7. −2  
8. 5\frac{4}{7}  
9. −1\frac{1}{3}

Write the fraction or mixed number as a decimal.

10. \frac{2}{9}  
11. 1\frac{4}{5}  
12. −\frac{13}{15}  
13. −9\frac{5}{8}

Write the decimal as a fraction or mixed number.

14. 0.4  
15. 0.324  
16. 0.7\overline{8}  
17. 2\overline{6}

18. **Swim Teams** Of the 20 students on the girls’ swim team, 9 are seniors. Of the 24 students on the boys’ swim team, 10 are seniors. On which team is the fraction of students who are seniors greater?

19. **Error Analysis** Describe and correct the error in writing the repeating decimal 5.07878... using a bar.

\[ 5.0\overline{78} = 5.078 \]

Practice and Problem Solving

**Homework Help**

<table>
<thead>
<tr>
<th>Example</th>
<th>Exercises</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20-27</td>
</tr>
<tr>
<td>2</td>
<td>28-35</td>
</tr>
<tr>
<td>3</td>
<td>44-45</td>
</tr>
<tr>
<td>4</td>
<td>36-43</td>
</tr>
<tr>
<td>5</td>
<td>49-56</td>
</tr>
<tr>
<td>6</td>
<td>45, 57-60</td>
</tr>
</tbody>
</table>

Show that the number is rational by writing it as a quotient of two integers.

20. 24  
21. −29  
22. 5\frac{7}{18}  
23. −\frac{1}{8}

24. 1  
25. −2\frac{3}{7}  
26. 0.3  
27. 0.87

Write the fraction or mixed number as a decimal.

28. \frac{1}{5}  
29. −\frac{7}{8}  
30. −\frac{5}{3}  
31. \frac{19}{6}

32. 3\frac{4}{25}  
33. −\frac{13}{11}  
34. 8\frac{5}{44}  
35. −13\frac{7}{10}

Write the decimal as a fraction or mixed number.

36. 0.54  
37. 0.63  
38. 7.6  
39. 2.093

40. −0.85  
41. 0.019  
42. −5.895  
43. −1.102
44. **Leaves** You and a friend are collecting leaves. In your collection of 45 leaves, 4 are oak leaves. In your friend’s collection of 36 leaves, 3 are oak leaves. Whose collection has a greater fraction of oak leaves?

45. **Recycling** The table shows monthly amounts of trash and recycled trash at a school.

<table>
<thead>
<tr>
<th>Month</th>
<th>Total trash (lb)</th>
<th>Recycled trash (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nov.</td>
<td>350</td>
<td>112</td>
</tr>
<tr>
<td>Dec.</td>
<td>315</td>
<td>119</td>
</tr>
<tr>
<td>Jan.</td>
<td>270</td>
<td>189</td>
</tr>
<tr>
<td>Feb.</td>
<td>330</td>
<td>234</td>
</tr>
<tr>
<td>Mar.</td>
<td>300</td>
<td>214</td>
</tr>
</tbody>
</table>

a. For each month, find the fraction of trash that was recycled.

b. **Compare** Use a calculator to write the fractions in part (a) as decimals. Order the decimals from least to greatest. In which month was the fraction of trash that was recycled the greatest?

c. **Writing** As of January 1, a new recycling plan was introduced at the school. What effect do you think the plan had on recycling efforts in January and the months that followed? Explain.

**Copy and complete the statement using always, sometimes, or never.**

46. An integer is _?_ a rational number.

47. A fraction can _?_ be written as a terminating decimal.

48. A repeating decimal is _?_ a rational number.

**Write the decimal as a fraction or mixed number.**

49. \(0.\bar{8}\)  

50. \(0.\bar{7}\)

51. \(-0.\bar{4}\)  

52. \(-9.\bar{6}\)

53. \(0.1\bar{2}\)

54. \(-1.\bar{36}\)

55. \(0.8\bar{97}\)

56. \(2.7\bar{07}\)

**Order the numbers from least to greatest.**

57. \(-2, \frac{7}{8}, 0.8, 2.1, 1\frac{1}{3}\)

58. \(0.7, -1, \frac{5}{4}, \frac{4}{3}, -2.3, -\frac{9}{2}\)

59. \(0.21, 2.3, \frac{8}{3}, -0.1, -\frac{1}{5}, 0.\bar{2}\)

60. \(0.3, 0.\bar{3}, 0.30, -0.3, -0.\bar{3}\)

**Extended Problem Solving** The table shows the number of at bats and hits that players on a softball team had in three games.

<table>
<thead>
<tr>
<th>Player</th>
<th>Game 1</th>
<th>Game 2</th>
<th>Game 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maria</td>
<td>4 at bats</td>
<td>5 at bats</td>
<td>4 at bats</td>
</tr>
<tr>
<td>Laura</td>
<td>4 at bats</td>
<td>5 at bats</td>
<td>4 at bats</td>
</tr>
<tr>
<td>Jenny</td>
<td>4 at bats</td>
<td>3 hits</td>
<td>4 at bats</td>
</tr>
</tbody>
</table>

a. Find the total number of at bats and the total number of hits for each player for the three games.

b. **Analyze** A player’s batting average is the total number of hits divided by the total number of at bats. The batting average is usually expressed as a decimal rounded to the nearest thousandth. Find each player’s batting average for the three games.

c. **Apply** Rank the players based on batting averages. Explain.
62. **Measurement** You have a rope that is \(4 \frac{1}{3}\) feet long. Your friend has a rope that is \(1 \frac{1}{2}\) yards long. Who has the longer rope?

63. **Critical Thinking** Try using a calculator to find a decimal value for \(\frac{1}{17}\). What do you notice? Then use long division to write \(\frac{1}{17}\) as a terminating or repeating decimal. Explain the calculator result you obtained.

64. **Critical Thinking** Let \(a\) and \(b\) represent nonzero integers. Find a rational number in the form \(\frac{a}{b}\) so that \(-1.7 < \frac{a}{b} < -\frac{5}{3}\). Explain how you found the number.

65. **Challenge** Write the decimal 0.32\(\overline{1}\) as a fraction.

### Mixed Review

#### Simplify the expression. (Lesson 2.3)

66. \(k - 9 - (2 + k)\)

67. \(m + 5 - 2(m + 7)\)

#### Find the least common multiple of the numbers. (Lesson 4.4)

68. 240, 340

69. 18, 60

70. 55, 77

71. 27, 189

#### Chemistry A common number used for calculations in chemistry is Avogadro’s number, which is approximately equal to \(6.02 \times 10^{23}\). Write this number in standard form. (Lesson 4.7)

### Standardized Test Practice

73. **Multiple Choice** Which number is not equivalent to \(\frac{40}{66}\)?

A. \(\frac{20}{33}\)

B. \(\frac{60}{99}\)

C. 0.6

D. 0.60

74. **Multiple Choice** Which number is greater than \(-1.5\)?

F. \(-1.\overline{5}\)

G. \(-\frac{3}{2}\)

H. \(-1.4\overline{5}\)

I. \(-\frac{7}{2}\)

75. **Short Response** Write 0.47\(\overline{5}\) as a fraction. Describe the steps you take to write the fraction.

### Brain Game

**Rational Number Riddle**

Order the fractions from least to greatest. The corresponding letters spell out the answer to the riddle.
Astronomy  One night, \( \frac{77}{100} \) of the moon’s visible surface is illuminated. The next night, an additional \( \frac{9}{100} \) is illuminated. What fraction of the moon’s visible surface is illuminated on the second night?

Adding and Subtracting Like Fractions

Words  To add or subtract fractions with the same denominator, write the sum or difference of the numerators over the denominator.

Numbers  \( \frac{4}{9} + \frac{1}{9} = \frac{5}{9} \)  \( \frac{9}{11} - \frac{2}{11} = \frac{7}{11} \)

Algebra  \( \frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}, \ c \neq 0 \)  \( \frac{a}{c} - \frac{b}{c} = \frac{a - b}{c}, \ c \neq 0 \)

Example 1  Adding Like Fractions

To find the fraction of the moon’s visible surface that is illuminated on the second night, as described above, find the sum of \( \frac{77}{100} \) and \( \frac{9}{100} \).

\[
\frac{77}{100} + \frac{9}{100} = \frac{77 + 9}{100} = \frac{86}{100} = \frac{43}{50}
\]

Write sum of numerators over denominator.

Add. Then simplify.

Answer  On the second night, \( \frac{43}{50} \) of the visible surface is illuminated.
Study Strategy

When you perform operations with negative fractions, be sure to assign a negative sign in front of a fraction to the numerator of the fraction. For instance, in part (a) of Example 2, \(-\frac{4}{7}\) is written as \(-\frac{4}{7}\).

Example 2  Subtracting Like Fractions

a. \(-\frac{4}{7} - \frac{2}{7} = \frac{-4 - 2}{7}
\)
\[= \frac{-6}{7} = -\frac{6}{7}\]
Write difference of numerators over denominator.

Subtract.

b. \(\frac{1}{10} - \left(-\frac{3}{10}\right) = \frac{1}{10} + \frac{3}{10}
\)
\[= \frac{1 + 3}{10} = \frac{4}{10} = \frac{2}{5}\]
To subtract \(-\frac{3}{10}\) add \(\frac{3}{10}\).

Write sum of numerators over denominator.

Add. Then simplify.

Checkpoint

Find the sum or difference.

1. \(\frac{3}{8} + \frac{2}{8}\)
2. \(-\frac{1}{6} + \frac{5}{6}\)
3. \(\frac{2}{15} - \frac{7}{15}\)
4. \(\frac{1}{12} - \left(-\frac{7}{12}\right)\)

Mixed Numbers To add or subtract mixed numbers, you can first write the mixed numbers as improper fractions.

Example 3  Adding and Subtracting Mixed Numbers

a. \(5\frac{5}{9} + 2\frac{7}{9} = \frac{50}{9} + \frac{25}{9}\)
Write mixed numbers as improper fractions.

\[= \frac{50 + 25}{9} = \frac{75}{9}\]
Write sum of numerators over denominator.

Add.

\[= \frac{25}{3} = 8\frac{1}{3}\]
Simplify. Then write fraction as a mixed number.

b. \(-10\frac{6}{13} - 6\frac{8}{13} = -\frac{136}{13} - \frac{86}{13}\)
Write mixed numbers as improper fractions.

\[= -\frac{136 - 86}{13} = -\frac{222}{13} = -17\frac{1}{13}\]
Write difference of numerators over denominator.

Subtract. Then write fraction as a mixed number.

Checkpoint

Find the sum or difference.

5. \(2\frac{3}{4} + 1\frac{3}{4}\)
6. \(-6\frac{2}{3} + 3\frac{1}{3}\)
7. \(4\frac{1}{5} - 2\frac{3}{5}\)
8. \(-3\frac{2}{7} - 6\frac{3}{7}\)
Example 4  Simplifying Variable Expressions

a. \( \frac{3a}{20} + \frac{5a}{20} = \frac{3a + 5a}{20} \)
   Write sum of numerators over denominator.
   \( = \frac{8a}{20} \)
   Add. Divide out common factor.
   \( = \frac{2a}{5} \)
   Simplify.

b. \( -\frac{8}{3b} - \left( -\frac{2}{3b} \right) = -\frac{8}{3b} + \frac{2}{3b} \)
   To subtract \( -\frac{2}{3b} \), add \( \frac{2}{3b} \).
   \( = -\frac{8 + 2}{3b} \)
   Write sum of numerators over denominator.
   \( = -\frac{10}{3b} \)
   Add. Divide out common factor.
   \( = -\frac{2}{b} = -\frac{2}{b} \)
   Simplify.

5.2  Exercises

Guided Practice

Vocabulary Check
1. Copy and complete: To find the sum of two fractions with the same denominator, write the sum of the \( \_ \_ \_ \) over the denominator.

2. Explain how to simplify the expression \( \frac{5m}{3} + \left( -\frac{2m}{3} \right) \).

Skill Check
Find the sum or difference.

3. \( \frac{7}{9} + \frac{1}{9} \)
4. \( -\frac{2}{7} + \frac{5}{7} \)
5. \( \frac{3}{8} - \frac{5}{8} \)
6. \( \frac{9}{13} + 9 - \frac{8}{13} \)
7. \( -3 \frac{7}{16} - 8 \frac{11}{16} \)
8. \( 1 \frac{3}{14} - 10 \frac{5}{14} \)

Guided Problem Solving
9. Crafts  You have \( 5 \frac{1}{4} \) feet of ribbon. You want to cut one piece that is \( 3 \frac{3}{4} \) feet long and one that is \( 1 \frac{3}{4} \) feet long. Do you have enough ribbon?
   1. Write \( 3 \frac{3}{4} \) and \( 1 \frac{3}{4} \) as improper fractions.
   2. Find the sum of the improper fractions.
   3. Compare the sum in Step 2 with \( 5 \frac{1}{4} \) to determine whether you have enough ribbon.
Find the sum or difference.

10. \( \frac{3}{5} + \frac{4}{5} \)  
11. \( \frac{12}{19} + \frac{8}{19} \)  
12. \( -\frac{17}{27} - \frac{13}{27} \)  
13. \( \frac{3}{7} - \left( -\frac{6}{7} \right) \)

14. \( \frac{13}{15} + \left( -\frac{8}{15} \right) \)  
15. \( -\frac{21}{26} + \frac{15}{26} \)  
16. \( \frac{9}{22} - \frac{19}{22} \)  
17. \( -\frac{6}{17} - \frac{12}{17} \)

18. \( 4\frac{1}{4} - 5\frac{3}{4} \)  
19. \( 3\frac{4}{5} + \left( -\frac{8}{5} \right) \)  
20. \( \frac{6}{10} + \frac{7}{10} \)  
21. \( \frac{1}{3} - \left( -\frac{2}{3} \right) \)

22. \( 8\frac{9}{11} - 3\frac{6}{11} \)  
23. \( -5\frac{5}{18} - \frac{17}{18} \)  
24. \( 3\frac{7}{16} - 8\frac{11}{16} \)  
25. \( 1\frac{3}{14} - 10\frac{5}{14} \)

26. **Error Analysis** Describe and correct the error in adding \(-\frac{3}{7}\) and \(\frac{2}{7}\).

\[ \frac{-3}{7} + \frac{2}{7} = \frac{-3 + 2}{7} = \frac{-1}{7} \]

\[ \neq \frac{-1}{14} \]

27. **Homework** One day, you studied math for \(\frac{3}{4}\) hour and English for \(\frac{3}{4}\) hour. What was the total time that you studied both subjects?

28. **Baking** A blueberry muffin recipe calls for \(1\frac{2}{3}\) cups of flour. A banana muffin recipe calls for \(2\frac{2}{3}\) cups of flour. How much flour do you need to make both recipes?

29. \( \frac{5x}{8} + \frac{x}{8} \)
30. \( \frac{r}{13} + \frac{12t}{13} \)
31. \( -\frac{11}{6p} + \frac{17}{6p} \)
32. \( \frac{29}{12s} + \frac{19}{12s} \)
33. \( \frac{2n}{15} - \frac{7n}{15} \)
34. \( \frac{m}{21} - \frac{5m}{21} \)
35. \( -\frac{5}{18a} - \frac{23}{18a} \)
36. \( -\frac{15}{4d} - \frac{21}{4d} \)

37. **Carpentry** You are making a shelf from a board that is \(12\frac{3}{4}\) inches long. You want to cut the board so that it is \(10\frac{1}{4}\) inches long. What length should you cut from the board?

38. \( \frac{3}{9} + \frac{7}{9} + \frac{4}{9} \)
39. \( \frac{3}{10} + \frac{5}{10} + \left( -\frac{7}{10} \right) \)
40. \( -\frac{7}{9} + \frac{2}{9} + \left( -\frac{4}{9} \right) \)
41. \( \frac{1}{5} - \left( -\frac{3}{5} \right) + \frac{2}{5} \)
42. \( -\frac{17}{31} - \frac{21}{31} - \frac{27}{31} \)
43. \( -\frac{13}{14} - \frac{11}{14} - \frac{9}{14} \)

44. **Shot Put** The school record for the shot put is 45 feet, \(3\frac{3}{8}\) inches. Your personal record is 42 feet, \(6\frac{7}{8}\) inches. How much farther must you throw the shot put to match the school record?
45. **Critical Thinking** Find two fractions, one positive and one negative, having the same denominator and a sum of $\frac{1}{2}$.  

Solve the equation. Check your solution.  

46. \( x + \frac{3}{7} = \frac{5}{7} \)  

47. \( y + \frac{8}{11} = \frac{2}{11} \)  

48. \( -\frac{7}{12} + z = -\frac{5}{12} \)  

49. \( m + 2\frac{4}{9} = 5\frac{2}{9} \)  

50. \( 7\frac{3}{8} = n + 6\frac{5}{8} \)  

51. \( -1\frac{5}{13} + t = 4\frac{10}{13} \)

52. **Lobsters** A lobster periodically sheds its shell and grows a new shell. During this process, which is called molting, the weight of the lobster increases, as shown in the table.

<table>
<thead>
<tr>
<th></th>
<th>Lobster Weights (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before molting</td>
<td>1(\frac{1}{4})</td>
</tr>
<tr>
<td>After 1 molting</td>
<td>1(\frac{3}{4})</td>
</tr>
<tr>
<td>After 2 moltings</td>
<td>2(\frac{2}{4})</td>
</tr>
<tr>
<td>After 3 moltings</td>
<td>3(\frac{3}{4})</td>
</tr>
<tr>
<td>After 4 moltings</td>
<td>5(\frac{1}{4})</td>
</tr>
</tbody>
</table>

a. How many pounds did the lobster gain after each molting?  
b. How many pounds in all did the lobster gain after four moltings?  
c. Suppose the lobster gains 2\(\frac{1}{4}\) pounds after molting one more time. How much does it weigh then?

53. **Challenge** Solve the equation \(\frac{5}{8} + \frac{7x}{3} = \frac{8x}{3}\). Explain how you found the solution.

**Mixed Review**

54. **Watch** You buy a watch and a battery for $57.99. The battery costs $2.99. Write and solve an equation to find the cost of the watch. *(Lesson 2.5)*

Write the fraction in simplest form. *(Lesson 4.3)*

55. \(\frac{15x^3}{5x^2}\)  

56. \(\frac{120t^2}{140t^5}\)  

57. \(\frac{65m^4}{80m^2}\)  

58. \(\frac{54a^5}{78a^2}\)  

Find the least common multiple of the monomials. *(Lesson 4.4)*

59. \(18m, 3mn\)  

60. \(5t, 20s^2t\)  

61. \(12a^3b, 6a\)  

62. \(9vw, 36v^2w^2\)

**Standardized Test Practice**

63. **Multiple Choice** Find the difference \(\frac{13}{16} - \left(\frac{7}{16}\right)\).  

A. \(-1\frac{1}{4}\)  

B. \(-\frac{3}{8}\)  

C. \(\frac{3}{8}\)  

D. \(1\frac{1}{4}\)

64. **Multiple Choice** Simplify the expression \(\frac{16}{3y} - \frac{28}{3y}\).  

F. \(-\frac{12}{3y}\)  

G. \(-\frac{4}{y}\)  

H. 0  

I. \(\frac{44}{3y}\)

65. **Short Response** A rectangular picture frame is made of wooden strips that are \(\frac{3}{4}\) inch wide. The outside edge of the frame is \(8\frac{1}{4}\) inches long and \(6\frac{3}{4}\) inches wide. Can a rectangular picture that is 7 inches long and \(5\frac{1}{2}\) inches wide fit inside the frame? Explain your answer.
5.3 Combining Fractions with Different Denominators

**Goal**
Use area models to add and subtract fractions with different denominators.

**Materials**
- paper
- colored pencils

---

**Investigate**

**Use area models to add and subtract fractions.**

To model finding the sum $\frac{1}{4} + \frac{2}{3}$, follow the steps below.

1. Draw area models for $\frac{1}{4}$ and $\frac{2}{3}$, as shown.

   ![Fraction Models](image1)

2. Redraw the models so they have the same number of equal parts.

   ![Redrawn Models](image2)

3. Combine the shaded parts to find the sum.

   $\frac{1}{4} + \frac{2}{3} = \frac{11}{12}$

To model finding the difference $\frac{3}{5} - \frac{1}{2}$, follow the steps below.

1. Draw area models for $\frac{3}{5}$ and $\frac{1}{2}$, as shown.

   ![Fraction Models](image3)

2. Redraw the models so they have the same number of equal parts.

   ![Redrawn Models](image4)

3. Find the difference of the numbers of shaded parts in the two models.

   $\frac{3}{5} - \frac{1}{2} = \frac{1}{10}$

---

**Draw Conclusions**

**Use area models to find the sum or difference.**

1. $\frac{3}{4} + \frac{1}{6}$
2. $\frac{1}{2} + \frac{2}{7}$
3. $\frac{2}{3} - \frac{1}{4}$
4. $\frac{5}{6} - \frac{2}{5}$

5. **Critical Thinking**
   Show how you can use an area model to find the difference $2 - \frac{2}{5}$. 
Adding and Subtracting Unlike Fractions

**BEFORE**
You worked with like fractions.

**NOW**
You’ll add and subtract unlike fractions.

**WHY?**
So you can find the range of salamander lengths, as in Ex. 34.

**Hiking** You are hiking between two campsites in the Chesapeake and Ohio Canal National Historic Park. The distance between the campsites is $10\frac{1}{5}$ miles. You have already hiked $5\frac{3}{4}$ miles. How many more miles do you have to hike? Example 3 answers this question by finding the difference of two mixed numbers.

To add or subtract fractions with different denominators, begin by using the LCD of the fractions to write equivalent fractions that have the same denominator.

---

**Example 1 Adding and Subtracting Fractions**

### a.
\[
\frac{5}{12} + \frac{1}{3} = \frac{5}{12} + \frac{4}{12}
\]

Write $\frac{1}{3}$ using LCD.

Write sum of numerators over denominator.

Add.

Simplify.

\[
= \frac{5 + 4}{12} = \frac{9}{12} = \frac{3}{4}
\]

### b.
\[
\frac{5}{6} - \frac{7}{9} = \frac{15}{18} - \frac{14}{18}
\]

Write fractions using LCD.

Write difference of numerators over denominator.

Subtract.

\[
= \frac{-15 - 14}{18} = \frac{-29}{18} = -1\frac{11}{18}
\]
Example 2

Adding Mixed Numbers

\[-\frac{4}{5} + \left( -\frac{2}{11} \right) = \frac{-22}{5} + \left( -\frac{28}{11} \right)\]

Write mixed numbers as improper fractions.

\[= \frac{-242}{55} + \left( -\frac{140}{55} \right)\]

Write fractions using LCD.

\[= \frac{-382}{55} = \frac{-6 \frac{2}{55}}\]

Write sum of numerators over denominator.

Add. Then write fraction as a mixed number.

\[\checkmark\] Checkpoint

Find the sum or difference.

1. \[-\frac{2}{3} + \frac{1}{4} \]

2. \[\frac{3}{10} - \frac{4}{5} \]

3. \[-\frac{4}{15} - \frac{9}{10} \]

4. \[3 \frac{5}{9} + 2 \frac{1}{6} \]

5. \[6 \frac{7}{10} + \left( -1 \frac{1}{5} \right) \]

6. \[-2 \frac{1}{3} + 6 \frac{3}{5} \]

Example 3

Subtracting Mixed Numbers

How many more miles do you need to hike before you reach the next campsite in the Chesapeake and Ohio Canal National Historic Park, as described on page 231?

Solution

Your total hiking distance is \(10 \frac{1}{5}\) miles. You have already hiked \(5 \frac{3}{4}\) miles. To find the remaining distance, subtract.

\[10 \frac{1}{5} - 5 \frac{3}{4} = \frac{51}{5} - \frac{23}{4}\]

Write mixed numbers as improper fractions.

\[= \frac{204}{20} - \frac{115}{20}\]

Write fractions using LCD.

\[= \frac{204 - 115}{20}\]

Write difference of numerators over denominator.

\[= \frac{89}{20} = 4 \frac{9}{20}\]

Subtract. Then write fraction as a mixed number.

\[\text{Answer} \quad \text{You need to hike} \quad 4 \frac{9}{20} \text{ miles, or about} \quad 4 \frac{1}{2} \text{ miles.}\]

\[\checkmark\] Checkpoint

Find the difference.

7. \[5 \frac{4}{11} - 2 \frac{2}{3}\]

8. \[-1 \frac{3}{7} - 2 \frac{3}{14}\]

9. \[\frac{3}{8} - \left( -1 \frac{2}{3} \right)\]
Example 4  Simplifying an Expression

Simplify the expression \( \frac{a}{2} - \frac{a}{6} \).

\[
\frac{a}{2} - \frac{a}{6} = \left( \frac{a}{2} \cdot \frac{3}{3} \right) - \frac{a}{6}
\]

Write \( \frac{a}{2} \) using LCD.

\[
= \frac{3a}{6} - \frac{a}{6}
\]

Multiply.

\[
= \frac{3a - a}{6}
\]

Write difference of numerators over denominator.

\[
= \frac{2a}{6}
\]

Subtract.

\[
= \frac{2a}{6} \cdot \frac{1}{3}
\]

Divide out common factor.

\[
= \frac{a}{3}
\]

Simplify.

5.3 Exercises

More Practice, p. 807

Guided Practice

Vocabulary Check  1. What is the LCD of \( \frac{2}{3} \) and \( \frac{1}{2} \)?

2. Explain how to add two fractions with different denominators.

Skill Check  Find the sum or difference.

3. \( -\frac{1}{4} + \frac{1}{8} \)

4. \( -\frac{3}{4} - \frac{1}{3} \)

5. \( -\frac{3}{5} + \frac{7}{15} \)

6. \( 2 \frac{7}{12} - 9 \frac{2}{3} \)

Simplify the expression.

7. \( \frac{a}{15} + \frac{a}{6} \)

8. \( \frac{b}{8} + \frac{b}{12} \)

9. \( \frac{5a}{3} - \frac{a}{6} \)

10. \( -\frac{d}{5} - \frac{5d}{6} \)

Guided Problem Solving  11. Lumber  Newly cut lumber contains a lot of moisture. Before the wood is used for carpentry or construction, it is usually dried. Suppose a freshly cut board weighs 10 \( \frac{1}{2} \) pounds. After drying, the board weighs 4 \( \frac{2}{3} \) pounds. What was the weight of the water that evaporated?

1  Write 10 \( \frac{1}{2} \) and 4 \( \frac{2}{3} \) as improper fractions.

2  Rewrite the improper fractions using the LCD of the fractions.

3  Find the difference of the improper fractions from Step 2.
Find the sum or difference.

12. \(\frac{1}{12} + \frac{3}{16}\)  
13. \(\frac{5}{6} + \left(\frac{-2}{3}\right)\)  
14. \(-\frac{7}{10} + \frac{7}{20}\)  
15. \(-\frac{1}{3} - \frac{5}{18}\)

16. \(-\frac{4}{15} - \frac{7}{25}\)  
17. \(\frac{5}{8} - \frac{11}{14}\)  
18. \(-\frac{6}{7} + \left(-\frac{16}{21}\right)\)  
19. \(\frac{1}{5} - \left(-\frac{2}{11}\right)\)

Evaluate the expression when \(m = \frac{5}{12}\) and \(n = \frac{7}{9}\).

20. \(m + n\)  
21. \(m - n\)  
22. \(n - m\)  
23. \(-m - n\)

Find the sum or difference.

24. \(5 \frac{1}{4} + 1 \frac{2}{5}\)  
25. \(-3 \frac{3}{4} + 10 \frac{7}{8}\)  
26. \(6 \frac{7}{18} - 8 \frac{21}{54}\)  
27. \(2 \frac{5}{13} - \left(-\frac{1}{2}\right)\)

28. \(-4 \frac{7}{10} - 9 \frac{7}{15}\)  
29. \(3 \frac{1}{2} - \left(-\frac{2 \frac{1}{3}}{3}\right)\)  
30. \(-1 \frac{5}{12} + 4 \frac{5}{14}\)  
31. \(15 \frac{1}{6} - 7 \frac{3}{10}\)

32. **Snow** On one day it snows \(2\frac{1}{2}\) inches. On the next day it snows \(2\frac{1}{4}\) inches, and on the third day it snows \(4\frac{1}{8}\) inches. What was the total amount of snowfall over the three-day period?

33. **Geometry** The width of a rectangle is \(2\frac{3}{8}\) inches. The rectangle is \(1\frac{3}{4}\) inches longer than it is wide. Find the length of the rectangle and the perimeter of the rectangle.

34. **Salamanders** Texas blind salamanders have been found in lengths varying from \(3\frac{1}{4}\) inches to \(5\frac{3}{8}\) inches. Find the range of these lengths.

35. **Extended Problem Solving** A catalog gives the information below about hats. Head size is the distance around a person’s head.

<table>
<thead>
<tr>
<th>Hat size</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
<th>Extra Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head size (in.)</td>
<td>21(\frac{1}{8})</td>
<td>21(\frac{1}{2})</td>
<td>21(\frac{7}{8})</td>
<td>22(\frac{1}{4})</td>
</tr>
</tbody>
</table>

**a. Analyze** For each hat size, find the range in head sizes.

**b. Apply** The catalog says that if your head size is between two hat sizes, you should buy the larger hat size. You are ordering hats for friends whose head sizes (in inches) are 22\(\frac{1}{2}\), 21\(\frac{3}{4}\), 21\(\frac{5}{8}\), 23\(\frac{3}{4}\), 22\(\frac{1}{8}\), and 22\(\frac{3}{8}\). How many hats of each size should you buy?

**c. Writing** If you assume that customers always measure head size to the nearest \(\frac{1}{8}\) inch, how would you revise the table to include all possible head sizes between 21 inches and 24 inches?
Simplify the expression.

36. \( \frac{d}{6} + \frac{2d}{9} \)

37. \( \frac{-y}{5} + \frac{y}{7} \)

38. \( \frac{3a}{2} - \frac{a}{6} \)

39. \( \frac{-9r}{11} - \frac{r}{8} \)

40. \( \frac{4z}{7} - \frac{7z}{4} \)

41. \( \frac{-x}{8} + \frac{x}{12} \)

42. \( \frac{-5c}{3} - \frac{4c}{15} \)

43. \( \frac{-5w}{12} + \frac{7w}{9} \)

44. **Horses** Of the three different types of horses on a ranch, \( \frac{1}{4} \) are Arabians, \( \frac{2}{5} \) are Thoroughbreds, and the rest are Morgans. What fraction of the horses are Morgans?

Evaluate the expression.

45. \( \frac{1}{2} + \frac{1}{4} + 2 + \frac{1}{2} \)

46. \( -\frac{2}{3} + 1\frac{5}{6} - \frac{3}{4} \)

47. \( \frac{3}{4} + \frac{11}{12} - \frac{3}{8} \)

48. \( \frac{1}{2} - \frac{3}{8} + \frac{4}{5} \)

49. \( -\frac{3}{5} - \frac{2}{15} - \frac{7}{10} \)

50. \( \frac{13}{64} - \left( -\frac{3}{16} \right) + 1\frac{1}{8} \)

51. **Critical Thinking** Can you use 48 as a common denominator when you find the sum of \( \frac{7}{8} \) and \( \frac{5}{12} \)? Will you get the same answer that you do if you use the least common denominator of the fractions? Compare the steps you would use to find the sum using each common denominator.

52. **Challenge** Find a value of \( x \) so that the sum \( \frac{1}{x} + \frac{3}{2x} \) is equal to 1. Explain how you found your answer.

---

**Mixed Review** (p. 780)

53. \( 8 \times \frac{3}{4} \)

54. \( \frac{5}{8} \times 16 \)

55. \( \frac{6}{7} \times 21 \)

56. \( 20 \times \frac{3}{5} \)

Find the product or quotient. Write your answer using exponents. (Lesson 4.5)

57. \( b^3 \cdot b^8 \)

58. \( c^2 \cdot c^5 \)

59. \( \frac{d^5}{d^7} \)

60. \( \frac{3a^6}{a^2} \)

61. **Track and Field** The school record for the javelin throw is 186 feet, 2\( \frac{1}{4} \) inches. Your personal record for the javelin throw is 172 feet, 3\( \frac{3}{4} \) inch. Suppose you want to match the school record. By how much do you need to increase the distance you can throw the javelin? (Lesson 5.2)

---

**Standardized Test Practice**

62. **Multiple Choice** Find the sum \( -6\frac{1}{2} + \frac{5}{8} \).
   
   A. \( -7\frac{1}{16} \)  
   B. \( -7\frac{1}{8} \)  
   C. \( -5\frac{7}{8} \)  
   D. \( -5\frac{7}{16} \)

63. **Multiple Choice** Simplify the expression \( \frac{x}{4} - \frac{5x}{6} \).
   
   F. \( -\frac{4x}{12} \)  
   G. \( -\frac{7x}{12} \)  
   H. \( \frac{4x}{12} \)  
   I. \( \frac{7x}{12} \)
Write the fraction or mixed number as a decimal.

1. \( \frac{1}{12} \)  
2. \( \frac{42}{56} \)  
3. \( -\frac{7}{4} \)  
4. \( 1\frac{6}{11} \)

Write the decimal as a fraction or mixed number.

5. \( 0.55 \)  
6. \( -4.22 \)  
7. \( 0.\overline{8} \)  
8. \( 0.\overline{54} \)

Find the sum or difference.

9. \( \frac{2}{15} + \frac{7}{15} \)  
10. \( \frac{1}{6} + \frac{5}{6} \)  
11. \( \frac{11}{12} - \frac{7}{12} \)  
12. \( -\frac{13}{30} + \frac{17}{30} \)
13. \( -\frac{1}{4} + \frac{2}{9} \)  
14. \( \frac{2}{3} - \frac{9}{14} \)  
15. \( \frac{11}{28} - \frac{25}{42} \)  
16. \( -\frac{3}{4} + \left(-\frac{17}{25}\right) \)
17. \( -4\frac{9}{10} - 2\frac{3}{10} \)  
18. \( 3\frac{1}{4} + 5\frac{3}{4} \)  
19. \( -1\frac{1}{4} + 1\frac{1}{18} \)  
20. \( -10\frac{1}{2} - 14\frac{3}{5} \)

Simplify the expression.

21. \( \frac{9d}{12} - \frac{d}{12} \)  
22. \( \frac{7}{3a} + \frac{5}{3a} \)  
23. \( -\frac{7c}{9} + \frac{c}{6} \)  
24. \( \frac{b}{14} - \frac{b}{22} \)

25. **Jogging** You are jogging on a trail around a pond. The distance around the pond is \( 1\frac{5}{16} \) miles. So far, you have jogged \( \frac{3}{8} \) mile. How much farther do you need to jog before you have gone exactly once around the pond?

---

**Magic Square**

Arrange the fractions \( \frac{1}{10}, \frac{3}{20}, \frac{1}{5}, \frac{1}{4}, \frac{3}{10}, \frac{7}{20}, \frac{2}{5} \), and \( \frac{9}{20} \) in the square so that the sum of the numbers in each row, column, and diagonal is \( \frac{3}{4} \).
Multiplying Fractions

You added and subtracted fractions. You’ll multiply fractions and mixed numbers. So you can adjust ingredients in a recipe, as in Ex. 36.

You can use an area model to find the product of two fractions, such as \( \frac{3}{5} \cdot \frac{1}{4} \).

The area model suggests the following rule for multiplying fractions.

**Multiplying Fractions**

**Words** The product of two or more fractions is equal to the product of the numerators over the product of the denominators.

**Numbers** \( \frac{3}{5} \cdot \frac{4}{7} = \frac{3 \cdot 4}{5 \cdot 7} = \frac{12}{35} \)

**Algebra** \( \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \) where \( b \neq 0 \) and \( d \neq 0 \)

**Example 1**

Assign negative sign to numerator.

Use rule for multiplying fractions.

Divide out common factors.

Multiply.

\[
\frac{7}{10} \cdot \left( -\frac{4}{21} \right) = \frac{7}{10} \cdot \left( -\frac{4}{21} \right)
\]

\[
= \frac{7 \cdot (-4)}{10 \cdot 21}
\]

\[
= \frac{-28}{210}
\]

\[
= -\frac{2}{15}
\]
Example 2  Multiplying a Mixed Number and an Integer

**Emmys** Each year, the Academy of Television Arts and Sciences presents gold-plated Emmy awards for programs and individuals in the television industry. Each Emmy statue weighs $4 \frac{3}{4}$ pounds. In 2002, 63 statues were awarded. What was the combined weight of all the statues?

**Solution**

$$\text{Combined weight} = \text{Statue weight} \times \text{Number of statues}$$

$$= \frac{3}{4} \cdot 63 \quad \text{Substitute values.}$$

$$= \frac{19}{4} \cdot \frac{63}{1} \quad \text{Write numbers as improper fractions.}$$

$$= \frac{19 \cdot 63}{4 \cdot 1} \quad \text{Use rule for multiplying fractions.}$$

$$= \frac{1197}{4} \quad \text{Multiply.}$$

$$= 299 \frac{1}{4} \quad \text{Write fraction as a mixed number.}$$

**Answer** The combined weight of the statues was $299 \frac{1}{4}$ pounds.

**Checkpoint**

Find the product.

1. $\frac{2}{3} \cdot \frac{7}{8}$
2. $\left(-\frac{5}{12}\right) \left(\frac{3}{10}\right)$
3. $10 \cdot \left(-2 \frac{3}{11}\right)$
4. $\left(\frac{3}{4}\right)(-12)$

Example 3  Multiplying Mixed Numbers

$$-2 \frac{3}{4} \cdot 3 \frac{1}{5} = \frac{-11}{4} \cdot \frac{16}{5} \quad \text{Write mixed numbers as improper fractions.}$$

$$= \frac{-11 \cdot 16}{4 \cdot 5} \quad \text{Use rule for multiplying fractions.}$$

$$= \frac{-44}{1} \text{ Divide out common factor.}$$

$$= -8 \frac{4}{5} \quad \text{Multiply.}$$

$$= -8 \frac{4}{5} \quad \text{Write fraction as a mixed number.}$$

**Checkpoint**

Find the product.

5. $4 \frac{7}{8} \cdot 5 \frac{2}{3}$
6. $-3 \frac{2}{7} \cdot 1 \frac{1}{2}$
7. $-3 \frac{3}{5} \cdot \left(-1 \frac{5}{9}\right)$
8. $4 \frac{1}{8} \cdot \left(-1 \frac{2}{3}\right)$
**Example 4**  Simplifying Expressions

Simplify the expression.

a. \( \frac{m}{3} \cdot \left( -\frac{12}{5} \right) \)

Solution

\[
\begin{align*}
\text{a. } & \quad \frac{m}{3} \cdot \left( -\frac{12}{5} \right) = \frac{m \cdot (-12)}{3 \cdot 5} \\
& \quad = \frac{-4m}{5} = -\frac{4m}{5} \\
& \quad \text{Use rule for multiplying fractions.} \\
& \quad \text{Divide out common factor.} \\
& \quad \text{Multiply.}
\end{align*}
\]

b. \( \frac{n^2}{10} \cdot \frac{5n^3}{9} \)

\[
\begin{align*}
\text{b. } & \quad \frac{n^2}{10} \cdot \frac{5n^3}{9} = \frac{n^2 \cdot 5n^3}{10 \cdot 9} \\
& \quad = \frac{n^{2+3}}{18} \\
& \quad = \frac{n^5}{18} \\
& \quad \text{Use rule for multiplying fractions.} \\
& \quad \text{Divide out common factor.} \\
& \quad \text{Product of powers property} \\
& \quad \text{Add exponents.}
\end{align*}
\]

**Checkpoint**

Simplify the expression.

9. \( \frac{3v}{4} \cdot \frac{v^5}{9} \)  
10. \( \frac{x^4}{6} \cdot \left( -\frac{16x}{3} \right) \)  
11. \( \frac{3z^5}{25} \cdot \frac{2z^4}{15} \)  
12. \( -\frac{4y^2}{21} \cdot \frac{7y^3}{16} \)

## 5.4 Exercises

**Guided Practice**

**Vocabulary Check**

1. Copy and complete: The product of two or more fractions is equal to the product of the \( \underline{\text{?}} \) over the product of the \( \underline{\text{?}} \).

2. Show how to simplify the expression \( \frac{a^2}{7} \cdot \frac{7a}{2} \).

**Skill Check**

Find the product.

3. \( \frac{3}{4} \cdot \frac{5}{8} \)  
4. \( \left( -\frac{7}{12} \right) \left( -\frac{4}{21} \right) \)  
5. \( \frac{5}{6} \cdot (-8) \)  
6. \( -2 \frac{1}{2} \cdot 1 \frac{3}{4} \)

7. **Error Analysis** Describe and correct the error in simplifying the expression \( \frac{c^2}{7} \cdot \frac{4c^4}{5} \).
Find the product.

8. \(\frac{5}{6} \cdot \frac{3}{8}\)
9. \(\frac{3}{10} \cdot \left(-\frac{5}{12}\right)\)
10. \(-\frac{9}{28} \cdot \frac{49}{54}\)
11. \(-\frac{35}{38} \cdot \left(-\frac{19}{40}\right)\)
12. \(32 \cdot \frac{17}{24}\)
13. \(-\frac{18}{9}\)
14. \(\frac{5}{16} \cdot (-36)\)
15. \(-\frac{25}{28} \cdot (-21)\)
16. \(4\frac{14}{15} \cdot \frac{7}{38}\)
17. \(6\frac{1}{14} \cdot 6\frac{37}{51}\)
18. \(-\frac{7}{11} \cdot \frac{5}{24}\)
19. \(-\frac{3}{20} \cdot \left(-\frac{14}{23}\right)\)

20. Pineapples  About \(\frac{4}{5}\) of the weight of a pineapple is water. About how much water would you expect to find in \(2\frac{1}{2}\) pounds of pineapple?

21. Painting  You want to paint a wall that is \(8\frac{3}{4}\) feet high and \(11\frac{1}{4}\) feet long. You have a can of paint that will cover 200 square feet with one coat.
   a. Find the area of the wall.
   b. Interpret  If you want to apply two coats of paint, do you have enough paint? Explain.

22. Sewing  You are making 20 fleece jackets for a craft fair. For each jacket, you need \(1\frac{7}{8}\) yards of fleece. If fleece costs \$9 per yard, how much money will you spend?

Simplify the expression.

23. \(\frac{a}{5} \cdot \frac{3a}{11}\)
24. \(\frac{16b}{7} \cdot \frac{35b^3}{4}\)
25. \(-\frac{11c^2}{6} \cdot \frac{8c^7}{3}\)
26. \(-\frac{d^5}{13} \cdot \left(-\frac{3d^7}{4}\right)\)
27. \(-\frac{4x}{7} \cdot \left(-\frac{2x}{5}\right)\)
28. \(\frac{ab}{4} \cdot \frac{2a^5b^2}{9}\)

29. Extended Problem Solving  The tread depth of the tires on your family’s new car is \(\frac{3}{8}\) inch. You predict that, as a result of driving the car, the change in tire tread depth will be about \(-\frac{3}{64}\) inch per year.
   a. Analyze  Write a variable expression for the tire tread depth after \(y\) years of driving. Use your expression to make a table showing the tire tread depth after 1, 2, and 3 years.
   b. Apply  Tires should be replaced when the tread depth is \(\frac{1}{16}\) inch. Extend your table from part (a) to find the approximate number of years the car can be driven before the tires need to be replaced.

Evaluate the expression.

30. \(\frac{2}{3} \cdot \left(-\frac{9}{10}\right) \cdot \frac{7}{12}\)
31. \(-\frac{3}{4} \cdot \left(-\frac{8}{15}\right) - \frac{2}{5}\)
32. \(\frac{99}{8} \cdot \frac{2}{17} + \frac{27}{34}\)
33. \(3 + \frac{5}{6} \cdot \left(-\frac{3}{20}\right)\)
34. \(-2 \cdot \frac{7}{8} + \left(-\frac{5}{28}\right)\)
35. \(\frac{5}{11} + \frac{5}{22} \cdot \left(-\frac{8}{33}\right)\)
36. **Recipe** The recipe shown makes 60 sugar cookies. You want to bake 90 cookies.
   
   a. Write a fraction comparing the number of cookies you want to bake with the number of cookies the recipe makes.
   
   b. **Writing** How much of each ingredient will you need to make 90 cookies? Explain how you got your answer.

---

### In Exercises 37–40, use the following example to find a quotient of two numbers that are written in scientific notation.

**Example**

Finding a Quotient of Numbers in Scientific Notation

\[
\frac{6.6 \times 10^3}{1.5 \times 10^4} = \frac{6.6}{1.5} \times \frac{10^3}{10^4} = 4.4 \times 10^{-1}
\]

- **Write quotient as a product of two fractions.**
- **Divide. Use quotient of powers rule.**
- **Subtract exponents.**

37. \(\frac{7.2 \times 10^5}{3.6 \times 10^3}\)
38. \(\frac{8.4 \times 10^5}{3.0 \times 10^8}\)
39. \(\frac{2.4 \times 10^2}{1.2 \times 10^6}\)
40. \(\frac{5.4 \times 10^5}{1.2 \times 10^7}\)

---

41. **Challenge** Find the next three numbers in the following pattern:

\[\frac{2}{3}, \frac{2}{5}, \frac{6}{25}, \frac{18}{125}, \ldots\]

Explain your reasoning.

---

**Mixed Review**

(Esson 1.3)

42. \(72 \div [6 - 14 - 1]\)
43. \(3 \cdot [22 - (16 + 4)]\)
44. \(5 \cdot [49 \div 7 + 2]\)

(Essons 2.6, 2.7)

45. \(-14y = 42\)
46. \(33w = 39.6\)
47. \(-37.7 = -5.8z\)

(Esson 3.5)

48. \(\frac{a}{-10} < -7\)
49. \(\frac{b}{2} > -1\)
50. \(13 \geq \frac{c}{7}\)

---

**Standardized Test Practice**

51. **Multiple Choice** Find the product \(\frac{13}{36} \times \left(-\frac{1}{5}\right)\).
   
   A. \(-2\frac{5}{6}\)
   B. \(-2\frac{13}{180}\)
   C. \(-1\frac{209}{216}\)
   D. \(2\frac{5}{6}\)

52. **Multiple Choice** Which product is greater than 1?
   
   F. \(-\frac{4}{5} \cdot 5\frac{2}{3}\)
   G. \(-\frac{3}{8} \cdot \left(-\frac{6}{7}\right)\)
   H. \(\frac{1}{4} \cdot 2\frac{1}{2}\)
   I. \(1\frac{1}{7} \cdot 1\frac{1}{3}\)
5.5 Dividing Rational Numbers

Goal
Divide rational numbers.

Materials
- paper
- pencil

Investigate

Apply patterns observed in dividing whole numbers to dividing rational numbers.

1. Copy and complete the table. How are the dividend and the divisor of each expression related to the dividend and divisor of the expression $1000 \div 8$? What do you notice about the quotients?

<table>
<thead>
<tr>
<th>Expression</th>
<th>Simplified expression</th>
<th>Quotient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1000 \div 8$</td>
<td>$1000 \div 8$</td>
<td>?</td>
</tr>
<tr>
<td>$\frac{1}{2}(1000) \div \frac{1}{2}(8)$</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$\frac{1}{4}(1000) \div \frac{1}{4}(8)$</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$\frac{1}{8}(1000) \div \frac{1}{8}(8)$</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

2. Use the pattern you observed in Step 1 to find the quotient $\frac{3}{4} \div \frac{2}{3}$. Copy and complete the table by following the arrow. Begin by finding a fraction to replace the red fraction in the table. That is, find a fraction that makes the following statement true.

$\frac{3}{4} \div \frac{2}{3} = \ ?$

<table>
<thead>
<tr>
<th>Expression</th>
<th>Simplified expression</th>
<th>Quotient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{4} \div \frac{2}{3}$</td>
<td>$\frac{3}{4} \div \frac{2}{3}$</td>
<td>?</td>
</tr>
<tr>
<td>$\frac{3}{4} \div \frac{2}{3}$</td>
<td>$\frac{3}{4} \div \frac{2}{3}$</td>
<td>?</td>
</tr>
</tbody>
</table>

Draw Conclusions

In Exercises 1–8, find the quotient using the method shown above.

1. $\frac{3}{5} \div \frac{1}{2}$
2. $\frac{4}{5} \div \frac{1}{3}$
3. $\frac{7}{9} \div \frac{3}{10}$
4. $\frac{11}{12} \div \frac{5}{6}$
5. $\frac{5}{9} \div \frac{2}{7}$
6. $\frac{4}{13} \div \frac{5}{8}$
7. $\frac{6}{11} \div \frac{1}{5}$
8. $\frac{3}{16} \div \frac{2}{5}$

9. Copy and complete: For all integers $a$ and all nonzero integers $b$, $c$, and $d$, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$.

10. Critical Thinking Show how you could find the quotient $\frac{3}{5} \div \frac{7}{8}$ by multiplying the dividend and the divisor by a number that makes the divisor equal to 1.
Dividing Fractions

**B E F O R E**  You multiplied fractions and mixed numbers.  
**N o w**  You’ll divide fractions and mixed numbers.  
**W H Y ?**  So you can find how many book covers you can print, as in Ex. 36.

Two nonzero numbers whose product is 1 are **reciprocals**. The pairs of numbers below are examples of reciprocals.

<table>
<thead>
<tr>
<th>Number</th>
<th>Reciprocal</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1/5</td>
<td>5 \cdot \frac{1}{5} = 1</td>
</tr>
<tr>
<td>\frac{2}{7}</td>
<td>\frac{7}{2}</td>
<td>\frac{2}{7} \cdot \frac{7}{2} = 1</td>
</tr>
<tr>
<td>\frac{5}{8}</td>
<td>\frac{-8}{5}</td>
<td>\frac{5}{8} \cdot \frac{-8}{5} = 1</td>
</tr>
<tr>
<td>0.1</td>
<td>10</td>
<td>0.1(10) = 1</td>
</tr>
</tbody>
</table>

As you may have seen in the activity on page 242, you can use reciprocals when dividing rational numbers.

### Study Strategy

To find the reciprocal of a decimal, you can write the decimal as a fraction. For example, because 0.1 = \frac{1}{10}, the reciprocal of 0.1 is \frac{10}{1} or 10.

### Using Reciprocals to Divide

**Words**  To divide by any nonzero number, multiply by its reciprocal.

**Numbers**  \[
\frac{2}{9} \div \frac{3}{7} = \frac{2}{9} \cdot \frac{7}{3} = \frac{14}{27}
\]

**Algebra**  \[
\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}, \text{ where } b \neq 0, c \neq 0, \text{ and } d \neq 0
\]

### Example 1  Dividing a Fraction by a Fraction

\[
\frac{-\frac{2}{5}}{\frac{4}{7}} = -\frac{2}{5} \cdot \frac{7}{4}
\]

Multiply by reciprocal.

\[
= -\frac{14}{20}
\]

Use rule for multiplying fractions.

Divide out common factor.

\[
= -\frac{7}{10}
\]

Multiply.

✓ **Check**  To check, multiply the quotient by the divisor:

\[
-\frac{7}{10} \cdot \frac{4}{7} = -\frac{2}{5}
\]

Solution checks.
Example 2  \textbf{Dividing a Mixed Number by a Mixed Number}

\[ 4 \frac{1}{6} ÷ \left(-\frac{2}{3}\right) = \frac{25}{6} ÷ \left(-\frac{5}{3}\right) \]

Write mixed numbers as improper fractions.

\[ = \frac{25}{6} \cdot \left(-\frac{3}{5}\right) \]

Multiply by reciprocal.

\[ = \frac{5 \cdot (-3)}{2 \cdot 6 \cdot (-5)} \]

Use rule for multiplying fractions.
Divide out common factors.

\[ = \frac{-5}{2} = -2 \frac{1}{2} \]

Multiply. Then write fraction as a mixed number.

\textbf{Checkpoint}

Find the quotient.

1. \( \frac{7}{12} ÷ \frac{2}{3} \)
2. \( \frac{-4}{9} ÷ \left(-\frac{8}{11}\right) \)
3. \( \frac{3}{8} ÷ 9 \frac{1}{6} \)
4. \( -5 \frac{1}{4} ÷ 2 \frac{2}{5} \)

Example 3  \textbf{Dividing a Whole Number by a Mixed Number}

\textbf{Woodworking} You want to join strips of wood that are 15 inches long and \(1\frac{5}{8}\) inches wide to make a cutting board that is at least 12 inches wide. How many strips are needed?

\textbf{Solution}

Divide to find how many strips are needed.

\[
\text{Number of strips} = \frac{\text{Cutting board width}}{\text{Strip width}}
\]

\[ = \frac{12}{1} ÷ \frac{1\frac{5}{8}}{\frac{13}{8}} \]

Substitute values.

\[ = \frac{12}{1} ÷ \frac{13}{8} \]

Write numbers as improper fractions.

\[ = \frac{12}{1} \cdot \frac{8}{13} \]

Multiply by reciprocal.

\[ = \frac{12 \cdot 8}{1 \cdot 13} \]

Use rule for multiplying fractions.

\[ = \frac{96}{13} \]

Multiply.

\[ = 7 \frac{5}{13} \]

Write fraction as a mixed number.

\textbf{Answer} Because a whole number of strips is needed, you should use 8 strips to make sure that the cutting board is at least 12 inches wide.
Guided Practice

**Vocabulary Check**
1. Explain why 0.25 and 4 are reciprocals.
2. Describe the steps you would take to find the quotient $\frac{2}{5} ÷ 1\frac{2}{3}$.

**State the reciprocal of the number.**
3. 8  
4. $-\frac{2}{3}$  
5. 0.75  
6. 2.5

**Skill Check**
Find the quotient.
7. $\frac{8}{11} ÷ \frac{33}{40}$  
8. $\frac{2}{15} ÷ \left(-\frac{8}{25}\right)$  
9. $-\frac{4}{9} ÷ \frac{16}{11}$  
10. $6\frac{1}{3} ÷ \left(-\frac{3}{8}\right)$
11. $1\frac{3}{14} ÷ 13$  
12. $12 ÷ \left(-\frac{5}{12}\right)$  
13. $\frac{3}{13} ÷ 15$  
14. $7\frac{1}{5} ÷ \left(-1\frac{3}{10}\right)$

15. **Error Analysis** Describe and correct the error in finding the quotient $6\frac{1}{4} ÷ \frac{1}{2}$.

$$6\frac{1}{4} ÷ \frac{1}{2} = \frac{25}{4} ÷ \frac{1}{2}$$
$$= \frac{25}{8} = 3\frac{1}{8}$$

Practice and Problem Solving

**Homework Help**

<table>
<thead>
<tr>
<th>Example</th>
<th>Exercises</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16–19</td>
</tr>
<tr>
<td>2</td>
<td>20–23, 29</td>
</tr>
<tr>
<td>3</td>
<td>24–28</td>
</tr>
</tbody>
</table>

**Online Resources**
- More Examples
- eTutorial Plus

**Find the quotient.**
16. $\frac{13}{18} ÷ \frac{20}{27}$  
17. $\frac{15}{16} ÷ \left(-\frac{25}{36}\right)$  
18. $\frac{32}{45} ÷ \frac{48}{35}$  
19. $\frac{-18}{19} ÷ \left(-\frac{9}{17}\right)$
20. $7\frac{7}{5} ÷ 1\frac{11}{45}$  
21. $6\frac{6}{13} ÷ 2\frac{28}{39}$  
22. $8\frac{4}{15} ÷ \left(-\frac{2}{5}\right)$  
23. $-1\frac{17}{55} ÷ 1\frac{11}{70}$
24. $34 ÷ \left(-\frac{4}{5}\right)$  
25. $27 ÷ \left(-\frac{3}{11}\right)$  
26. $-\frac{40}{77} ÷ (-44)$  
27. $-\frac{16}{21} ÷ 18$

28. **Bookmarks** You want to cut bookmarks that are 6 inches long and $2\frac{3}{8}$ inches wide from a sheet of decorative paper that is 13 inches long and 6 inches wide. If you cut the bookmarks as shown, what is the maximum number of bookmarks that you can cut from the paper?
29. **School Fair** At a school fair, the field for a three-legged race is \(31 \frac{1}{2}\) feet across. Each lane is \(5 \frac{1}{4}\) feet across. How many lanes are there?

**Evaluate the expression.**

30. \(\frac{6}{7} \div \frac{2}{3} - \frac{3}{7}\)  
31. \(\frac{5}{9} - \left(\frac{4}{9} - \frac{3}{18}\right)\)  
32. \(-\frac{8}{15} + \left(\frac{7}{25} + \frac{9}{15}\right)\)

33. \(5 \frac{1}{17} \div 1 \frac{9}{34} - \frac{19}{30}\)  
34. \(\frac{29}{35} \div \left(\frac{11}{21} + \frac{7}{15}\right)\)  
35. \(\frac{70}{61} \div \frac{21}{122} - \frac{55}{13}\)

36. **Extended Problem Solving** Book covers are printed on large sheets of paper. A cover is printed multiple times on a sheet. In parts (a)–(c), assume that a sheet measures \(21 \frac{1}{2}\) inches by 36 inches.

a. **Analyze** A particular book cover is \(7 \frac{1}{2}\) inches high and \(11 \frac{1}{8}\) inches wide. What is the greatest number of covers that can be printed on a sheet? Show how the covers should be arranged.

b. **Analyze** Another book cover is 8 inches high and \(9 \frac{1}{2}\) inches wide. What is the greatest number of these covers that can be printed on a sheet? Show how the covers should be arranged.

c. **Apply** Suppose you need to print 12,000 covers of each of the books in parts (a) and (b). How many sheets do you need for each book?

37. **Challenge** Solve the equation \(\frac{6}{11}x = 12\). Explain how you solved it.

**Mixed Review**

Solve the equation. Check your solution. (Lesson 3.1)

38. \(9m + 5 = 4\)  
39. \(1 - n = -5\)  
40. \(0 = \frac{m}{5} + 3\)  
41. \(1 = 2 + \frac{n}{7}\)

Use the LCD to determine which fraction is greater. (Lesson 4.4)

42. \(\frac{3}{13}, \frac{1}{4}\)  
43. \(\frac{7}{15}, \frac{9}{20}\)  
44. \(\frac{9}{17}, \frac{25}{51}\)  
45. \(\frac{3}{8}, \frac{29}{84}\)

Simplify the expression. (Lessons 5.2, 5.3)

46. \(\frac{a}{13} + \left(-\frac{6d}{65}\right)\)  
47. \(\frac{3b}{22} - \frac{7b}{22}\)  
48. \(-\frac{c}{11} - \frac{10c}{11}\)  
49. \(\frac{5d}{18} + \frac{7d}{9}\)

**Standardized Test Practice**

50. **Extended Response** After reading that the average women’s shoe size is \(8 \frac{1}{2}\), a shoe store owner records the sizes of women’s shoes sold in one morning. The sizes sold were \(6 \frac{1}{2}, 9 \frac{1}{2}, 5 \frac{1}{2}, 10, 11, 8 \frac{1}{2}, 9,\) and \(8\).

a. For each shoe size, find the deviation from the average stated above by subtracting the average from the shoe size. What does a positive deviation from the average indicate? What does a negative deviation from the average indicate?

b. Find the mean of the deviations from part (a). What might the store owner conclude? Explain your thinking.
Using Multiplicative Inverses to Solve Equations

**Vocabulary**

multiplicative inverse, p. 247

**BEFORE**

You used reciprocals to divide fractions.

**Now**

You’ll use multiplicative inverses to solve equations.

**WHY?**

So you can find the width of a U.S. flag, as in Ex. 19.

**Caves** Stalactites are icicle-shaped stone formations found on cave ceilings. They form from minerals deposited by dripping water. Suppose a stalactite is 10 inches long and is growing at a rate of about \( \frac{1}{8} \) inch per decade. How long will it take for the stalactite to reach a length of 1 foot? In Example 3, you’ll see how to answer this question by writing and solving an equation.

To solve an equation that has a fractional coefficient, you can multiply each side of the equation by the fraction’s multiplicative inverse. The multiplicative inverse of a nonzero number is the number’s reciprocal.

**Multiplicative Inverse Property**

**Words** The product of a number and its multiplicative inverse is 1.

**Numbers** \( \frac{3}{5} \cdot \frac{5}{3} = 1 \)

**Algebra** \( \frac{a}{b} \cdot \frac{b}{a} = 1 \), where \( a \neq 0 \), \( b \neq 0 \)

**Study Strategy**

When you solve an equation with fractional coefficients, remember to check your solution by substituting the value of the variable in the original equation.

**Example 1** Solving a One-Step Equation

\[ \frac{4}{7}x = -12 \]

**Original equation**

\[ \frac{7}{4} \left( \frac{4}{7} \right)x = \frac{7}{4}(-12) \]

**Multiply each side by multiplicative inverse of \( \frac{4}{7} \)**

\[ 1x = \frac{7}{4}(-12) \]

**Multiplicative inverse property**

\[ x = -21 \]

**Multiply.**

**Answer** The solution is \(-21\).
Example 2  

Solving a Two-Step Equation

\[-\frac{11}{15}x + \frac{4}{5} = \frac{1}{3}\]  

Original equation

\[-\frac{11}{15}x + \frac{4}{5} - \frac{4}{5} = \frac{1}{3} - \frac{4}{5}\]  

Subtract \(\frac{4}{5}\) from each side.

\[-\frac{11}{15}x = \frac{1}{3} - \frac{4}{5}\]  

Simplify.

\[-\frac{11}{15}x = \frac{5}{15} - \frac{12}{15}\]  

Write fractions using LCD.

\[-\frac{11}{15}x = -\frac{7}{15}\]  

Subtract.

\[\frac{15}{11}\left(-\frac{11}{15}\right)x = -\frac{15}{11}\left(-\frac{7}{15}\right)\]  

Multiply each side by multiplicative inverse of \(-\frac{11}{15}\).

\[x = \frac{7}{11}\]  

Multiply.

Checkpoint

Solve the equation. Check your solution.

1. \(\frac{5}{6}m = 20\)
2. \(-16 = \frac{3}{4}n - 20\)
3. \(-\frac{2}{3}p + \frac{1}{2} = \frac{5}{6}\)

Example 3  

Writing and Solving a Two-Step Equation

Find how long it will take the stalactite described on page 247 to reach a length of 1 foot.

Solution

Write a verbal model. Let \(x\) represent the number of decades it will take the stalactite to reach a length of 1 foot.

<table>
<thead>
<tr>
<th>Current length</th>
<th>+ Growth rate</th>
<th>(\times) Number of decades</th>
<th>= New length</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>(\frac{1}{8})</td>
<td>(x)</td>
<td>12</td>
</tr>
</tbody>
</table>

Write equation. Write 1 foot as 12 inches.

\[10 + \frac{1}{8}x - 10 = 12 - 10\]  

Subtract 10 from each side.

\[\frac{1}{8}x = 2\]  

Simplify.

\[8\left(\frac{1}{8}x\right) = 8(2)\]  

Multiply each side by multiplicative inverse of \(\frac{1}{8}\).

\[x = 16\]  

Multiply.

Answer  The stalactite will be 1 foot long after 16 decades, or 160 years.

In the Real World  

Caves  The photo above was taken in La Grotte de Gournier, a cave in France. Cave explorers have found that the deepest point in this cave is 680 meters beneath Earth’s surface. Give the depth of the cave in kilometers. Then use the fact that 1 km = 0.62 mi to find the depth of the cave in miles. Is the cave more than or less than half a mile deep?
Guided Practice

**Vocabulary Check**
1. What is the multiplicative inverse of a nonzero number?

2. Describe how to solve the equation \( \frac{5}{6}x = \frac{2}{7} \).

**Skill Check**

Solve the equation. Check your solution.

3. \( \frac{4}{9}x = -16 \)

4. \( -\frac{7}{12}x = 28 \)

5. \( -\frac{2}{3}x = \frac{6}{7} \)

6. \( \frac{1}{3}x + 5 = 11 \)

7. \( \frac{7}{8}x - 9 = 5 \)

8. \( -\frac{3}{4}x + \frac{3}{8} = \frac{27}{32} \)

**Guided Problem Solving**

9. **Ants** The diagram shows the distances of two ants from the edge of a picnic table. Ant A travels in a straight line at a speed of \( \frac{3}{4} \) inch per second. Ant B travels in a straight line at a speed of \( \frac{7}{8} \) inch per second.

Which ant will reach the edge first?

1. Use the formula distance = rate \( \times \) time to write an equation for each ant.
2. Solve each equation from Step 1.
3. Compare your solutions to determine which ant will reach the edge first.

Practice and Problem Solving

10. **Error Analysis** Describe and correct the error in solving the equation \( \frac{1}{6}x = \frac{2}{3} \).

   \[ \frac{1}{6}x = \frac{2}{3} \]
   
   \[ 6 \left( \frac{1}{6}x \right) = 6 \left( \frac{2}{3} \right) \]
   
   \[ x = 4 \]

Solve the equation. Check your solution.

11. \( \frac{2}{3}x = 12 \)

12. \( \frac{3}{8}x = 15 \)

13. \( -\frac{5}{12}x = 25 \)

14. \( -\frac{1}{6}x = 8 \)

15. \( \frac{5}{7}x = -\frac{9}{14} \)

16. \( \frac{2}{5}x = -\frac{8}{15} \)

17. \( -\frac{17}{22}x = \frac{4}{11} \)

18. \( -\frac{10}{21}x = \frac{2}{3} \)
19. **United States Flag** The length of the United States flag is $1 \frac{9}{10}$ times the width of the flag. A particular U.S. flag is 5 feet long. Write and solve an equation to find the width of the flag.

20. **Writing** Solve the equation $\frac{3}{7}x = 5$ by using a multiplicative inverse.

Then solve the equation by dividing each side of the equation by $\frac{3}{7}$.

Compare these two methods of solving the equation. How are they alike? How are they different?

**Solve the equation. Check your solution.**

21. $\frac{4}{5}x + 7 = 31$  
22. $\frac{7}{11}x + (-17) = 4$  
23. $4 + \left(-\frac{3}{5}\right)x = 16$

24. $\frac{2}{13} = \frac{8}{13}x + \frac{4}{13}$  
25. $-\frac{8}{17} = \frac{11}{17} - \frac{5}{17}x$  
26. $\frac{13}{15}x - \frac{7}{9} = -\frac{1}{5}$

27. $\frac{5}{14} + \frac{2}{7}x = 1 \frac{5}{42}$  
28. $\frac{7}{8}x - \frac{9}{10} = -\frac{1}{8}$  
29. $\frac{5}{48} = \frac{5}{6} + \frac{5}{16}x$

30. **Geometry** The figure shown is composed of two rectangles. The area of the figure is 1 square inch.

   a. Find the area of the red rectangle.

   b. Write an expression for the area of the blue rectangle.

   c. Write an equation relating the sum of the areas in parts (a) and (b) to the total area of the figure. Solve the equation to find the value of $x$.

31. **Panama Canal** Locks on the Panama Canal are used to move a ship from a higher elevation to a lower elevation. When a ship enters a lock chamber on the canal, water is allowed to spill out of the lock chamber into the next lock chamber until the water levels in the two chambers are equal. Suppose the water level in one lock chamber is 72 feet. As water spills out of the chamber, the depth changes at a rate of about $-3\frac{1}{2}$ feet per minute until the water level is 41 feet. How many minutes does it take for the depth to change from 72 feet to 41 feet?

32. **Baseball Game** At a college baseball game, $\frac{4}{5}$ of the spectators are home team fans. The rest of the spectators are opposing team fans. There are 750 opposing team fans.

   a. Find the fraction of spectators who are opposing team fans.

   b. Find the total number of spectators at the game.

   c. One third of the home team fans at the game attend the college. How many home team fans attend the college?

33. **Critical Thinking** Suppose you want to solve the equation $\left(-2\frac{1}{2}\right)x = \frac{8}{15}$ by using the multiplicative inverse of $-2\frac{1}{2}$. What is the multiplicative inverse of $-2\frac{1}{2}$?
34. **Challenge** The following is based on a famous problem about Diophantus, a Greek mathematician from the third century B.C.

Let \( x \) be the number of years Diophantus lived. Find how long he lived by using the following facts about him to write and solve an equation.

- One sixth of his life was spent in boyhood.
- One twelfth of his life was spent as a youth.
- After \( \frac{1}{7} \) more of his life passed, he got married.
- Five years after getting married, he had a son.
- His son lived \( \frac{1}{2} \) as long as Diophantus lived.
- The son died four years before Diophantus died.

**Mixed Review**

**Algebra Basics** Solve the inequality. Graph your solution. ([Lesson 3.6])

35. \(-17 + 2y > 11\)  
36. \(5x - 23 < 12\)  
37. \(-6z + 13 \leq 31\)

38. **Commuting** Of the 1458 students at school A, 324 students take the bus. Of the 2123 students at school B, 242 take the bus. At which school is the fraction of students who take the bus greater? ([Lesson 5.1])

**Find the sum or difference.** ([Lesson 5.3])

39. \(\frac{1}{3} + \frac{3}{4}\)  
40. \(\frac{1}{6} + \left(\frac{-5}{8}\right)\)  
41. \(\frac{-4}{9} - \frac{7}{12}\)  
42. \(\frac{3}{7} - \frac{9}{14}\)

**Standardized Test Practice**

43. **Multiple Choice** What is the solution of the equation \(\frac{7}{12}x - 5 = -7\frac{1}{3}\)?

A. \(-4\)  
B. \(-1\frac{13}{36}\)  
C. \(-\frac{1}{4}\)  
D. 4

44. **Short Response** Two thirds of the lockers at a health club are rented to members at a rate of $6 per month. The other 48 lockers are available free to members on a first-come, first-serve basis. How much money does the club make from renting lockers each month?

---

**Brain Game**

**CD Sort**

Your friend has a collection of CDs: \(\frac{2}{3}\) of the CDs have booklets, \(\frac{1}{2}\) of the CDs are singles, and \(\frac{1}{4}\) are singles with booklets. All but two of the CDs are either singles or have booklets or both. How many CDs does your friend have?
5.6 Solving Equations with Fractions

**Goal** Use a calculator to solve an equation with a fractional coefficient.

**Example**

*Use a calculator to solve the following problem.*

Your horses eat about $\frac{2}{3}$ of a bale of hay every day. You have 33 bales of hay. After how many days will you need to buy more hay?

To represent the situation, use the equation $\frac{2}{3}d = 33$, where $d$ is the number of days. To solve this equation, you can multiply each side by the multiplicative inverse of $\frac{2}{3}$. To obtain this multiplicative inverse on a calculator, use these keystrokes:

**Keystrokes**

\[
\left( \frac{2}{3} \right)^{-1} = \frac{3}{2} \]

1. Now find the product of the multiplicative inverse and 33.

**Keystrokes**

\[
\frac{3}{2} \times 33 = \frac{99}{2} = 49 \frac{1}{2} \]

**Answer** You will run out of hay in $49 \frac{1}{2}$ days. To be sure your horses have enough hay, you need to buy more after 49 days.

**Draw Conclusions**

*Use a calculator to solve the equation.*

1. $\frac{3}{4}x = 27$  
2. $\frac{5}{16}x = -55$  
3. $\frac{2}{7}x = -26$  
4. $-\frac{9}{11}x = 39$

5. **Sewing** Use a calculator to solve the following problem:

The stitches made by a sewing machine are each $\frac{3}{16}$ inch long and lie end to end in a line. How many stitches are there in a line of stitches that is 21 inches long?
Equations and Inequalities with Rational Numbers

You used reciprocals to solve equations. You’ll use the LCD to solve equations and inequalities. So you can find the original price of a sale item, as in Ex. 39.

So far you have followed these steps to solve equations with fractions:

- Undo any addition or subtraction in order to get the variable term alone on one side of the equation.
- Multiply both sides of the equation by the multiplicative inverse of the coefficient of the variable term.

Another way to solve an equation with fractions is to clear fractions by multiplying each side of the equation by the LCD of the fractions. The resulting equation is equivalent to the original equation.

**Example 1**  
* Solving an Equation by Clearing Fractions

$$\frac{-5}{6}x + \frac{1}{2} = \frac{3}{4}$$  

**Original equation**

$$12\left(\frac{-5}{6}x + \frac{1}{2}\right) = 12\left(\frac{3}{4}\right)$$  

**Multiply each side by LCD of fractions.**

$$12\left(\frac{-5}{6}x\right) + 12\left(\frac{1}{2}\right) = 12\left(\frac{3}{4}\right)$$  

**Use distributive property.**

$$-10x + 6 = 9$$  

**Simplify.**

$$-10x + 6 - 6 = 9 - 6$$  

**Subtract 6 from each side.**

$$-10x = 3$$  

**Simplify.**

$$\frac{-10x}{-10} = \frac{3}{-10}$$  

**Divide each side by -10.**

$$x = -\frac{3}{10}$$  

**Simplify.**

**Checkpoint**

Solve the equation by first clearing the fractions.

1. \(\frac{1}{2}x + \frac{7}{10} = \frac{4}{5}\)  
2. \(\frac{3}{8}x - \frac{2}{3} = \frac{7}{12}\)  
3. \(-\frac{2}{9} = \frac{3}{4}x - \frac{1}{6}\)

**Solving Equations with Decimals**  
As shown in Example 2 on page 254, you can clear decimals from an equation.
Example 2  **Solving an Equation by Clearing Decimals**

**Solve the equation** $2.3 = 5.14 + 0.8m$.

Because the greatest number of decimal places in any of the terms with decimals is 2, multiply each side of the equation by $10^2$, or 100.

\[
2.3 = 5.14 + 0.8m \\
100(2.3) = 100(5.14 + 0.8m) \\
230 = 514 + 80m \\
230 - 514 = 514 + 80m - 514 \\
-284 = 80m \\
\frac{-284}{80} = \frac{80m}{80} \\
-3.55 = m
\]

Write original equation.

Multiply each side by 100.

Use distributive property. Simplify.

Subtract 514 from each side.

Simplify.

Divide each side by 80.

Simplify.

**Solving Inequalities** You can use the methods you have learned for solving equations with fractional coefficients to solve inequalities.

Example 3  **Solving an Inequality with Fractions**

**Shopping** A sign in a clothing store says to take $\frac{1}{3}$ off the marked price of a shirt. You have $20 in cash and a $5 gift certificate. What are the original prices of the shirts you can afford to buy?

**Solution**

Write a verbal model. Let $x$ represent the original prices of the shirts you can afford to buy.

<table>
<thead>
<tr>
<th>Original price</th>
<th>$-\frac{1}{3}$ of original price</th>
<th>Gift certificate amount</th>
<th>$\leq$ Cash on hand</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x - \frac{1}{3}x - 5 \leq 20$</td>
<td>Substitute.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1 - \frac{1}{3})x - 5 \leq 20</td>
<td>Combine like terms.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{2}{3}x - 5 \leq 20$</td>
<td>Simplify.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{2}{3}x - 5 + 5 \leq 20 + 5$</td>
<td>Add 5 to each side.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{3}{2}\left(\frac{2}{3}x\right) \leq \frac{3}{2}(25)$</td>
<td>Multiply each side by multiplicative inverse of $\frac{2}{3}$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x \leq 37.50$</td>
<td>Simplify.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answer** You can afford a shirt whose original price is $37.50 or less.
Example 4  

**Solving an Inequality by Clearing Fractions**

\[
-\frac{3}{4}m - \frac{1}{8} \leq -\frac{1}{4} \\
8\left(-\frac{3}{4}m - \frac{1}{8}\right) \leq 8\left(-\frac{1}{4}\right) \\
8\left(-\frac{3}{4}m\right) - 8\left(\frac{1}{8}\right) \leq 8\left(-\frac{1}{4}\right) \\
-6m - 1 \leq -2 \\
-6m - 1 + 1 \leq -2 + 1 \\
-6m \leq -1 \\
\frac{-6m}{-6} \geq \frac{-1}{-6} \\
m \geq \frac{1}{6}
\]

Original inequality  
Multiply each side by LCD of fractions.  
Distributive property  
Simplify.  
Add 1 to each side.  
Simplify.  
Divide each side by -6, 
Reverse inequality symbol.  
Simplify.

5.7  
**Exercises**  
More Practice, p. 807

**Guided Practice**

**Vocabulary Check**  
1. Copy and complete: To clear the fractions in an equation, multiply each side of the equation by the _?_ of the fractions.

2. To clear the decimals in an equation, how do you determine what power of 10 to multiply each side of the equation by?

**Skill Check**  
Solve the equation by first clearing the fractions or the decimals.

3. \(\frac{2}{3}n + 17 = \frac{5}{6}\)  
4. \(\frac{2}{5} - \frac{5}{8}n = -4\)  
5. \(\frac{3}{4}n - \frac{1}{2} = -\frac{7}{4}\)

6. \(2.3m - 11 = -29.4\)  
7. \(5.3m - 6 = -27.2\)  
8. \(-1.2m + 1.25 = 0.77\)

Solve the inequality by first clearing the fractions.

9. \(\frac{7}{13}x - 1 > \frac{1}{2}\)  
10. \(\frac{4}{5} \geq \frac{2}{3} - \frac{2}{7}x\)  
11. \(\frac{8}{15}x - \frac{17}{30} \leq \frac{7}{10}\)

12. **Error Analysis**  
   Describe and correct the error in clearing the fractions in the equation
   \[
   \frac{2}{3}x + 5 = \frac{5}{2}
   \]
   \[
   6\left(\frac{2}{3}x\right) + 5 = 6\left(\frac{5}{2}\right)
   \]
   \[
   4x + 5 = 15
   \]

Lesson 5.7  
Equations and Inequalities with Rational Numbers  
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Practice and Problem Solving

Homework Help

Example 1 Exercises 13-21, 40
Example 2 Exercises 22-31, 41
Example 3 Exercises 32-39
Example 4 Exercises 32-39

Online Resources
CLASSZONE.COM
• More Examples
• eTutorial Plus

Solve the equation by first clearing the fractions.

13. \( \frac{1}{2} t + \frac{3}{4} = \frac{5}{16} \)  
14. \( \frac{5}{6} x + \frac{2}{9} = -\frac{7}{12} \)  
15. \( \frac{3}{4} = \frac{5}{6} a + \frac{2}{9} \)

16. \( \frac{5}{8} = \frac{1}{10} + \frac{5}{14} m \)  
17. \( -\frac{41}{60} + \frac{17}{20} p = \frac{29}{30} \)  
18. \( \frac{3}{8} = -\frac{4}{1} x - \frac{3}{5} \)

19. \( -\frac{3}{2} - \frac{5}{6} = -\frac{4}{9} \)  
20. \( -\frac{3}{5} z - 4 = -\frac{77}{20} \)  
21. \( 4w + \frac{2}{7} = -\frac{4}{5} \)

Solve the equation by first clearing the decimals.

22. \( 6.2x + 3.7 = 22.3 \)  
23. \( 7.8y + 6 = 23.16 \)  
24. \( 10.7w + 4 = 47.87 \)

25. \( 2 = -6.4z + 10 \)  
26. \( -3.3x + 6.5 = 1.55 \)  
27. \( 1.6b - 3 = -9.4 \)

28. \( -1.7w - 4 = 2.63 \)  
29. \( 2.875y + 9 = 12.45 \)  
30. \( 4.125c + 5 = -9.85 \)

31. **Saving Money** You want to save $400 for a camping trip. You have $64.96 in your savings account. Each week you deposit your paycheck from your part-time job. Each paycheck is for $69.80. How many paychecks must you deposit to reach your goal of $400?

32. \( -\frac{4}{11} z - 1 > -\frac{8}{11} \)  
33. \( \frac{1}{5} k + 14 \leq \frac{2}{9} \)  
34. \( -\frac{31}{4} < -13 + \frac{7}{8} f \)

35. \( \frac{1}{7} r + \frac{53}{56} \geq \frac{6}{7} \)  
36. \( \frac{5}{6} r - \frac{1}{5} < -\frac{8}{15} \)  
37. \( \frac{1}{3} + \frac{1}{13} d \geq \frac{17}{39} \)

38. **Fundraiser** Your class is selling gift wrap for a school fundraiser. One fourth of the money collected will be used to pay for the gift wrap. Your class wants to raise at least $675 after paying for the gift wrap. How much money does your class need to collect?

39. **Sale Price** A store displays the sign shown. You want to buy a belt that costs $8 and a pair of jeans. You have $18. Write and solve an inequality to find the original prices of the jeans you can afford to buy.

40. **Pets** Each morning you feed your dog \( \frac{3}{4} \) cup of dry dog food. At night you feed him \( \frac{1}{3} \) cup of dry dog food. You buy a bag of dog food that contains 40 cups. How many days will the bag last?

41. **Physics** The speed of sound in air depends on temperature. The relationship between the speed of sound and the air temperature is given by the equation \( v = 331.4 + 0.6T \), where \( v \) is the speed of sound in meters per second and \( T \) is the air temperature in degrees Celsius. During a storm, the speed of sound was measured at 343.37 meters per second. What was the air temperature?
42. **Writing** Compare and contrast the method of using multiplicative inverses with the method of clearing fractions when solving an equation like $\frac{2}{3}x - 1 = \frac{5}{6}$.

43. **Costumes** You buy $12\frac{1}{2}$ yards of material to make costumes for a school play. Each costume consists of a matching hat and cape. You need $1\frac{3}{4}$ feet of the material for each hat. You need $3\frac{1}{2}$ feet of material for each cape. How many costumes can you make?

44. **Critical Thinking** Can you clear fractions in an equation by multiplying each side of the equation by a common denominator other than the LCD? Give an example to explain your reasoning.

45. **Money** You are visiting Canada. You have $21.25 in Canadian currency, and the rest of your money is in U.S. currency. You want to exchange your U.S. currency for Canadian currency. For every dollar you have in U.S. currency, you can get $1.557 in Canadian currency. You want to buy a souvenir that costs $25.50 in Canadian currency. How much money in U.S. currency do you need to exchange to have enough to buy the souvenir?

46. **Challenge** Solve the equation $\frac{4}{9}(\frac{1}{3}x + 6) = \frac{5}{18}x + \frac{1}{3}$. Show the steps you take.

**Mixed Review**

*Find the product or quotient.*  
(Lesson 1.7)

47. $-3(-40)$  
48. $-5(11)$  
49. $-180 \div 5$  
50. $90 \div (-6)$

*Simplify the expression.*  
(Lesson 5.3)

51. $\frac{r}{5} + \frac{7r}{9}$  
52. $\frac{s}{7} - \frac{s}{3}$  
53. $\frac{5t}{2} - \frac{t}{6}$  
54. $\frac{3d}{10} - \frac{8d}{15}$

55. **Rain Gauge** The water level in a rain gauge is $2\frac{3}{4}$ inches. A steady rain raises the water level by $\frac{1}{8}$ inch each hour. When the rain stops, the gauge reads 4 inches. How many hours did the rain last?  
(Lesson 5.6)

**Standardized Test Practice**

56. **Multiple Choice** Which graph represents the solution of the inequality $-\frac{3}{8}x > -9$?

- A.  
  ![Graph A](image1)
- B.  
  ![Graph B](image2)
- C.  
  ![Graph C](image3)
- D.  
  ![Graph D](image4)

57. **Multiple Choice** Which number is not a solution of the inequality $2 - \frac{5}{7}m \geq -3$?

- F. $-9$  
- G. $-7$  
- H. $7$  
- I. $9$
Chapter Review

Vocabulary Review

rational number, p. 219  repeating decimal, p. 219  multiplicative inverse, p. 247
terminating decimal, p. 219  reciprocals, p. 243

1. Give an example of a number that is an integer but not a whole number.
2. Give an example of a number that is a rational number but not a whole number.
3. Give an example of a rational number that is a whole number and an integer.
4. Which number is greater, the repeating decimal 0.7 or the decimal 0.77? Why?
5. Give an example of two numbers that are reciprocals.
6. How is the term multiplicative inverse related to the term reciprocal?

5.1 Rational Numbers

Goal

Write, compare, and order rational numbers.

Example Write the fraction as a decimal.

a. \(-\frac{39}{1000} = -0.039\)

b. \(\frac{3}{11} = 0.272727\ldots = 0.\overline{27}\)

Example Order \(\frac{19}{20}, -3.06, -0.8, \frac{9}{20}, \text{ and } -\frac{54}{25}\) from least to greatest.

Order the numbers on a number line. Write improper fractions as mixed numbers.

From least to greatest, the numbers are \(-3.06, -\frac{54}{25}, -0.8, \frac{9}{20}, \text{ and } \frac{19}{20}\).

Example Write the fraction or mixed number as a decimal.

7. \(\frac{3}{11}\)
8. \(-\frac{7}{80}\)
9. \(-2\frac{3}{5}\)
10. \(4\frac{2}{90}\)

11. Order \(-5.24, 5.5, \frac{134}{25}, -\frac{263}{25}, \text{ and } 5\frac{9}{20}\) from least to greatest.
5.2 Adding and Subtracting Like Fractions

**Goal**
Add and subtract fractions and mixed numbers with the same denominator.

**Example** Find the sum $-6\,\frac{5}{8} + \left(-4\,\frac{1}{8}\right)$.

$$
-6\,\frac{5}{8} + \left(-4\,\frac{1}{8}\right) = \frac{-53}{8} + \frac{-33}{8}
$$

Write mixed numbers as improper fractions.

$$
= \frac{-53 + (-33)}{8}
$$

Write sum of numerators over denominator.

$$
= \frac{-86}{8}
$$

Add.

$$
= \frac{-43}{4} = -10\,\frac{3}{4}
$$

Simplify. Then write improper fraction as a mixed number.

**Find the sum or difference.**

12. $\frac{1}{12} + \left(-\frac{5}{12}\right)$

13. $-\frac{4}{7} - \frac{5}{7}$

14. $7\,\frac{2}{9} - 3\,\frac{8}{9}$

15. $9\,\frac{7}{10} + 5\,\frac{3}{10}$

5.3 Adding and Subtracting Unlike Fractions

**Goal**
Add and subtract fractions and mixed numbers with different denominators.

**Example** Find the difference $4\,\frac{5}{18} - 6\,\frac{8}{9}$.

$$
4\,\frac{5}{18} - 6\,\frac{8}{9} = \frac{77}{18} - \frac{62}{9}
$$

Write mixed numbers as improper fractions.

$$
= \frac{77}{18} - \frac{124}{18}
$$

Write $\frac{62}{9}$ using LCD.

$$
= \frac{77 - 124}{18}
$$

Write difference of numerators over denominator.

$$
= \frac{-47}{18}
$$

Subtract.

$$
= -\frac{21}{18}
$$

Write improper fraction as a mixed number.

**Find the sum or difference.**

16. $\frac{7}{12} - \frac{5}{24}$

17. $-\frac{8}{21} + \frac{9}{14}$

18. $\frac{3}{17} - \frac{15}{34}$

19. $2\,\frac{3}{4} + 2\,\frac{5}{6}$

20. $-13\,\frac{9}{14} + 21\,\frac{17}{28}$

21. $9\,\frac{14}{15} - 18\,\frac{5}{21}$
5.4 Multiplying Fractions

**Goal**
Multiply fractions and mixed numbers.

**Example** Find the product \(-\frac{4}{5} \cdot \frac{25}{42}\).

\[
\frac{4}{5} \cdot \frac{25}{42} = \frac{-2 \cdot 25}{5 \cdot 21} = \frac{-10}{21}
\]
Use rule for multiplying fractions. Divide out common factors. Multiply.

**Example** Simplify the expression.

\[
\frac{m^3 \cdot 2m}{8 \cdot 5} = \frac{m^3 \cdot 2m}{4 \cdot 5} = \frac{m^{3+1}}{20} = \frac{m^4}{20}
\]

Find the product.

22. \(\frac{18}{19} \cdot \frac{38}{27}\)  
23. \(\frac{2}{15} \cdot \frac{5}{8}\)  
24. \(-3 \cdot \frac{1}{17} \cdot \left(-\frac{3}{4}\right)\)  
25. \(\frac{63}{4} \cdot \left(-\frac{7}{9}\right)\)

Simplify the expression.

26. \frac{a^3}{2} \cdot \frac{2a}{9}\)  
27. \frac{3b^2}{4} \cdot \frac{16b}{21}\)  
28. \(-\frac{12n^3}{5} \cdot \frac{n^4}{3}\)  
29. \(-\frac{55}{4} \cdot \frac{12s^4}{25}\)

5.5 Dividing Fractions

**Goal**
Divide fractions and mixed numbers.

**Example** Find the quotient \(\frac{2}{3} \div \left(-\frac{6}{7}\right)\).

\[
\frac{2}{3} \div \left(-\frac{6}{7}\right) = \frac{2}{3} \cdot \left(-\frac{7}{6}\right) = \frac{1 \cdot (-7)}{3 \cdot 6} = \frac{-7}{18}
\]
Multiply by reciprocal. Use rule for multiplying fractions. Divide out common factor. Multiply.

Find the quotient.

30. \(-\frac{6}{7} \div \frac{36}{77}\)  
31. \(-\frac{21}{58} \div \frac{3}{16}\)  
32. \(16 \div \frac{2}{3} \div 2\)  
33. \(-\frac{3}{11} \div \frac{17}{55}\)
5.6 Using Multiplicative Inverses to Solve Equations

**Goal**
Use multiplicative inverses to solve equations with fractional coefficients.

**Example** Solve the equation \( \frac{4}{5}t = -\frac{8}{11} \).

\[
\begin{align*}
\frac{4}{5}t &= -\frac{8}{11} & \text{Write original equation.} \\
\quad \quad \frac{5}{4} \left(\frac{4}{5}t\right) &= \frac{5}{4} \left(-\frac{8}{11}\right) & \text{Multiply each side by multiplicative inverse of } \frac{4}{5}. \\
\quad t &= \frac{5}{4} \left(-\frac{8}{11}\right) & \text{Multiplicative inverse property} \\
\quad t &= -\frac{10}{11} & \text{Multiply.}
\end{align*}
\]

**Solve the equation.**

34. \( \frac{14}{27}x = -\frac{7}{12} \)  
35. \( \frac{5}{8}x = \frac{10}{17} \)  
36. \( \frac{3}{8} - \frac{3}{4}x + 1 \)  
37. \( \frac{5}{6}x - \frac{1}{4} = -\frac{11}{24} \)

5.7 Equations and Inequalities with Rational Numbers

**Goal**
Solve equations and inequalities with rational numbers.

**Example** Solve the equation \( -\frac{8}{9}x + \frac{1}{6} = \frac{49}{54} \) by first clearing the fractions.

\[
\begin{align*}
-\frac{8}{9}x + \frac{1}{6} &= \frac{49}{54} & \text{Write original equation.} \\
54 \left(-\frac{8}{9}x + \frac{1}{6}\right) &= 54 \left(\frac{49}{54}\right) & \text{Multiply each side by LCD of fractions.} \\
54 \left(-\frac{8}{9}x\right) + 54 \left(\frac{1}{6}\right) &= 54 \left(\frac{49}{54}\right) & \text{Distributive property} \\
-48x + 9 &= 49 & \text{Simplify.} \\
-48x + 9 - 9 &= 49 - 9 & \text{Subtract 9 from each side.} \\
-48x &= 40 & \text{Simplify.} \\
\frac{-48x}{-48} &= \frac{40}{-48} & \text{Divide each side by } -48. \\
x &= -\frac{5}{6} & \text{Simplify.}
\end{align*}
\]

**Solve the equation by first clearing the fractions.**

38. \( \frac{3}{7}x + \frac{5}{14} = \frac{19}{42} \)  
39. \( \frac{5}{16} = -\frac{3}{32} + \frac{7}{8}x \)  
40. \( -\frac{3}{4}x - \frac{5}{8} = -\frac{1}{56} \)
Chapter Test

Write the fraction or mixed number as a decimal.

1. $\frac{-7}{125}$  
2. $10\frac{4}{9}$  
3. $\frac{-2}{27}$  
4. $\frac{37}{10,000}$

Write the decimal as a fraction or mixed number.

5. 11.85  
6. $-7.52$  
7. 0.7  
8. 0.63

Find the sum or difference.

9. $-\frac{15}{24} + \frac{19}{24}$
10. $-4\frac{1}{3} - 8\frac{1}{3}$
11. $\frac{4}{5} - \frac{11}{15}$
12. $-3\frac{5}{7} + 1\frac{2}{9}$

13. **Birds** The Northern junco, a bird found in Alaska and Canada, varies in length from $5\frac{1}{2}$ inches to $6\frac{3}{4}$ inches. Find the range of the lengths of the Northern junco.

Simplify the expression.

14. $\frac{4m}{21} + \frac{17m}{21}$
15. $\frac{3r}{22} - \frac{5r}{44}$
16. $\frac{-7n}{18} - \frac{11n}{30}$
17. $-\frac{4z}{35} - \frac{8z}{25}$

18. **Geometry** The side lengths of a triangle are $3\frac{5}{8}$ inches, $4\frac{5}{16}$ inches, and 2 inches. What is the perimeter of the triangle?

Find the product or quotient.

19. $\frac{8}{9} \cdot \left(-\frac{3}{10}\right)$
20. $4 \div \left(-2\frac{4}{15}\right)$
21. $\frac{-9}{20} \cdot \frac{22}{3}$
22. $14\frac{5}{6} \div 2\frac{1}{8}$

Simplify the expression.

23. $\frac{4r^3}{15} \cdot \frac{5r^3}{12}$
24. $\frac{-7n^2}{12} \cdot \left(-\frac{18n}{49}\right)$
25. $\frac{-9t^2}{13} \cdot \frac{t}{12}$
26. $\frac{-25w^3}{42} \cdot \left(-\frac{3w}{10}\right)$

27. **Encyclopedia** An encyclopedia has 30 volumes. The total weight of these volumes is $71\frac{1}{4}$ pounds. Find the average weight of a volume.

Solve the equation. Check your solution.

28. $\frac{7}{8}m = -8$  
29. $\frac{9}{25} = \frac{3}{5}t$  
30. $\frac{-13}{20} = \frac{1}{4} + \frac{3}{5}t$  
31. $\frac{1}{7}g + 8 = 2$

Solve the equation or inequality by first clearing the fractions or decimals.

32. $9.2m + 1.4 = 12.9$  
33. $10 = 8.22w - 3.152$  
34. $\frac{2}{15}b - \frac{4}{5} < \frac{2}{3}$  
35. $\frac{1}{3}m - \frac{7}{18} = \frac{4}{9}$
Chapter Standardized Test

**Test-Taking Strategy** Mark test questions that you can’t answer on the first try. Move on to new questions and return later to the marked questions.

1. Which fraction is equivalent to $0.5\overline{2}$?
   - A. $\frac{4}{9}$  
   - B. $\frac{51}{99}$  
   - C. $\frac{52}{99}$  
   - D. $\frac{5}{9}$

2. Which list of numbers is in order from least to greatest?
   - A. $-3 \frac{3}{10}, -3.2, 7 \frac{7}{8}, 0.9, 0.9$  
   - B. $-3 \frac{3}{10}, -3.2, 7 \frac{7}{8}, 0.9, 0.9$  
   - C. $-3 \frac{3}{10}, -3.2, 7 \frac{7}{8}, 0.9, 0.9$  
   - D. $-3 \frac{3}{10}, -3.2, 7 \frac{7}{8}, 0.9, 0.9$

3. Simplify the expression $-\frac{12n}{35} - \frac{13n}{35}$.
   - A. $-\frac{25n}{35}$  
   - B. $-\frac{5n}{7}$  
   - C. $-\frac{n}{35}$  
   - D. $\frac{25n}{35}$

4. Find the sum $3 \frac{1}{4} + \left(-2 \frac{3}{8}\right)$.
   - A. $-5 \frac{5}{8}$  
   - B. $-\frac{7}{8}$  
   - C. $\frac{7}{8}$  
   - D. $5 \frac{5}{8}$

5. Find the product $-2 \frac{5}{8} \cdot \left(-5 \frac{3}{7}\right)$.
   - A. $-14 \frac{1}{4}$  
   - B. $-2 \frac{19}{28}$  
   - C. $2 \frac{19}{28}$  
   - D. $14 \frac{1}{4}$

6. Find the quotient $1 \frac{1}{12} \div 2 \frac{7}{16}$.
   - A. $\frac{2}{21}$  
   - B. $\frac{4}{9}$  
   - C. $2 \frac{7}{192}$  
   - D. $2 \frac{41}{64}$

7. What is the solution of the equation $\frac{4}{5} - 12 = \frac{3}{10}x$?
   - A. $-37 \frac{1}{3}$  
   - B. $-9 \frac{1}{3}$  
   - C. $9 \frac{1}{3}$  
   - D. $37 \frac{1}{3}$

8. Half the books you own are mysteries. One third are historical fiction. The other four books are science fiction. How many of your books are mysteries?
   - A. 4  
   - B. 8  
   - C. 12  
   - D. 24

9. Which number is not a solution of $\frac{2}{3}x + \frac{1}{6} \geq 1\frac{17}{18}$?
   - A. 2  
   - B. 3  
   - C. 4  
   - D. 5

10. **Short Response** You have a length of cloth that is $2 \frac{3}{4}$ yards long and $1 \frac{1}{4}$ yards wide. You want to cut the cloth into 20 squares that are $1 \frac{1}{4}$ feet long on each side. Do you have enough cloth to cut all the squares? Explain why or why not.

11. **Extended Response** A recipe for making 24 pretzels calls for using $1 \frac{1}{2}$ cups of milk. A baker wants to use the recipe to make pretzels to sell. He wants to buy enough milk to make 100 pretzels each day over the next 5 days.
   - a. How many cups of milk does the baker need for all the pretzels?
   - b. How many gallons of milk should he buy? Use the fact that 1 gallon = 16 cups. Explain your answer.
Making a Plan

Using Multiple Strategies

When you make a plan to solve a problem, be aware that there is often more than one strategy you can use to find the solution.

**Problem** You and your younger brother have volunteered to paint a wall of a community center as part of a community service project. You could paint the wall by yourself in 3 hours. Your brother (who has less experience painting) could paint the wall by himself in 6 hours. How long will it take to paint the wall if you and your brother work together?

1. **Use estimation.**

   An upper limit on the time you and your brother take to paint the wall is the time it would take the faster painter to do the job working alone. Because you are the faster painter and can do the job in 3 hours, you can estimate that you and your brother will paint the wall in less than 3 hours.

2. **Draw a diagram.**

   Your work rate when painting is \( \frac{1}{3} \) of the wall per hour. Your brother’s work rate is \( \frac{1}{6} \) of the wall per hour. Use these rates to draw a diagram showing how much of the wall is painted after each hour. The diagram below shows that you and your brother will take 2 hours to paint the wall.

   After 1 hour:
   
   ![Diagram showing 1/3 + 1/6 of the wall painted after 1 hour]

   After 2 hours:
   
   ![Diagram showing 2/3 + 1/3 of the wall painted after 2 hours]

3. **Solve an equation.**

   A third way to solve the problem is to use an equation. Let \( t \) represent the time (in hours) that you and your brother take to paint the wall. Make a table to find how much work each person does in \( t \) hours.

<table>
<thead>
<tr>
<th></th>
<th>Work rate ( \times ) Time = Work done</th>
</tr>
</thead>
<tbody>
<tr>
<td>You</td>
<td>( \frac{1}{3} ) ( \times ) ( t ) = ( \frac{t}{3} )</td>
</tr>
<tr>
<td>Brother</td>
<td>( \frac{1}{6} ) ( \times ) ( t ) = ( \frac{t}{6} )</td>
</tr>
</tbody>
</table>

   The sum of your work and your brother’s work equals 1 whole wall painted. Use this fact to write and solve an equation.

   \[
   \frac{t}{3} + \frac{t}{6} = 1
   \]

   \[
   6 \left( \frac{t}{3} + \frac{t}{6} \right) = 6(1)
   \]

   \[
   6 \left( \frac{t}{3} \right) + 6 \left( \frac{t}{6} \right) = 6
   \]

   \[
   2t + t = 6
   \]

   \[
   3t = 6
   \]

   \[
   t = 2
   \]

   So, you and your brother will take 2 hours to paint the wall.
1. **Wallpapering** You and your sister are wallpapering a wall of your family’s living room. The diagram below shows how much of the wall each of you finished in 1 hour. What is the total time you and your sister spend wallpapering the wall?

   ![Wallpapering Diagram]

<table>
<thead>
<tr>
<th>You</th>
<th>Sister</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   In Exercises 2–5, use at least two strategies to solve the problem.

2. **Shingling a Roof** A contractor estimates that either she or her assistant could shingle the roof of a certain building in 8 days working alone. How long would the contractor and her assistant take to shingle the roof if they work together?

3. **Splitting Wood** Paul can split a cord of wood in 12 hours. His father can split a cord of wood in 4 hours. How long do Paul and his father take to split a cord of wood if they work together?

4. **Shoveling Snow** Sara can shovel the snow off her family’s driveway in 30 minutes. Her sister can shovel the driveway in 20 minutes. How long do Sara and her sister take to shovel the driveway if they work together?

5. **Mowing a Lawn** You and a friend run a lawn mowing service. You can mow your next-door neighbor’s lawn in 45 minutes. Your friend can mow the lawn in 1 hour. How long do you and your friend take to mow the lawn when working together?

6. **Filling a Sink** The faucet of a sink can fill the sink with water in 2 minutes. The drain can empty the sink in 3 minutes. If the faucet is turned on and the drain is left open, does the sink ever fill up with water? Justify your answer in two different ways.

7. **Painting** Look back at the painting problem on page 264. Suppose your sister helps you and your brother paint the wall. She can paint the wall by herself in 2 hours. Use a diagram and an equation to find how long the three of you take to paint the wall when working together.

8. **Biking** Heidi is riding her bike from her house to a park. After Heidi has traveled 400 feet, her brother Josh leaves the house on his bike to catch up to her. Heidi’s speed is 22 feet per second, while Josh’s speed is 27 feet per second. In parts (a) and (b), use the specified strategy to find how long Josh takes to catch up to Heidi.

   a. Copy and complete the table below. Continue adding rows to the table until you know the solution to the problem.

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Heidi’s distance traveled (ft)</th>
<th>Josh’s distance traveled (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>400</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>20</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

   b. Write and solve an equation.

9. **Car Wash** A science club holds a car wash to raise money. Two groups of students wash the cars. One group can wash 6 cars per hour. The other group can wash 4 cars per hour. How long do both groups take to wash a total of 30 cars? Use a diagram and an equation to find the answer.