Chapter 4 **Factors, Fractions, and Exponents**
- Find greatest common factors and least common multiples.
- Identify equivalent fractions and write fractions in simplest form.
- Use rules of exponents and scientific notation.

Chapter 5 **Rational Numbers and Equations**
- Write fractions as decimals and decimals as fractions.
- Perform operations with fractions and mixed numbers.
- Solve equations and inequalities with rational numbers.

Chapter 6 **Ratio, Proportion, and Probability**
- Write and compare ratios and rates.
- Write and solve proportions.
- Find theoretical and experimental probabilities.

Chapter 7 **Percents**
- Find and use equivalent decimals, fractions, and percents.
- Use proportions and the percent equation to solve percent problems.
- Solve problems involving percent of change.

From Chapter 7, p. 338
How many solar cars competed in a race?
Chapter 4

Factors, Fractions, and Exponents

Before

In previous chapters you’ve . . .
- Evaluated powers
- Compared and ordered integers
- Written and evaluated variable expressions

Now

In Chapter 4 you’ll study . . .
- Factoring numbers and monomials
- Finding common factors and common multiples
- Simplifying and comparing fractions
- Multiplying and dividing powers
- Writing numbers in scientific notation

Why?

So you can solve real-world problems about . . .
- coin collecting, p. 175
- community gardens, p. 180
- monarch butterflies, p. 186
- the Great Pyramid, p. 197
- geckos, p. 201
- compact discs, p. 208

How far away is this giant cloud of gas and dust?
Astronomy  New stars are forming in the Orion Nebula, a vast cloud of gas and dust nearly 15,100,000,000,000,000 kilometers from Earth. In this chapter, you will learn to use scientific notation to express large numbers like this one.

What do you think?  The distance from Earth to the Orion Nebula can be read as 15.1 quadrillion kilometers. How many zeros are there in 1 quadrillion?
Chapter Prerequisite Skills

PREREQUISITE SKILLS QUIZ

Preparing for Success To prepare for success in this chapter, test your knowledge of these concepts and skills. You may want to look at the pages referred to in blue for additional review.

1. **Vocabulary** Label the power, the base, and the exponent in the expression $9^3$.

   Write the mixed number as an improper fraction. (p. 778)
   
   2. $5 \frac{1}{3}$  
   3. $6 \frac{2}{5}$  
   4. $3 \frac{7}{9}$  
   5. $8 \frac{5}{6}$

   Find the product. (p. 780)
   
   6. $20 \times \frac{3}{5}$  
   7. $32 \times \frac{7}{8}$  
   8. $\frac{2}{3} \times 27$  
   9. $\frac{9}{10} \times 50$

   Write the power in words and as a repeated multiplication. Then evaluate the power. (p. 10)
   
   10. $5^4$  
   11. $12^3$  
   12. $(1.3)^3$  
   13. $(0.2)^2$

NOTETAKING STRATEGIES

**Note Worthy**

You will find a notetaking strategy at the beginning of each chapter. Look for additional notetaking and study strategies throughout the chapter.

**RECORDING THE PROCESS** When copying examples in class, be sure to write a verbal description next to each step in a calculation. Then you can refer to the example when solving similar exercises.

**Calculation step:**

\[-5x - 8 = -23\]
\[-5x - 8 + 8 = -23 + 8\]
\[-5x = -15\]
\[-\frac{5x}{-5} = \frac{-15}{-5}\]
\[x = 3\]

**Verbal description:**

Write original equation.  
Add 8 to each side.  
Simplify.  
Divide each side by -5.  
Simplify.

The strategy above will be helpful in Lesson 4.5 when you are simplifying variable expressions with powers.
A prime number is a whole number that is greater than 1 and has exactly two whole number factors, 1 and itself. A composite number is a whole number that is greater than 1 and has more than two whole number factors. A multiple of a number is the product of the number and any nonzero whole number.

**Investigate**

Use patterns to determine if a number is prime or composite.

1. Write the whole numbers from 2 to 60 in rows of 6 as shown.

2. Start with the number 2. Circle it and cross out every multiple of 2 after 2. (The first few multiples of 2 have been crossed out for you.)

3. Move to the next number that is not crossed out. Circle it and cross out all other multiples of that number.

4. Repeat Step 3 until every number is either crossed out or circled.

**Draw Conclusions**

1. **Writing** What can you say about the numbers that have been crossed out? What can you say about the numbers that have been circled? Use the words prime and composite in your answers.

2. **Critical Thinking** All the numbers in the sixth column are crossed out because they are all multiples of 6. Explain why all the numbers in the second column (except for 2), in the third column (except for 3), and in the fourth column are crossed out.

3. **Predict** Suppose you repeated the activity but arranged the numbers in rows of 10 instead of 6. Predict which columns would contain only crossed-out numbers. Then check your prediction.
Factors and Prime Factorization

Before

You found the product of two or more numbers.

Now

You'll write the prime factorization of a number.

Why?

So you can count ways to display a firefly collection, as in Ex. 56.

Yearbook

You are working on your school yearbook. Each page will have 24 student photos. The photos will be arranged in a rectangular display with the same number of photos in each row. How many ways can you arrange the photos so that there are no more than 10 photos in any row or column?

You can use factors to determine the number of possible displays. In this chapter, finding the factors of a given whole number means finding whole numbers that divide the given number without a remainder. For example, two factors of 50 are 5 and 10.

Example 1

Writing Factors

For the yearbook described above, each possible display will consist of 24 photos. Because there will be the same number of photos in each row, the number of photos in each row will be a factor of 24.

1) Write 24 as a product of two whole numbers in all possible ways.

- 1 \cdot 24
- 2 \cdot 12
- 3 \cdot 8
- 4 \cdot 6

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

2) Use the factors to find all the rectangular displays with no more than 10 photos in any row or column.

- 3 \text{ rows of 8 photos}
- 6 \text{ rows of 4 photos}
- 8 \text{ rows of 3 photos}
- 4 \text{ rows of 6 photos}

Answer

There are 4 possible displays.

Checkpoint

Write all the factors of the number.

1. 30  2. 31  3. 45  4. 87
**Prime and Composite Numbers**  A **prime number** is a whole number that is greater than 1 and has exactly two whole number factors, 1 and itself. A **composite number** is a whole number that is greater than 1 and has more than two whole number factors. The number 1 is neither prime nor composite.

<table>
<thead>
<tr>
<th>Number</th>
<th>Factors</th>
<th>Prime or composite?</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>1, 2, 3, 4, 6, 8, 12, 24</td>
<td>Composite</td>
</tr>
<tr>
<td>41</td>
<td>1, 41</td>
<td>Prime</td>
</tr>
<tr>
<td>51</td>
<td>1, 3, 17, 51</td>
<td>Composite</td>
</tr>
<tr>
<td>89</td>
<td>1, 89</td>
<td>Prime</td>
</tr>
<tr>
<td>121</td>
<td>1, 11, 121</td>
<td>Composite</td>
</tr>
</tbody>
</table>

**Prime Factorization**  When you write a number as a product of prime numbers, you are writing its **prime factorization**. You can use a diagram called a **factor tree** to write the prime factorization of a number.

**Example 2**  **Writing a Prime Factorization**

Write the prime factorization of 630.

One possible factor tree:

```
       630
      /  \
     30   21
    / \  /  \  
   6   5   3   7
  / \ / \ / \  
2  3 5 3 7
```

Write original number.
Write 630 as $30 \cdot 21$.
Write 30 as $6 \cdot 5$. Write 21 as $3 \cdot 7$.
Write 6 as $2 \cdot 3$.

Another possible factor tree:

```
       630
      /  \    
     63   10
    /  \     
   9   7     2   5
  / \ / \  / \  
3 3 7 3 7 2 5
```

Write original number.
Write 630 as $63 \cdot 10$.
Write 63 as $9 \cdot 7$. Write 10 as $2 \cdot 5$.
Write 9 as $3 \cdot 3$.

Both trees give the same result: $630 = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7 = 2 \cdot 3^2 \cdot 5 \cdot 7$.

**Answer**  The prime factorization of 630 is $2 \cdot 3^2 \cdot 5 \cdot 7$.

**Checkpoint**  Tell whether the number is **prime or composite**. If it is composite, write its prime factorization.

5. 32  6. 56  7. 59  8. 83
9. 101  10. 175 11. 180 12. 420
Factoring Monomials

A monomial is a number, a variable, or the product of a number and one or more variables raised to whole number powers.

<table>
<thead>
<tr>
<th>Monomials</th>
<th>Not monomials</th>
</tr>
</thead>
<tbody>
<tr>
<td>7x</td>
<td>7 + x</td>
</tr>
<tr>
<td>25mn²</td>
<td>25m − n²</td>
</tr>
<tr>
<td>24y²z²</td>
<td>24 + y³ + z²</td>
</tr>
</tbody>
</table>

To factor a monomial, write the monomial as a product of prime numbers and variables with exponents of 1.

Example 3

Factoring a Monomial

Factor the monomial \(28xy^3\).

\[
28xy^3 = 2 \cdot 2 \cdot 7 \cdot x \cdot y^3
\]

Write \(28\) as \(2 \cdot 2 \cdot 7\).

\[
= 2 \cdot 2 \cdot 7 \cdot x \cdot y \cdot y \cdot y
\]

Write \(y^3\) as \(y \cdot y \cdot y\).

Checkpoint

Factor the monomial.

13. \(6ab\)  
14. \(15n^3\)  
15. \(3x^3y^2\)  
16. \(36s^4r\)

4.1 Exercises

Guided Practice

Vocabulary Check
1. Describe how to write the prime factorization of a number.
2. Explain why 34 is a composite number.

Skill Check
Write all the factors of the number.

3. \(16\)  
4. \(32\)  
5. \(29\)  
6. \(55\)

Tell whether the number is prime or composite.

7. \(9\)  
8. \(15\)  
9. \(17\)  
10. \(23\)

Write the prime factorization of the number.

11. \(10\)  
12. \(18\)  
13. \(25\)  
14. \(39\)

15. Error Analysis Describe and correct the error in writing the prime factorization of 60.

\[60 = 3 \cdot 4 \cdot 5\]
Write all the factors of the number.

16. 8 17. 53 18. 12 19. 33
20. 36 21. 60 22. 71 23. 144

Tell whether the number is prime or composite.

24. 7 25. 16 26. 21 27. 19
28. 121 29. 51 30. 84 31. 141

Copy and complete the factor tree. Then write the prime factorization of the number.

32. \[\begin{array}{c}
8 \\
\hline
2 \cdot ? \cdot ? \\
\hline
2 \cdot 2 \cdot ? \cdot 13
\end{array}\]
33. \[\begin{array}{c}
180 \\
\hline
9 \cdot ? \\
\hline
3 \cdot ? \cdot 5 \cdot ?
\end{array}\]

Write the prime factorization of the number.

34. 26 35. 58 36. 63 37. 85
38. 120 39. 160 40. 154 41. 195
42. 202 43. 210 44. 217 45. 225

46. **Coin Collecting** The U.S. Mint began issuing state quarters in 1999. There will be one state quarter for each of the 50 states. You are collecting the state quarters and want to design a rectangular display with the same number of quarters in each row. How many ways can you arrange your display?

47. **Writing** Give an expression that is a monomial and tell why it is an example of a monomial. Then give an expression that is *not* a monomial and tell why it is not an example of a monomial.

Factor the monomial.

48. 11cd 49. 19m^3 50. 3f^6 51. 21ab
52. 5xy^2 53. 35rs^5 54. 2y^4z^3 55. 40m^2n

56. **Fireflies** There are 69 species of flashing fireflies, also known as lightning bugs, in the United States. A museum is designing a rectangular display of these 69 species with the same number of fireflies in each row. How many displays are possible?

57. **Critical Thinking** Explain why all two-digit whole numbers with 5 as the ones’ digit are composite.

Use the prime factorization of the number to list all of its factors.

58. 240 59. 335 60. 500 61. 201

List all the factors of the monomial.

62. 6ab^2 63. 52w 64. 2r^3s 65. 7xyz
66. **Extended Problem Solving** A geologist has collected 102 different types of silicate minerals. The geologist has taken a photograph of each mineral and wants to make a display of the photographs.

   a. **Calculate** How many rectangular arrangements of the photographs are possible?

   b. The geologist wants no more than 15 photographs in any row or column. How many rectangular arrangements satisfying this requirement are possible?

   c. **Analyze** The geologist decreases the number of photographs in the display to 96. How many rectangular arrangements, with no more than 15 photographs in any row or column, are now possible?

67. **Conjecture** The square of an integer is called a *perfect square*. Write the prime factorizations, with exponents, for these perfect squares: 4, 9, 16, 25, 36, and 64. Make a conjecture about the exponents in the prime factorization of a perfect square.

68. **Perfect Numbers** A *perfect number* is a number that is the sum of all its factors except for itself. The smallest perfect number is 6, because \( 6 = 1 + 2 + 3 \). The next perfect number is between 20 and 30. Find the next perfect number.

69. **Critical Thinking** If 18 is a factor of a number, what other numbers must also be factors of that number? Give examples to support your answer.

70. **Challenge** What is the least whole number that has exactly 7 factors, including 1 and itself? Explain your answer.

---

**Mixed Review**

**Algebra Basics** Solve the equation. Check your solution. *(Lessons 2.5, 2.6)*

71. \( a + 24 = 16 \)  
72. \( 33 + b = 58 \)  
73. \( c - 14 = 18 \)  
74. \( d - 10 = 10 \)

75. \( 6r = 48 \)  
76. \( -10s = 50 \)  
77. \( \frac{f}{9} = -7 \)  
78. \( \frac{u}{-2} = -14 \)

**Write the verbal sentence as an equation. Then solve the equation.** *(Lesson 3.3)*

79. Fifteen plus a number is equal to 21 minus the number.

80. Two times the sum of 3 and a number is equal to 5 plus the number.

81. Eight plus a number is equal to \(-3\) times the number.

---

**Standardized Test Practice**

82. **Multiple Choice** For which value of \( x \) is the value of the expression \( 7x + 1 \) a prime number?

   A. 0  
   B. 1  
   C. 3  
   D. 4

83. **Multiple Choice** Which expression is the prime factorization of 252?

   F. \( 2^2 \cdot 3^2 \cdot 7^2 \)  
   G. \( 2^2 \cdot 3^2 \cdot 7 \)  
   H. \( 2 \cdot 3^2 \cdot 7 \)  
   I. \( 2 \cdot 3^2 \cdot 7^2 \)

84. **Short Response** The area of a rectangle is 54 square inches. The length and width are whole numbers of inches. Find all possible dimensions of the rectangle. Which dimensions result in the rectangle having the greatest perimeter?
Greatest Common Factor

**Vocabulary**
- common factor, p. 177
- greatest common factor (GCF), p. 177
- relatively prime, p. 178

**Music Choir** A choir director wants to divide a choir into smaller groups. The choir has 24 sopranos, 60 altos, and 36 tenors. Each group will have the same number of each type of voice. What is the greatest number of groups that can be formed? How many sopranos, altos, and tenors will be in each group?

A **common factor** is a whole number that is a factor of two or more nonzero whole numbers. The greatest of the common factors is the **greatest common factor (GCF)**.

**Example 1** Finding the Greatest Common Factor

For the choir described above, the greatest number of groups that can be formed is given by the GCF of 24, 60, and 36. You can use one of two methods to find the GCF.

**Method 1** List the factors of each number. Identify the greatest number that is on every list.

Factors of 24: 1, 2, 3, 4, 6, (12), 24
Factors of 60: 1, 2, 3, 4, 5, 6, 10, (12), 15, 20, 30, 60
Factors of 36: 1, 2, 3, 4, 6, 9, (12), 18, 36

The common factors are 1, 2, 3, 4, 6, and 12. The GCF is 12.

**Method 2** Write the prime factorization of each number. The GCF is the product of the common prime factors.

\[
\begin{align*}
24 &= 2 \cdot 2 \cdot 2 \cdot 3 \\
60 &= 2 \cdot 2 \cdot 3 \cdot 5 \\
36 &= 2 \cdot 2 \cdot 3 \cdot 3
\end{align*}
\]

The common prime factors are 2, 2, and 3. The GCF is the product \(2 \cdot 2 \cdot 3 = 12\).

**Answer** The greatest number of groups that can be formed is 12. Each group will have 24 \(\div 12 = 2\) sopranos, 60 \(\div 12 = 5\) altos, and 36 \(\div 12 = 3\) tenors.

**Checkpoint**

Find the greatest common factor of the numbers.

1. 12, 30
2. 21, 42
3. 16, 32, 40
4. 27, 45, 90

Lesson 4.2 Greatest Common Factor
Relatively Prime Two or more numbers are **relatively prime** if their greatest common factor is 1.

### Example 2  Identifying Relatively Prime Numbers

Find the greatest common factor of the numbers. Then tell whether the numbers are relatively prime.

a. $24, 45$

b. $35, 54$

**Solution**

a. List the factors of each number. Identify the greatest number that the lists have in common.

**Factors of 24:** $1, 2, 3, 4, 6, 8, 12, 24$

**Factors of 45:** $1, 3, 5, 9, 15, 45$

The GCF is 3. So, the numbers are not relatively prime.

b. Write the prime factorization of each number.

$35 = 5 \cdot 7$

$54 = 2 \cdot 3 \cdot 3 \cdot 3$

There are no common prime factors. However, two numbers always have 1 as a common factor. So, the GCF is 1, and the numbers are relatively prime.

### Checkpoint

Find the greatest common factor of the numbers. Then tell whether the numbers are relatively prime.

5. $18, 33$

6. $39, 50$

7. $110, 77$

8. $21, 160$

9. **Critical Thinking** Suppose you divide two numbers by their greatest common factor. What is the relationship between the resulting quotients?

---

**Monomials and the GCF** You can find the greatest common factor of two or more monomials by factoring each monomial.

### Example 3  Finding the GCF of Monomials

Find the greatest common factor of $18xy^2$ and $28x^2y^2$.

Factor the monomials. The GCF is the product of the common factors.

$18xy^2 = 2 \cdot 3 \cdot 3 \cdot x \cdot y \cdot y$

$28x^2y^2 = 2 \cdot 2 \cdot 7 \cdot x \cdot x \cdot y \cdot y$

**Answer** The GCF is $2xy^2$.

### Checkpoint

Find the greatest common factor of the monomials.

10. $6x, 15x$

11. $20x^2, 36x$

12. $32y^2, 6x^2y$

13. $7xy^3, 28xy^2$
Guided Practice

Vocabulary Check
1. What does it mean for a number to be a common factor of two numbers?
2. Find two pairs of relatively prime numbers from 5, 10, 16, and 25.

Skill Check
Find the greatest common factor of the numbers. Then tell whether the numbers are relatively prime.
3. 7, 28
4. 34, 38
5. 11, 51
6. 32, 81

Find the greatest common factor of the monomials.
7. 18c, 4c
8. r, r^4
9. 5m, 20m^3
10. 3x^2, 15x^3

Guided Problem Solving
11. Art Supplies To celebrate a grand opening, the owner of an art supplies store is making free gift bags for customers. The owner has 225 pastel crayons, 75 paintbrushes, and 120 tubes of oil paint. Each gift bag must be identical. What is the greatest number of gift bags the owner can make?

1. Write the prime factorization of each number.
2. What are the common prime factors of the numbers? What is the GCF of the numbers?
3. What does the GCF represent in this situation?

Practice and Problem Solving

Find the greatest common factor of the numbers.
12. 28, 42
13. 21, 99
14. 34, 85
15. 12, 36
16. 32, 55
17. 54, 89
18. 76, 86
19. 120, 960

Find the greatest common factor of the numbers. Then tell whether the numbers are relatively prime.
20. 9, 26
21. 11, 55
22. 12, 33
23. 77, 51
24. 58, 60
25. 121, 280
26. 64, 144
27. 28, 84

Find the greatest common factor of the monomials.
28. 16x, 36x
29. 18m^2, 7m
30. 18k, 15k^3
31. 2x, 8x^2, 6x^3
32. **Music Camp** A summer music camp has 88 participants. The camp has 32 vocalists, 16 drummers, 24 guitarists, and 16 bassists. What is the greatest number of identical bands that can be formed using all the participants? How many vocalists will be in each band?

33. **Flower Bouquets** The science club is selling flowers for a fundraiser. The club wants to make bouquets from 4 types of flowers. The circle graph shows how many flowers of each type the club has. What is the greatest number of identical bouquets that can be made? What will each bouquet contain?

```
Types of Flowers

<table>
<thead>
<tr>
<th>Flower</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freesia</td>
<td>21</td>
</tr>
<tr>
<td>Iris</td>
<td>42</td>
</tr>
<tr>
<td>Lily</td>
<td>56</td>
</tr>
<tr>
<td>Daisy</td>
<td>63</td>
</tr>
</tbody>
</table>
```

Tell whether the numbers are relatively prime.

34. 115, 207
35. 224, 243
36. 152, 171

Find the greatest common factor of the monomials.

37. \(12m^2n^3, 70m^3n\)
38. \(72a^3b^2, 86a\)
39. \(44m^2n, 48mn^2\)
40. \(a^3b^2, ab^3\)
41. \(3x, 7xy^2\)
42. \(4rs^2, 27st^3\)
43. \(18wx^2, 45wx\)
44. \(12y^2, 15y^3, 5y\)
45. \(rs^3, s^3t, r^2st^2\)

46. **Community Garden** You want to cover the walkway of a community garden with square clay tiles. The space you want to cover is a rectangle 42 inches wide by 72 inches long. Assuming you want to cover the space exactly without cutting any tiles, what is the greatest side length you can use for the tiles?

47. **Bracelets** You want to make woven plastic bracelets. You have 3 pieces of plastic lacing with lengths 45 cm, 75 cm, and 60 cm. You need to cut the lacing into pieces of the same length. What is the greatest possible length each piece can be, without any lacing being wasted?

48. **Critical Thinking** The greatest common factor of 30 and a number \(n\) is 6. Find a possible value for \(n\). Are there other possible values for \(n\)? Explain.

49. **Extended Problem Solving** In the future, scientists may want to make a unit of time that is convenient for people living on both Earth and Mars. The new unit of time, called the space-hour, should divide evenly into the number of minutes in each planet’s day. Under the current Earth definition of minutes, Earth has 1440 minutes per day, and Mars has approximately 1480 minutes per day.

a. **Analyze** What is the greatest number of minutes that could be in a space-hour?

b. **Apply** How many space-hours would there be each day on Earth? on Mars?

c. **Idea** A spacecraft that uses current technology can take 210 days to travel from Earth to Mars. Use a calculator to find how long this trip would be in space-hours.
50. **Writing** If \(a\) and \(b\) are nonzero whole numbers and \(a\) is a factor of \(b\), what is the GCF of \(a\) and \(b\)? Explain your thinking and give three numerical examples to support your answer.

51. **Critical Thinking** If \(a\) and \(b\) are relatively prime numbers and \(b\) and \(c\) are relatively prime numbers, are \(a\) and \(c\) relatively prime numbers? Give examples to support your answer.

52. **Challenge** Consider the pattern \(2x, 6x^2, 18x^3, 54x^4, \ldots\). What are the next two monomials in the pattern? What is the GCF of all the monomials in the pattern? What is the GCF of all the monomials in the pattern excluding the first monomial?

---

**Mixed Review**

**Find the sum or difference. (p. 779)**

53. \(\frac{2}{9} + \frac{5}{9}\)

54. \(\frac{3}{7} + \frac{3}{7}\)

55. \(\frac{14}{15} - \frac{8}{15}\)

56. \(\frac{11}{20} - \frac{3}{20}\)

**Find the product. (p. 780)**

57. \(60 \times \frac{3}{10}\)

58. \(28 \times \frac{1}{4}\)

59. \(\frac{5}{12} \times 36\)

60. \(\frac{3}{7} \times 49\)

**Write the prime factorization of the number. (Lesson 4.1)**

61. 125

62. 70

63. 52

64. 200

---

**Standardized Test Practice**

65. **Multiple Choice** Which numbers are **not** relatively prime?
   - A. 32, 65
   - B. 34, 69
   - C. 63, 91
   - D. 26, 85

66. **Short Response** You are making first-aid kits to go camping. You have 48 bandages, 15 squares of gauze, 6 tubes of antibiotic ointment, and 6 ice packs. What is the greatest number of identical first-aid kits that you can make? How many of each item will each first-aid kit contain?

---

**Brain Game**

Each number in the third column of the table is the greatest common factor of the numbers in the same row. Each number in the first two columns has exactly one digit that is different from the number above it and exactly one digit that is different from the number below it.

Copy the table and fill in each of the blanks with a number that satisfies the conditions.

<table>
<thead>
<tr>
<th>First number</th>
<th>Second number</th>
<th>GCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>945</td>
<td>?</td>
<td>105</td>
</tr>
<tr>
<td>?</td>
<td>435</td>
<td>15</td>
</tr>
<tr>
<td>648</td>
<td>432</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>532</td>
<td>14</td>
</tr>
</tbody>
</table>

---

Lesson 4.2  Greatest Common Factor
**Equivalent Fractions**

**Vocabulary**
- equivalent fractions, p. 182
- simplest form, p. 183

**Before**
- You wrote fractions and mixed numbers.

**Now**
- You’ll write equivalent fractions.

**Why?**
- So you can compare the life stages of butterflies, as in Ex. 48.

A fraction is a number of the form $\frac{a}{b}$, where $a$ is the numerator and $b$ is the denominator. The value of $b$ cannot be 0.

The number lines show the graphs of two fractions, $\frac{1}{3}$ and $\frac{2}{6}$. From the number lines, you can see that these fractions represent the same number. Two fractions that represent the same number are called equivalent fractions. You can use the following rule to write equivalent fractions.

**Equivalent Fractions**

**Words**
To write equivalent fractions, multiply or divide the numerator and the denominator by the same nonzero number.

**Algebra**
For all numbers $a$, $b$, and $c$, where $b \neq 0$ and $c \neq 0$,

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c} \text{ and } \frac{a}{b} = \frac{a \div c}{b \div c}.$$  

**Numbers**

$$\frac{1}{3} = \frac{1 \cdot 2}{3 \cdot 2} = \frac{2}{6} \quad \frac{2}{6} = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$$

**Example 1**  
**Writing Equivalent Fractions**

Write two fractions that are equivalent to $\frac{8}{12}$.

Multiply or divide the numerator and the denominator by the same nonzero number.

$$\frac{8}{12} = \frac{8 \cdot 3}{12 \cdot 3} = \frac{24}{36} \quad \text{Multiply numerator and denominator by 3.}$$

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3} \quad \text{Divide numerator and denominator by 4.}$$

**Answer**
The fractions $\frac{24}{36}$ and $\frac{2}{3}$ are equivalent to $\frac{8}{12}$.
**Checkpoint**

Write two fractions that are equivalent to the given fraction.

1. $\frac{5}{10}$
2. $\frac{6}{9}$
3. $\frac{12}{20}$
4. $\frac{18}{24}$

**Simplest Form** A fraction is in **simplest form** when its numerator and its denominator are relatively prime. To write a fraction in simplest form, divide the numerator and the denominator by their GCF.

---

**Example 2**

**Writing a Fraction in Simplest Form**

Write $\frac{12}{30}$ in simplest form.

Write the prime factorizations of the numerator and denominator.

$12 = 2^2 \cdot 3$
$30 = 2 \cdot 3 \cdot 5$

The GCF of 12 and 30 is $2 \cdot 3 = 6$.

$\frac{12}{30} = \frac{12}{30} \div 6$

**Divide numerator and denominator by GCF.**

$= \frac{2}{5}$

**Simplify.**

---

**Example 3**

**Simplifying a Fraction**

**Geography** The map at the left shows the 48 contiguous states in the United States. (The word *contiguous* means “connected without a break.”) Of the 48 contiguous states, 21 are coastal states. These states border the Pacific Ocean, the Atlantic Ocean, or the Gulf of Mexico.

Write the fraction, in simplest form, of the contiguous states that are coastal states.

**Solution**

$$\frac{\text{Number of coastal states}}{\text{Number of contiguous states}} = \frac{21}{48}$$

Write fraction.

$$= \frac{21}{48} \div 3$$

Divide numerator and denominator by GCF, 3.

$$= \frac{7}{16}$$

Simplify.

**Answer** Of the contiguous states, $\frac{7}{16}$ are coastal states.

---

**Checkpoint**

Write the fraction in simplest form.

5. $\frac{4}{14}$
6. $\frac{8}{36}$
7. $\frac{27}{42}$
8. $\frac{28}{49}$

---

Lesson 4.3    Equivalent Fractions
Variable Expressions  To simplify fractions that contain variables, factor the numerator and the denominator. Then divide out common factors. In this book, you should assume that any variable in the denominator of a fraction is not equal to zero.

Example 4  **Simplifying a Variable Expression**

Write \(\frac{10xy}{15y^2}\) in simplest form.

\[
\frac{10xy}{15y^2} = \frac{2 \cdot 5 \cdot x \cdot y}{3 \cdot 5 \cdot y \cdot y} \quad \text{Factor numerator and denominator.}
\]

\[
= \frac{\cancel{2} \cdot \cancel{5} \cdot x \cdot \cancel{y}}{3 \cdot \cancel{5} \cdot \cancel{y} \cdot y} \quad \text{Divide out common factors.}
\]

\[
= \frac{2x}{3y} \quad \text{Simplify.}
\]

### 4.3 Exercises

**Guided Practice**

**Vocabulary Check**

1. What does it mean for a fraction to be in simplest form?

2. Explain how to find fractions that are equivalent to \(\frac{3}{7}\).

**Write two fractions that are equivalent to the given fraction.**

3. \(\frac{12}{16}\)

4. \(\frac{15}{18}\)

5. \(\frac{8}{14}\)

6. \(\frac{10}{25}\)

**Write the fraction in simplest form.**

7. \(\frac{16}{38}\)

8. \(\frac{35}{40}\)

9. \(\frac{21a^3}{11a}\)

10. \(\frac{6b}{24b^2}\)

**Guided Problem Solving**

**Film Ratings**  The table shows the number of rated films that are owned by a film library. What fraction of the films were rated G?

<table>
<thead>
<tr>
<th>Rating</th>
<th>G</th>
<th>PG</th>
<th>PG-13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of films</td>
<td>30</td>
<td>55</td>
<td>163</td>
</tr>
</tbody>
</table>

1. Find the total number of films owned by the library.

2. Write a fraction for the number of films that were rated G.

3. Simplify the fraction from Step 2.
Write two fractions that are equivalent to the given fraction.

12. \(\frac{6}{12}\)  
13. \(\frac{5}{15}\)  
14. \(\frac{14}{16}\)  
15. \(\frac{18}{21}\)

16. \(\frac{16}{20}\)  
17. \(\frac{3}{27}\)  
18. \(\frac{7}{10}\)  
19. \(\frac{5}{8}\)

Write the fraction in simplest form.

20. \(\frac{32}{36}\)  
21. \(\frac{25}{35}\)  
22. \(\frac{46}{72}\)  
23. \(\frac{8}{30}\)

24. \(\frac{54}{60}\)  
25. \(\frac{36}{45}\)  
26. \(\frac{39}{42}\)  
27. \(\frac{48}{76}\)

28. **Anatomy** The human skeleton can be divided into two systems. The axial system has 80 bones. It consists of the skull, spine, and ribs. The appendicular system has 126 bones. It consists of the shoulders, pelvis, and limbs.

a. What fraction of the body’s bones are in the axial system? Give your answer in simplest form.

b. What fraction of the body’s bones are in the appendicular system? Give your answer in simplest form.

Write the fraction in simplest form.

29. \(\frac{6a}{6a^2}\)  
30. \(\frac{4mn^3}{10n^2}\)  
31. \(\frac{27bcde}{12b}\)  
32. \(\frac{5x^2y^2}{40xy}\)

33. \(\frac{36w}{60w^2}\)  
34. \(\frac{42r^3}{56r^2}\)  
35. \(\frac{77x^3}{6x}\)  
36. \(\frac{49r^2}{7l^3}\)

37. **Checkers** Checkers is a game for two players. A checkerboard has 64 squares. Each player begins with 12 pieces. Players capture each other’s pieces.

a. What fraction of the squares hold pieces at the start of the game?

b. Later in the game, one player has 5 pieces on the board. The other player has 3 pieces on the board. Now what fraction of the squares hold pieces?

Use a number line to determine whether the fractions are equivalent.

38. \(\frac{1}{4}, \frac{2}{10}\)  
39. \(\frac{3}{4}, \frac{14}{16}\)  
40. \(\frac{5}{8}, \frac{10}{16}\)

Write the fractions in simplest form. Tell whether they are equivalent.

41. \(\frac{12}{15}, \frac{26}{30}\)  
42. \(\frac{18}{20}, \frac{45}{50}\)  
43. \(\frac{9}{24}, \frac{15}{48}\)

44. \(\frac{63}{84}, \frac{45}{60}\)  
45. \(\frac{49}{63}, \frac{21}{27}\)  
46. \(\frac{30}{36}, \frac{57}{72}\)

47. **Critical Thinking** Consider the fractions \(\frac{-12}{27}, \frac{-25}{-35}\), and \(\frac{-33}{55}\). Explain how to simplify each of the fractions.
48. **Monarch Butterflies** Monarch butterflies go through four stages of life: egg, caterpillar, pupa, and butterfly. A regular monarch lives as a butterfly for about 5 weeks. However, migrating monarchs (born in early fall) live as butterflies for up to 30 weeks.

<table>
<thead>
<tr>
<th>Length of Monarch’s Life Stages in Weeks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monarch type</td>
</tr>
<tr>
<td>--------------</td>
</tr>
<tr>
<td>Regular</td>
</tr>
<tr>
<td>Migrating</td>
</tr>
</tbody>
</table>

a. What fraction of a regular monarch’s life is spent as a butterfly?

b. What fraction of a migrating monarch’s life is spent as a butterfly?

Tell what value of \( x \) makes the fractions equivalent.

49. \( \frac{5}{6} \cdot \frac{x}{24} \)

50. \( \frac{7}{9} \cdot \frac{28}{x} \)

51. \( \frac{x}{12} \cdot \frac{80}{192} \)

52. \( \frac{3}{8} \cdot \frac{2 + x}{32} \)

53. **Critical Thinking** Consider the expression \( \frac{8x^2y}{6x^2y^2} \).

a. First evaluate the expression when \( x = 2 \) and \( y = 3 \). Then simplify.

b. Now return to the original expression. First simplify the expression. Then evaluate it when \( x = 2 \) and \( y = 3 \).

c. **Analyze** Compare your results from parts (a) and (b). Which method requires less work? Explain your answer.

d. **Analyze** Now repeat parts (a) and (b) with the values \( x = 3 \) and \( y = 4 \). Compare your results.

54. **Challenge** Does adding the same nonzero number to the numerator and denominator of a fraction produce an equivalent fraction? If so, explain why. If not, tell whether it ever produces an equivalent fraction.

**Mixed Review**

Evaluate the expression when \( x = 4 \) and \( y = -9 \). (Lesson 1.4)

55. \(|x| + |y|\)

56. \(|-19| + |y|\)

57. \(x + |-14|\)

Identify the property that the statement illustrates. (Lesson 2.1)

58. \(n + p = p + n\)

59. \(1 \cdot \frac{5}{6} = \frac{5}{6}\)

60. \(16 + 0 = 16\)

Find the greatest common factor of the monomials. (Lesson 4.2)

61. \(2x, 8x^2\)

62. \(9m^2, 27m^3\)

63. \(10r, 25r^4\)

**Standardized Test Practice**

64. **Multiple Choice** Which fraction is not equivalent to \( \frac{39}{52} \)?

A. \( \frac{36}{48} \)  
B. \( \frac{3}{4} \)  
C. \( \frac{78}{104} \)  
D. \( \frac{31}{42} \)

65. **Multiple Choice** Which fraction is not in simplest form?

A. \( \frac{13}{65} \)  
B. \( \frac{8}{17} \)  
C. \( \frac{9}{16} \)  
D. \( \frac{15}{37} \)
Least Common Multiple

**Vocabulary**
- multiple, p. 187
- common multiple, p. 187
- least common multiple (LCM), p. 187
- least common denominator (LCD), p. 188

**Before**

You found the GCF of two numbers.

**Now**

You’ll find the least common multiple of two numbers.

**WHY?**

So you can design a fitness schedule, as in Ex. 38.

**Agriculture**

Crop rotation is a system in which farmers vary the crops they plant in their fields each year. Suppose a farmer grows alfalfa in a certain field every 6 years. In another field, the farmer grows alfalfa every 10 years. This year, the farmer is growing alfalfa in both fields. In how many years will the farmer grow alfalfa in both fields again?

A **multiple** of a whole number is the product of the number and any nonzero whole number. A multiple that is shared by two or more numbers is a **common multiple**. Some of the common multiples of 8 and 12 are shown in blue below.

- **Multiples of 8**: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, . . .
- **Multiples of 12**: 12, 24, 36, 48, 60, 72, 84, 96, . . .

The least of the common multiples of two or more numbers is the **least common multiple (LCM)**. The LCM of 8 and 12 is 24.

**Example 1**

**Finding the Least Common Multiple**

For the crop rotation system described above, the number of years until the farmer grows alfalfa in both fields again is given by the LCM of 6 and 10. You can use one of two methods to find the LCM.

**Method 1**

List the multiples of each number. Identify the least number that is on both lists.

- **Multiples of 6**: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60
- **Multiples of 10**: 10, 20, 30, 40, 50, 60

The LCM of 6 and 10 is 30.

**Method 2**

Find the common factors of the numbers.

\[
\begin{align*}
6 & = 2 \cdot 3 \\
10 & = 2 \cdot 5 \\
\end{align*}
\]

The common factor is 2.

Multiply all of the factors, using each common factor only once.

\[
\text{LCM} = 2 \cdot 3 \cdot 5 = 30
\]

**Answer**

The farmer will grow alfalfa in both fields again in 30 years.

**Checkpoint**

Find the least common multiple of the numbers.

1. 16, 24
2. 20, 25
3. 6, 8, 20
4. 15, 30, 50

**Lesson 4.4** Least Common Multiple
Example 2  Finding the Least Common Multiple of Monomials

Find the least common multiple of $9xy^2$ and $15x^2y$.

\[
\begin{align*}
9xy^2 &= 3 \cdot 3 \cdot x \cdot y \cdot y \\
15x^2y &= 3 \cdot 5 \cdot x \cdot x \cdot y \\
\text{LCM} &= 3 \cdot x \cdot y \cdot 3 \cdot 5 \cdot x \cdot y = 45x^2y^2
\end{align*}
\]

**Answer**  The least common multiple of $9xy^2$ and $15x^2y$ is $45x^2y^2$.

**Least Common Denominator**  The **least common denominator (LCD)** of two or more fractions is the least common multiple of the denominators. You can use the LCD to compare and order fractions.

Example 3  Comparing Fractions Using the LCD

**Winter Sports**  Last year, a winter resort had 144,000 visitors, including 45,000 snowboarders. This year, the resort had 160,000 visitors, including 56,000 snowboarders. In which year was the fraction of snowboarders greater?

**Solution**

1) Write the fractions and simplify.

**Last year:**

\[
\frac{\text{Number of snowboarders}}{\text{Total number of visitors}} = \frac{45,000}{144,000} = \frac{5}{16}
\]

**This year:**

\[
\frac{\text{Number of snowboarders}}{\text{Total number of visitors}} = \frac{56,000}{160,000} = \frac{7}{20}
\]

2) Find the LCD of $\frac{5}{16}$ and $\frac{7}{20}$.

The LCM of 16 and 20 is 80. So, the LCD of the fractions is 80.

3) Write equivalent fractions using the LCD.

**Last year:**

\[
\frac{5}{16} = \frac{5 \cdot 5}{16 \cdot 5} = \frac{25}{80}
\]

**This year:**

\[
\frac{7}{20} = \frac{7 \cdot 4}{20 \cdot 4} = \frac{28}{80}
\]

4) Compare the numerators: $\frac{25}{80} < \frac{28}{80}$, so $\frac{5}{16} < \frac{7}{20}$.

**Answer**  The fraction of snowboarders was greater this year.

**Checkpoint**

Find the least common multiple of the monomials.

5. $15x^2$, $27x$  
6. $6m^2$, $10m^2$  
7. $14ab$, $21bc$  
8. $r^2$, $5rst$

Use the LCD to determine which fraction is greater.

9. $\frac{5}{6}$, $\frac{7}{9}$  
10. $\frac{5}{8}$, $\frac{13}{20}$  
11. $\frac{7}{12}$, $\frac{11}{15}$  
12. $\frac{5}{16}$, $\frac{3}{10}$
Example 4  Ordering Fractions and Mixed Numbers

Order the numbers $3\frac{4}{15}$, $3\frac{3}{10}$, and $\frac{19}{6}$ from least to greatest.

1. Write the mixed number as an improper fraction.
   
   $3\frac{4}{15} = \frac{3 \cdot 15 + 4}{15} = \frac{49}{15}$

2. Find the LCD of $\frac{49}{15}$, $\frac{33}{10}$, and $\frac{19}{6}$.
   
   The LCM of 15, 10, and 6 is 30. So, the LCD is 30.

3. Write equivalent fractions using the LCD.
   
   $\frac{49}{15} = \frac{49 \cdot 2}{15 \cdot 2} = \frac{98}{30}$
   
   $\frac{33}{10} = \frac{33 \cdot 3}{10 \cdot 3} = \frac{99}{30}$
   
   $\frac{19}{6} = \frac{19 \cdot 5}{6 \cdot 5} = \frac{95}{30}$

4. Compare the numerators.
   
   $\frac{95}{30} < \frac{98}{30}$ and $\frac{98}{30} < \frac{99}{30}$, so $\frac{19}{6} < \frac{49}{15}$ and $\frac{49}{15} < \frac{33}{10}$.

Answer  From least to greatest, the numbers are $\frac{19}{6}$, $3\frac{4}{15}$, and $3\frac{3}{10}$.

4.4  Exercises

4.4  Exercises

More Practice, p. 806

Guided Practice

Vocabulary Check

1. How are the terms least common multiple and least common denominator related?

2. Describe how you would use the LCD to compare $\frac{4}{7}$ and $\frac{7}{12}$.

Skill Check

Find the least common multiple of the numbers.

3. 3, 4  4. 4, 8  5. 18, 24  6. 10, 16

Find the least common multiple of the monomials.

7. $3s$, $s^2$  8. $x^4$, $x^2$  9. $15m^2$, $9m$  10. $8b$, $20b^2$

Use the LCD to determine which fraction is greater.

11. $\frac{3}{4}$, $\frac{5}{8}$  12. $\frac{2}{3}$, $\frac{13}{16}$  13. $\frac{2}{5}$, $\frac{3}{8}$  14. $\frac{3}{4}$, $\frac{7}{10}$

15. Error Analysis  Describe and correct the error in finding the LCM of 16 and 30.

$16 = 2^4$  30 = $2 \cdot 3 \cdot 5$

$\text{LCM} = 2^4 \cdot 3 \cdot 5 = 480$
Find the least common multiple of the numbers.

16. 9, 12  
17. 3, 8  
18. 4, 16  
19. 10, 15  
20. 21, 14  
21. 30, 36  
22. 55, 15  
23. 42, 66  
24. 3, 6, 12  
25. 8, 11, 36  
26. 10, 12, 14  
27. 16, 20, 30

Find the least common multiple of the monomials.

28. $5a^2$, $16a^3$  
29. $21w$, $9w^2$  
30. $17b^2$, $3b^3$  
31. $14x^4$, $21x^2$  
32. $60s^4$, $24s^3$  
33. $2n^3$, $8n^2$  
34. $25a$, $40a^2$  
35. $11s$, $33s^2$

36. **Visual Patterns** In the first pattern shown below, the green star repeats every 6 figures. In the second pattern, the green star repeats every 8 figures. How many figures after the first figure will both patterns have a green star?

[Diagram of patterns]

37. **Writing** Could you find the greatest common multiple of two numbers? Explain your thinking.

38. **Fitness** You lift weights every third day and take karate class every Monday. Today you have karate and are lifting weights. In how many days will you next lift weights and have karate on the same day?

Use the LCD to determine which fraction is greater.

39. $\frac{1}{4}$, $\frac{2}{7}$  
40. $\frac{2}{3}$, $\frac{5}{8}$  
41. $\frac{7}{10}$, $\frac{11}{15}$  
42. $\frac{3}{5}$, $\frac{6}{11}$  
43. $\frac{5}{12}$, $\frac{4}{15}$  
44. $\frac{7}{20}$, $\frac{9}{25}$  
45. $\frac{5}{18}$, $\frac{8}{21}$  
46. $\frac{11}{20}$, $\frac{42}{63}$

Order the numbers from least to greatest.

47. $\frac{7}{6}$, $\frac{11}{9}$, $\frac{1}{3}$  
48. $\frac{13}{4}$, $\frac{3}{2}$, $\frac{27}{8}$  
49. $\frac{8}{15}$, $\frac{1}{5}$, $\frac{3}{10}$  
50. $\frac{5}{11}$, $\frac{14}{33}$, $\frac{9}{22}$  
51. $\frac{3}{4}$, $\frac{4}{9}$, $\frac{7}{15}$  
52. $\frac{5}{7}$, $\frac{11}{10}$, $\frac{15}{12}$  
53. $\frac{12}{5}$, $\frac{25}{12}$, $\frac{43}{18}$  
54. $\frac{1}{3}$, $\frac{10}{7}$, $\frac{13}{33}$

55. **Critical Thinking** What is the least number for which the LCM of the number and 12 is 300? Explain your thinking.

Find the least common multiple of the monomials.

56. $24de^2$, $36d^3e$  
57. $x^3y$, $15xy^5$  
58. $10a^2b^2$, $20ab$  
59. $45gh^3$, $33g^4h$  
60. $xyz^3$, $x^2yz^2$  
61. $26ab^2$, $28ac^3$  
62. $11rst$, $15r^3t^2$  
63. $30df^2$, $40d^3ef$  
64. **Vice Presidents** During the period 1800–1900, 6 out of 23 U.S. Vice Presidents later became U.S. Presidents. During the period 1901–2000, 7 out of 21 Vice Presidents later became Presidents. During which period did a greater fraction of Vice Presidents become Presidents?
In Exercises 65–68, rewrite the variable expressions with a common denominator.

**Example**  
**Rewriting Variable Expressions**

To rewrite \( \frac{2a}{5b} \) and \( \frac{3}{4ab^2} \) with a common denominator, first find the LCD of the fractions.

The LCM of \( 5b \) and \( 4ab^2 \) is \( 20ab^2 \). So, the LCD is \( 20ab^2 \).

Then write equivalent fractions using the LCD.

\[
\frac{2a}{5b} = \frac{2a \cdot 4ab}{5b \cdot 4ab} = \frac{8a^2b}{20ab^2} \quad \frac{3}{4ab^2} = \frac{3 \cdot 5}{4ab^2 \cdot 5} = \frac{15}{20ab^2}
\]

65. \( \frac{x}{3} \cdot \frac{x}{4} \)  
66. \( \frac{x}{6y} \cdot \frac{y}{8x} \)  
67. \( \frac{3x^2}{4y^2} \cdot \frac{2}{5xy} \)  
68. \( \frac{3x}{2yz} \cdot \frac{5y}{4xz} \)

69. **Critical Thinking**  
Let \( a \) and \( b \) represent nonzero whole numbers.  
Find a fraction \( \frac{a}{b} \) such that \( \frac{1}{6} < \frac{a}{b} < \frac{2}{7} \), and \( b < 30 \).

70. **Challenge**  
Copy and complete the table for the given values of \( a \) and \( b \). Describe any relationships you notice between the product of the LCM and the GCF and the product of \( a \) and \( b \).

<table>
<thead>
<tr>
<th>Given numbers</th>
<th>Prime factorizations</th>
<th>LCM</th>
<th>GCF</th>
<th>LCM ( \cdot ) GCF</th>
<th>( a \cdot b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a = 6, b = 18 )</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>( a = 15, b = 35 )</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>( a = 6, b = 20 )</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>( a = 12, b = 60 )</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

**Mixed Review**

71. \( n^2 \)  
72. \( n^3 \)  
73. \( n^4 \)  
74. \( n^5 \)

**Write the prime factorization of the number.**  
75. 28  
76. 39  
77. 81  
78. 165

79. **Cookies**  
You are making gift boxes filled with cookies to give to friends.  
You have 64 peanut butter cookies, 80 chocolate chip cookies, and 56 sugar cookies. What is the greatest number of identical gift boxes that you can make?  

**Standardized Test Practice**

80. **Multiple Choice**  
Which expression is the least common multiple of the monomials \( 27w^3z \) and \( 75w^2z^2 \)?

A. \( 3w^2z \)  
B. \( 75w^4z^2 \)  
C. \( 675w^3z \)  
D. \( 675w^4z^2 \)

81. **Multiple Choice**  
Which list shows the fractions in order from least to greatest?

F. \( \frac{2}{9}, \frac{1}{6}, \frac{4}{25} \)  
G. \( \frac{3}{7}, \frac{11}{24}, \frac{9}{21} \)  
H. \( \frac{7}{20}, \frac{3}{8}, \frac{5}{12} \)  
I. \( \frac{2}{5}, \frac{19}{40}, \frac{21}{45} \)
Tell whether the number is prime or composite. If it is composite, write its prime factorization using exponents.

1. 46  
2. 57  
3. 61  
4. 89  

Factor the monomial.

5. $25m^3$  
6. $14n^4$  
7. $19a^2b$  
8. $64f^2g^2$  

Find the greatest common factor of the numbers. Then tell whether the numbers are relatively prime.

9. 9, 16  
10. 12, 51  
11. 18, 49  
12. 56, 75  

13. Soccer A soccer league has 180 members. The league consists of 24 eight-year-olds, 96 nine-year-olds, and 60 ten-year-olds. You want to divide the members into teams that have the same number of eight-year-olds, nine-year-olds, and ten-year-olds. What is the greatest number of teams that can be formed? How many ten-year-olds will be on each team?

Write the fraction in simplest form.

14. $\frac{18}{48}$  
15. $\frac{42}{81}$  
16. $\frac{32a}{8a^2}$  
17. $\frac{15b}{39b^4}$  

Find the least common multiple of the numbers.

18. 4, 11  
19. 10, 24  
20. 15, 45  
21. 30, 54  

Use the LCD to determine which fraction is greater.

22. $\frac{3}{8}$, $\frac{4}{9}$  
23. $\frac{7}{10}$, $\frac{18}{25}$  
24. $\frac{5}{12}$, $\frac{9}{20}$  
25. $\frac{11}{18}$, $\frac{13}{24}$  

Fraction Scramble

Rearrange the numerators and denominators in the five fractions shown to create five new fractions that are all equivalent to each other. Numerators can become denominators and vice versa. The question mark can be any positive integer.
4.5 Finding Rules of Exponents

**Goal**
Use patterns to discover rules for multiplying and dividing powers.

**Materials**
- pencil
- paper

**Investigate**

Use patterns to discover rules for multiplying and dividing powers.

Copy and complete each table.

### Products

<table>
<thead>
<tr>
<th>Expression</th>
<th>Expression written as repeated multiplication</th>
<th>Number of factors</th>
<th>Product as a power</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^4 \cdot 2^3$</td>
<td>$(2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2)$</td>
<td>7</td>
<td>$2^7$</td>
</tr>
<tr>
<td>$3^1 \cdot 3^4$</td>
<td>$(3) \cdot (3 \cdot 3 \cdot 3 \cdot 3)$</td>
<td>?</td>
<td>$3^5$</td>
</tr>
<tr>
<td>$5^2 \cdot 5^4$</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

### Quotients

<table>
<thead>
<tr>
<th>Expression</th>
<th>Expression written as repeated multiplication</th>
<th>Simplified expression</th>
<th>Number of factors</th>
<th>Quotient as a power</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{2^8}{2^3}$</td>
<td>$\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2}$</td>
<td></td>
<td>5</td>
<td>$2^5$</td>
</tr>
<tr>
<td>$\frac{3^5}{3^3}$</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>$3^2$</td>
</tr>
<tr>
<td>$\frac{5^7}{5^5}$</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

**Draw Conclusions**

1. **Critical Thinking** In the *Products* table, how are the exponents in the first and last columns related?

2. Use your answer to Exercise 1 to write the product $10^7 \cdot 10^4$ as a power.

3. **Critical Thinking** In the *Quotients* table, how are the exponents in the first and last columns related?

4. Use your answer to Exercise 3 to write the quotient $\frac{6^3}{6^7}$ as a power.
Rules of Exponents

**BEFORE**
You evaluated powers. You’ll multiply and divide powers. So you can estimate the number of stars, as in Ex. 58.

**Now**

Notice what happens when you multiply two powers with the same base.

\[
a^4 \cdot a^3 = (a \cdot a \cdot a) \cdot (a \cdot a) = a^4 + 3 = a^7
\]

This example suggests a rule for multiplying powers with the same base.

**Product of Powers Property**

**Words** To multiply powers with the same base, add their exponents.

**Algebra** \( a^m \cdot a^n = a^{m+n} \)

**Numbers** \( 4^3 \cdot 4^2 = 4^{3+2} = 4^5 \)

**Example 1**

**Using the Product of Powers Property**

**Lake Powell** Lake Powell, the reservoir behind the Glen Canyon Dam in Arizona, can hold about \(10^{12}\) cubic feet of water when full. There are about \(10^{27}\) water molecules in 1 cubic foot of water. About how many water molecules can the reservoir hold?

**Solution**

\[
\begin{align*}
\text{Number of water molecules in reservoir} &= \text{Cubic feet of water in reservoir} \cdot \text{Number of water molecules in a cubic foot} \\
&= 10^{12} \cdot 10^{27} \\
&= 10^{12 + 27} \\
&= 10^{39}
\end{align*}
\]

**Answer** Lake Powell can hold about \(10^{39}\) molecules of water.

**Checkpoint**

Find the product. Write your answer using exponents.

1. \(2^3 \cdot 2^2\)  
2. \(8^7 \cdot 8^5\)  
3. \(5 \cdot 5^2\)  
4. \(4^6 \cdot 4^4 \cdot 4^3\)
**Example 2**

Using the Product of Powers Property

- **a.** \( x^6 \cdot x^9 = x^{6+9} \)  
  Product of powers property  
  Add exponents.  
  \( = x^{15} \)

- **b.** \( 3x \cdot 5x^5 = 3 \cdot 5 \cdot x^1 \cdot x^5 \)  
  Commutative property of multiplication  
  Product of powers property  
  Add exponents.  
  \( = 3 \cdot 5 \cdot x^{1+5} \)  
  \( = 3 \cdot 5 \cdot x^6 \)  
  \( = 15x^6 \)  
  Multiply.

**Quotients of Powers**  There is a related rule you can use for dividing powers with the same base. The following example suggests this rule.

\[
\frac{a^5}{a^2} = \frac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a} = \frac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a} = \frac{a^1}{a^1} = \frac{a \cdot a \cdot a}{a} = a^5 - 2 = a^3
\]

5 factors  
2 factors  
3 factors

**Example 3**

Using the Quotient of Powers Property

- **a.** \( \frac{7^6}{7^2} = 7^{6-2} \)  
  Quotient of powers property  
  Subtract exponents.  
  \( = 7^4 \)

- **b.** \( \frac{4x^8}{10x^2} = \frac{4x^{8-2}}{10} \)  
  Quotient of powers property  
  Subtract exponents.  
  \( = \frac{4x^6}{10} \)  
  \( = \frac{2x^6}{5} \)  
  Divide numerator and denominator by 2.

**Checkpoint**

Find the product or quotient. Write your answer using exponents.

5. \( b^7 \cdot b^2 \)  
6. \( a \cdot a^5 \cdot a^2 \)  
7. \( 2n^{11} \cdot 6n^8 \)  
8. \( 2m^4 \cdot 7m^5 \)

9. \( \frac{6^3}{6^4} \)  
10. \( \frac{10^{11}}{10^7} \)  
11. \( \frac{z^8}{z^3} \)  
12. \( \frac{12n^5}{8n^2} \)
Example 4  
Using Both Properties of Powers

Simplify \( \frac{3m^5 \cdot m^2}{6m^3} \).

\[
\frac{3m^5 \cdot m^2}{6m^3} = \frac{3m^{5+2}}{6m^3} \\
= \frac{3m^7}{6m^3} \\
= \frac{3m^{7-3}}{6} \\
= \frac{3m^4}{6} \\
= \frac{m^4}{2}
\]

Product of powers property
Add exponents.
Quotient of powers property
Subtract exponents.
Divide numerator and denominator by 3.

✔ Checkpoint

Simplify.

13. \( \frac{a^4 \cdot 10a^3}{a^2} \)  
14. \( \frac{13b^4 \cdot b^4}{b} \)  
15. \( \frac{x \cdot 7x^5}{10x^4} \)  
16. \( \frac{12y^2 \cdot y^8}{16y^5} \)

4.5 Exercises

Guided Practice

Vocabulary Check

1. Copy and complete: To multiply two powers with the same base, _____ their exponents.

2. Give an example of an expression you could simplify using the quotient of powers property.

Skill Check

Find the product or quotient. Write your answer using exponents.

3. \( 4^2 \cdot 4^9 \)  
4. \( 5^3 \cdot 5^8 \)  
5. \( 6^7 \cdot 6 \)  
6. \( 7^3 \cdot 7^4 \cdot 7^2 \)

7. \( \frac{2^{12}}{2^7} \)  
8. \( \frac{5^{14}}{5^2} \)  
9. \( \frac{3^5}{3^2} \)  
10. \( \frac{10^9}{10^7} \)

Simplify.

11. \( m^4 \cdot m^3 \)  
12. \( 2x^7 \cdot 5x^2 \)  
13. \( \frac{x^{10}}{x^4} \)  
14. \( \frac{15y^7}{5y^3} \)

15. Error Analysis  Describe and correct the error in simplifying \( 2x^5 \cdot 2x^3 \).

\[
2x^5 \cdot 2x^3 = 2x^{5+4} = 2x^9
\]
Find the product or quotient. Write your answer using exponents.

16. $10^6 \cdot 10^7$  
17. $9^3 \cdot 9^3$  
18. $11^4 \cdot 11^4$  
19. $8 \cdot 8^3 \cdot 8^2$

20. $\frac{6^3}{6^2}$  
21. $\frac{8^{12}}{8^6}$  
22. $\frac{7^{20}}{7^4}$  
23. $\frac{9^{11}}{9}$

Simplify.

24. $a^4 \cdot a^8$  
25. $b^3 \cdot b^6$  
26. $3w^3 \cdot w^2$  
27. $z^7 \cdot 8z^4$

28. $3n^4 \cdot 6n^9$  
29. $4r^5 \cdot 2r$  
30. $x^2 \cdot x^6 \cdot x$  
31. $z^5 \cdot z^2 \cdot z^7$

32. $\frac{x^3}{x^4}$  
33. $\frac{7y^8}{y^5}$  
34. $\frac{24m^{11}}{18m^3}$  
35. $\frac{28s^{15}}{42s^{12}}$

36. The Great Pyramid The Great Pyramid in Egypt is composed of about $2^{21}$ limestone and granite blocks. The average mass of one of these blocks is about $2^{11}$ kilograms. Use the product of powers property to approximate the total mass in kilograms of the Great Pyramid.

Copy and complete the statement using $<$, $>$, or $=$.

37. $3^8$ ? $3^6 \cdot 3^2$  
38. $2^7$ ? $2 \cdot 2^5$  
39. $6^5$ ? $6^2 \cdot 6^2$

40. Computers Computer memory is measured in bytes. The table shows related units used to measure computer memory.

<table>
<thead>
<tr>
<th>Number of bytes</th>
<th>$10^3$</th>
<th>$10^6$</th>
<th>$10^9$</th>
<th>$10^{12}$</th>
<th>$10^{15}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name of unit</td>
<td>Kilobyte</td>
<td>Megabyte</td>
<td>Gigabyte</td>
<td>Terabyte</td>
<td>Petabyte</td>
</tr>
</tbody>
</table>

a. How many kilobytes are in a megabyte?

b. How many gigabytes are in a petabyte?

c. How many megabytes are in a petabyte?

41. Writing Explain why the product of powers property cannot be used to simplify $a^7 \cdot b^7$.

Find the missing exponent.

42. $\frac{a^3}{a^2} = a^5$  
43. $y^5 \cdot y^3 = y^7$  
44. $b^7 \cdot b^6 = b^{13}$  
45. $\frac{z^7}{z^3} = z^4$

Simplify.

46. $x^2 \cdot y^4 \cdot x^3$  
47. $4m^4(n^7 \cdot m)$  
48. $(4ab)(5a^2b^3)$  
49. $(p^3q^2)(p^4q^2)$

50. $\frac{14a^3b^4}{4ab}$  
51. $\frac{63m^5n^6}{27mn}$  
52. $\frac{24w^3z^3}{15w^2z^3}$  
53. $\frac{28c^{10}d^{13}}{24c^6d^8}$

54. $\frac{2x^6 \cdot 4x^3}{24x^5}$  
55. $\frac{3a \cdot 4a^4}{28a^2}$  
56. $\frac{6x^9 \cdot 8x^3}{27z^2}$  
57. $\frac{2w^6 \cdot 36w^8}{18w^4}$

58. Astronomy There are over 100 billion stars in our galaxy, the Milky Way. Scientists estimate there are about 100 billion galaxies in the universe. Recall that $1$ billion $= 10^9$. If every galaxy has about 100 billion stars, about how many stars are in the universe?
59. **Logical Reasoning** Consider the equation \( \frac{5^m}{5^n} = 5 \).

   a. Rewrite the left side of the equation using the quotient of powers property.

   b. Find a pair of integers \( m \) and \( n \) for which the equation is true.

   c. Are there other pairs of integers \( m \) and \( n \) for which the equation is true? Explain your reasoning.

60. Write \( 2 \cdot 2^n \) as a power of 2.

61. **Critical Thinking** Write three products of powers that are equal to \( 2^6 \). Then write three quotients of powers that are equal to \( 2^6 \).

62. Simplify the expression \( \frac{a^{m+n}}{a^n} \).

63. **Challenge** Find a value of \( n \) that makes \( 3^{4n} \cdot 3^{n+4} = 3^{14} \) a true statement. Explain how you found your answer.

---

**Mixed Review**

**Find the sum or difference. (Lessons 1.5, 1.6)**

64. \( -14 + 98 \)  
65. \( 26 + (-19) \)  
66. \( -89 - 23 \)  
67. \( 78 - (-34) \)

**Find the greatest common factor. (Lesson 4.2)**

68. \( 44x^3, 24x^2 \)  
69. \( 21xy, 25x^2 \)  
70. \( 42x^3y, 70xy^2 \)  
71. \( 100x^3, 75y^3 \)

**Find the least common multiple. (Lesson 4.4)**

72. \( 6x^2, 12xy^3 \)  
73. \( 3y, 5x^2y^2 \)  
74. \( 4x^3, 7xy^2 \)  
75. \( 9x^7y^3, 8xy \)

---

**Standardized Test Practice**

76. **Multiple Choice** Which expression is equivalent to \( \frac{24m^{18}}{36m^6} \)?

   A. \( \frac{24m^3}{36} \)  
   B. \( \frac{2m^6}{3} \)  
   C. \( \frac{24}{36m^{12}} \)  
   D. \( \frac{2m^{12}}{3} \)

77. **Multiple Choice** Which expression is equivalent to \( 36x^3 \cdot 9x^2 \)?

   F. \( 4x \)  
   G. \( 4x^5 \)  
   H. \( 324x \)  
   I. \( 324x^5 \)

---

**Ones’ Digit Wonder**

For powers of 3, the digits in the ones’ place follow a certain pattern. What is the pattern? What is the digit in the ones’ place for \( 3^{100} \)?
Consider the following pattern of powers of 2.

\[
\begin{align*}
2^3 &= 8 \\
2^2 &= 4 \\
2^1 &= 2 \\
2^0 &= 1 \\
2^{-1} &= \frac{1}{2} \\
2^{-2} &= \frac{1}{4} \\
\end{align*}
\]

As exponents decrease by 1, the values of the powers are halved.

By extending the pattern, you can conclude that \(2^0 = 1\), \(2^{-1} = \frac{1}{2}\), and \(2^{-2} = \frac{1}{4}\). Because \(\frac{1}{2} = \frac{1}{2^1}\) and \(\frac{1}{4} = \frac{1}{2^2}\), the pattern suggests the following definitions for negative and zero exponents.

**Negative and Zero Exponents**

For any nonzero number \(a\), \(a^0 = 1\).

For any nonzero number \(a\) and any integer \(n\), \(a^{-n} = \frac{1}{a^n}\).

**Example 1**

**Powers with Negative and Zero Exponents**

Write the expression using only positive exponents.

\[
\begin{align*}
a. \ 3^{-5} &= \frac{1}{3^5} & \text{Definition of negative exponent} \\
b. \ m^0n^{-4} &= 1 \cdot n^{-4} & \text{Definition of zero exponent} \\
&= \frac{1}{n^4} & \text{Definition of negative exponent} \\
c. \ 16x^{-6}y &= \frac{16y}{x^6} & \text{Definition of negative exponent}
\end{align*}
\]

**Checkpoint**

Write the expression using only positive exponents.

1. \(5^{-2}\) \hspace{1cm} 2. \(1,000,000^0\) \hspace{1cm} 3. \(3y^{-2}\) \hspace{1cm} 4. \(a^{-7}b^5\)
Rewriting Fractions You can use the prime factorization of a number to write a fraction as an expression involving negative exponents.

**Example 2**  
Rewriting Fractions

Write the expression without using a fraction bar.

a. \[ \frac{1}{16} \]  
b. \[ \frac{a^2}{c^3} \]

**Solution**

a. \[ \frac{1}{16} = \frac{1}{2^4} = 2^{-4} \]  
   Definition of negative exponent

b. \[ \frac{a^2}{c^3} = a^2 c^{-3} \]  
   Definition of negative exponent

Products and Quotients of Powers You can use the product of powers property and the quotient of powers property to find products and quotients that involve negative exponents.

**Example 3**  
Using Powers Properties with Negative Exponents

Find the product or quotient. Write your answer using only positive exponents.

a. \[ 5^{10} \cdot 5^{-6} \]  
b. \[ \frac{8n^{-3}}{n^2} \]

**Solution**

a. \[ 5^{10} \cdot 5^{-6} = 5^{10 + (-6)} = 5^4 \]  
   Product of powers property
   Add exponents.

b. \[ \frac{8n^{-3}}{n^2} = 8n^{-3-2} = 8n^{-5} \]  
   Quotient of powers property
   Subtract exponents.
   Definition of negative exponent

☐ **Checkpoint**

Write the expression without using a fraction bar.

5. \[ \frac{1}{25} \]  
6. \[ \frac{1}{1000} \]  
7. \[ \frac{2}{a^8} \]  
8. \[ \frac{x^7}{z^2} \]

Find the product or quotient. Write your answer using only positive exponents.

9. \[ 3^{-7} \cdot 3^{11} \]  
10. \[ 5^{-8} \cdot 5^{-7} \]  
11. \[ m^{-3} \cdot m^{-1} \]  
12. \[ a^{-2} \cdot a^{10} \]

13. \[ \frac{2^{-3}}{2^4} \]  
14. \[ \frac{7^2}{7^8} \]  
15. \[ \frac{5k^3}{k^{-3}} \]  
16. \[ \frac{b^{-4}}{b^{-6}} \]
Example 4  Solving Problems Involving Negative Exponents

Geckos  Geckos can easily climb smooth vertical surfaces. Biologists have discovered that tiny hairs are the reason that the feet of a gecko are so sticky. Each hair is about 100 micrometers long. A micrometer is $10^{-6}$ meter. What is the length of one hair in meters?

Solution

To find the length of one hair in meters, multiply the length of the hair in micrometers by the number of micrometers in one meter.

$$
100 \cdot 10^{-6} = 10^2 \cdot 10^{-6} \quad \text{Rewrite 100 as } 10^2.
= 10^2 + (-6) \quad \text{Product of powers property}
= 10^{-4} \quad \text{Add exponents.}
= \frac{1}{10^4} \quad \text{Definition of negative exponent}
= \frac{1}{10,000} \quad \text{Evaluate power.}
$$

Answer  The length of one hair is about $\frac{1}{10,000}$ meter.

4.6  Exercises

Vocabulary Check  1. Write $5^{-2}$ using a positive exponent.
2. If $a$ is nonzero, does the value of $a^0$ depend on the value of $a$? Explain.

Skill Check  Write the expression using only positive exponents.

3. $5^{-3}$  4. $3^{-5}$  5. $4a^{-6}$  6. $b^{-3}c^0$

Write the expression without using a fraction bar.

7. $\frac{1}{27}$  8. $\frac{1}{10^8}$  9. $\frac{4}{x^3}$  10. $\frac{11}{c^5}$

Find the product. Write your answer using only positive exponents.

11. $6^{-4} \cdot 6^7$  12. $3^{-2} \cdot 3^{-8}$  13. $x^{11} \cdot x^{-3}$  14. $z^{-5} \cdot z^{-1}$

Guided Problem Solving  15. How many nanoseconds are in a millisecond?

1) Write the quotient of the duration of a millisecond and the duration of a nanosecond.

2) Use the quotient of powers property to simplify the quotient in Step 1.

<table>
<thead>
<tr>
<th>Name of unit</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Millisecond</td>
<td>$10^{-3}$ sec</td>
</tr>
<tr>
<td>Microsecond</td>
<td>$10^{-6}$ sec</td>
</tr>
<tr>
<td>Nanosecond</td>
<td>$10^{-9}$ sec</td>
</tr>
</tbody>
</table>
Write the expression using only positive exponents.

16. \(13^{-6}\)  17. \(121^0\)  18. \(8^{-9}\)  19. \(20^{-4}\)
20. \(xy^0\)  21. \(18f^{-1}\)  22. \(6g^{-5}\)  23. \(c^3d^{-1}\)

Write the expression without using a fraction bar.

24. \(\frac{1}{25}\)  25. \(\frac{1}{19}\)  26. \(\frac{1}{10,000}\)  27. \(\frac{1}{64}\)
28. \(\frac{8}{c^5}\)  29. \(\frac{4}{d}\)  30. \(\frac{4y}{x^3}\)  31. \(\frac{9a^2}{b^6}\)

Find the product. Write your answer using only positive exponents.

32. \(3^4 \cdot 3^{-7}\)  33. \(5 \cdot 5^{-5}\)  34. \(10^{-2} \cdot 10^{-8}\)  35. \(13^0 \cdot 13^6\)
36. \(2s^{-3} \cdot s^3\)  37. \(5t^{-3} \cdot 3t^{-8}\)  38. \(4a^0 \cdot 7a^{-4}\)  39. \(b^{-5} \cdot b^{-9}\)

40. **Historical Documents** Scientists have discovered that nanoparticles of a substance called slaked lime can help preserve historical documents.

The diameters of these nanoparticles are less than \(\frac{1}{100,000,000,000}\) meter.

Write this number using a negative exponent.

41. **Critical Thinking** Explain how \(6^{-2}\) is different from \(6^2\).

42. **Writing** Explain why the rule \(a^{-n} = \frac{1}{a^n}\) does not apply to \(a = 0\).

Find the quotient. Write your answer using only positive exponents.

43. \(\frac{2^5}{2^8}\)  44. \(\frac{4^{-2}}{4^6}\)  45. \(\frac{16^{-3}}{16^{-8}}\)  46. \(\frac{15^3}{15^{-4}}\)
47. \(\frac{17a^3}{a^7}\)  48. \(\frac{15b^{-5}}{3b^4}\)  49. \(\frac{26w^{-4}}{13w^{-12}}\)  50. \(\frac{11g^2}{g^{-4}}\)

Use a calculator to evaluate the expression. If necessary, round the result to the nearest thousandth.

51. \((4.5)^{-3}\)  52. \((8.1)^{-2}\)  53. \((3.2)^{-4}\)  54. \((7.5)^{-3}\)

55. **Brittle Stars** The brittle star is a type of starfish. A certain species of brittle star has a skeleton that is covered in microscopic crystals. Scientists have discovered that these crystals act as lenses that allow the brittle star to sense light.

a. The surface of each crystal has an area of \(\frac{1}{1,000,000,000}\) square meter.

Write this number using a negative exponent.

b. Approximately \(10^4\) crystals cover the skeleton of a brittle star. What is the total area of all the crystals on a single brittle star? Write your answer using a negative exponent.

Find the quotient. Write your answer using only positive exponents.

56. \(\frac{a^6b^4}{a^5b^7}\)  57. \(\frac{c^2d^{11}}{c^6d^5}\)  58. \(\frac{m^8n^4}{m^3n^3}\)  59. \(\frac{x^7y^7}{x^10y^7}\)
60. **Extended Problem Solving** Inkjet printers spray droplets of ink onto paper. The volume of a single droplet is about 10 picoliters. Some printers spray as many as $10^9$ droplets to completely cover a square inch of paper.

a. Find the volume of ink in picoliters needed to completely cover a square inch of paper.

b. There are $10^{12}$ picoliters in a liter. Find the volume of ink in liters needed to completely cover a square inch of paper.

c. **Estimate** Find the area of a 8.5 inch by 11 inch piece of paper and round this number to the nearest power of 10. Then estimate the volume of ink (in liters) needed to completely cover an entire piece of paper.

d. **Apply** Suppose an inkjet cartridge contains 60 milliliters of ink. About how many pages can this cartridge print if each page is 8.5 inches by 11 inches and is covered completely in ink?

61. **Challenge** One way to develop the definitions of zero and negative exponents is to use the quotient of powers property.

a. Consider $\frac{a^n}{a^m}$. First, simplify using the quotient of powers property.

b. Then simplify the expression in part (a) in a different way: Write the numerator and denominator as a product of $a$’s and divide out common factors. Which definition of exponents have you developed?

c. Now consider $\frac{a^0}{a^n}$. First, simplify using the quotient of powers property.

d. Then simplify the expression in part (c) by using the definition of a zero exponent. Which definition of exponents have you developed?

---

**Mixed Review**

**Algebra Basics** Solve the equation using mental math. *(Lesson 2.4)*

62. $9 + x = 17$  
63. $8 - x = 3$  
64. $-3x = 36$  
65. $\frac{x}{-8} = 6$

66. **Amusement Parks** You must be at least 46 inches tall to ride the bumper cars at an amusement park. Write and graph an inequality to show the heights for which you can ride the bumper cars. *(Lesson 3.4)*

Find the product or quotient. Write your answer using exponents. *(Lesson 4.5)*

67. $3^2 \cdot 3^2$  
68. $5^4 \cdot 5$  
69. $\frac{2^3}{2^4}$  
70. $\frac{10^8}{10^5}$

71. **Multiple Choice** Which expression is not equivalent to $x^2 \cdot x^{-6}$?
   
   A. $x^{-4}$  
   B. $x^{-2} \cdot x^6$  
   C. $\frac{1}{x^4}$  
   D. $\frac{x^2}{x^6}$

72. **Multiple Choice** Which expression is equivalent to $\frac{24a^6}{3b^2}$?
   
   F. $24a^{-6}b^2$  
   G. $24a^6b^{-2}$  
   H. $8a^6b^{-2}$  
   I. $8a^{-6}b^2$

**Standardized Test Practice**

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**Lesson 4.6** Negative and Zero Exponents 203
**Scientific Notation**

**Before**
- You used properties of exponents.

**Now**
- You’ll write numbers using scientific notation.

**Why?**
- So you can calculate how much a whale eats, as in Ex. 53.

**Anatomy** The retina is a layer of the eyeball that contains rods and cones. Rods and cones are cells that absorb light and change it to electric signals that are sent to the brain. The human retina is about 0.00012 meter thick and contains about 120,000,000 rods and about 6,000,000 cones.

You can use *scientific notation* to write these numbers. Scientific notation is a shorthand way of writing numbers using powers of 10.

---

**Using Scientific Notation**

A number is written in **scientific notation** if it has the form

\[ c \times 10^n \]

where \( 1 \leq c < 10 \) and \( n \) is an integer.

<table>
<thead>
<tr>
<th>Standard form</th>
<th>Product form</th>
<th>Scientific notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>725,000</td>
<td>( 7.25 \times 100,000 )</td>
<td>( 7.25 \times 10^5 )</td>
</tr>
<tr>
<td>0.006</td>
<td>( 6 \times 0.001 )</td>
<td>( 6 \times 10^{-3} )</td>
</tr>
</tbody>
</table>

---

**Example 1**

**Writing Numbers in Scientific Notation**

**a.** The retina has 120,000,000 rods. Write this number in scientific notation.

- **Standard form** 120,000,000
- **Product form** \( 1.2 \times 100,000,000 \)
- **Scientific notation** \( 1.2 \times 10^8 \)

Exponent is 8.

**b.** The thickness of the human retina is 0.00012 meter. Write this number in scientific notation.

- **Standard form** 0.00012
- **Product form** \( 1.2 \times 0.0001 \)
- **Scientific notation** \( 1.2 \times 10^{-4} \)

Exponent is \(-4\).
Example 2  Writing Numbers in Standard Form

a. Write $3.2 \times 10^7$ in standard form.

<table>
<thead>
<tr>
<th>Scientific notation</th>
<th>Product form</th>
<th>Standard form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.2 \times 10^7$</td>
<td>$3.2 \times 10,000,000$</td>
<td>$32,000,000$</td>
</tr>
<tr>
<td>Exponent is 7.</td>
<td></td>
<td>Move decimal point 7 places to the right.</td>
</tr>
</tbody>
</table>

b. Write $8.69 \times 10^{-5}$ in standard form.

<table>
<thead>
<tr>
<th>Scientific notation</th>
<th>Product form</th>
<th>Standard form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8.69 \times 10^{-5}$</td>
<td>$8.69 \times 0.00001$</td>
<td>$0.00000869$</td>
</tr>
<tr>
<td>Exponent is $-5$.</td>
<td></td>
<td>Move decimal point 5 places to the left.</td>
</tr>
</tbody>
</table>

Checkpoint

Write the number in scientific notation.

1. 4100  
2. 0.000067  
3. 34,600,000  
4. 0.0000145

Write the number in standard form.

5. $7.1 \times 10^4$  
6. $1.93 \times 10^{-3}$  
7. $3.641 \times 10^{-6}$  
8. $5.59 \times 10^8$

Comparing Numbers To compare numbers written in scientific notation, first compare the powers of 10, then compare the decimal parts.

Example 3  Ordering Numbers Using Scientific Notation

Order $3.9 \times 10^6$, $3,800,000$, and $4.2 \times 10^5$ from least to greatest.

1. Write each number in scientific notation if necessary.
   
   $3,800,000 = 3.8 \times 10^6$

2. Order the numbers with different powers of 10.
   
   Because $10^5 < 10^6$, $4.2 \times 10^5 < 3.9 \times 10^6$ and $4.2 \times 10^5 < 3.8 \times 10^6$.

3. Order the numbers with the same power of 10.
   
   Because $3.8 < 3.9$, $3.8 \times 10^6 < 3.9 \times 10^6$.

4. Write the original numbers in order from least to greatest.
   
   $4.2 \times 10^5$; $3,800,000$; $3.9 \times 10^6$

Checkpoint

Order the numbers from least to greatest.

9. $2.4 \times 10^5$; $3.3 \times 10^4$; $49,000$  
10. $8.16 \times 10^5$; $635,000$; $4.08 \times 10^5$

11. $0.00017$; $1.9 \times 10^{-4}$; $2.8 \times 10^{-3}$  
12. $7.8 \times 10^{-3}$; $7.9 \times 10^{-3}$; $0.00056$
Example 4  Multiplying Numbers in Scientific Notation

**Plants** A wolffia plant is the smallest flowering plant in the world. One wolffia plant has a mass of about $1.5 \times 10^{-4}$ gram. At least $5 \times 10^3$ wolffia plants could fit in a thimble. What is the mass of $5 \times 10^3$ wolffia plants?

**Solution**

$$\text{Total mass} = \text{Mass of one plant} \times \text{Number of plants}$$

$$(1.5 \times 10^{-4})(5 \times 10^3)$$

$$(1.5 \times 5) \times (10^{-4} \times 10^3)$$

$$7.5 \times (10^{-4} \times 10^3)$$

$$7.5 \times 10^{-1} + 3$$

$$7.5 \times 10^{-1}$$

**Answer** The mass of $5 \times 10^3$ wolffia plants is $7.5 \times 10^{-1}$ gram, or 0.75 gram.

### 4.7 Exercises

*More Practice, p. 806*

**Guided Practice**

**Vocabulary Check**

1. Give an example of a number that is between 0 and 1 and is written in scientific notation.

2. Explain why $12.5 \times 10^7$ is *not* written in scientific notation.

**Skill Check**

**Write the number in scientific notation.**

3. 9,180,000  
4. 0.000062  
5. 723,000  
6. 0.00000002

**Write the number in standard form.**

7. $2.78 \times 10^7$  
8. $5.67 \times 10^{-3}$  
9. $4.15 \times 10^{-5}$  
10. $1.96 \times 10^5$

**11. Bicycle Chain** Scientists have made a tiny bicycle chain out of silicon links that are thinner than a human hair. The centers of the links are 0.00005 meter apart. Write this distance in scientific notation.

**12. Error Analysis** Describe and correct the error in comparing $6.5 \times 10^3$ and $6.4 \times 10^4$.

*Because $6.5 > 6.4$, $6.5 \times 10^3 > 6.4 \times 10^4$.*
Write the number in scientific notation.
13. 46,200,000  
14. 9,750,000  
15. 1700  
16. 8,910,000,000  
17. 104,000  
18. 0.00000062  
19. 0.000023  
20. 0.00095  
21. 0.0000106

Write the number in standard form.
22. 4.18 \times 10^4  
23. 5.617 \times 10^6  
24. 7.894 \times 10^8  
25. 3.8 \times 10^{-9}  
26. 9.83 \times 10^{-2}  
27. 6 \times 10^{-7}  
28. 1.03 \times 10^{-5}  
29. 2.28 \times 10^9  
30. 8.391 \times 10^4

In Exercises 31–33, write the number in scientific notation.
31. Population of Asia in 2001: 3,721,000,000  
32. Distance (in meters) to the star Vega: 239,000,000,000,000  
33. Time (in seconds) required for light to travel 1 meter: 0.0000000000334

In Exercises 34–36, write the number in standard form.
34. Distance (in centimeters) that the North Pacific plate slides along the San Andreas fault in 1 hour: 5.71 \times 10^{-4}  
35. Diameter (in meters) of a xylem cell in a redwood tree: 3.0 \times 10^{-5}  
36. Cruising speed (in miles per hour) of a supersonic jet: 1.336 \times 10^3

37. Critical Thinking  Your friend thinks that 4 \times 10^3 is twice as great as 2 \times 10^2. What error is your friend making? Explain your reasoning.

38. Writing  When a number between 0 and 1 is written in scientific notation, what can you say about the exponent? When a number greater than 1 is written in scientific notation, what can you say about the exponent?

39. Dust Mites  Dust mites are microscopic organisms that can be found in most natural and synthetic fibers. Dust mites are 0.00042 meter in length and 0.00028 meter in width. An average mattress contains 2,000,000 dust mites. Write these numbers in scientific notation.

Copy and complete the statement using <, >, or =.
40. 3.21 \times 10^3 \_ 321,000  
41. 91,600 \_ 9.61 \times 10^4  
42. 2.3 \times 10^{-6} \_ 1.3 \times 10^{-2}  
43. 0.00875 \_ 8.75 \times 10^{-4}

Find the product. Write your answer in scientific notation.
44. (2.5 \times 10^3)(3 \times 10^2)  
45. (6 \times 10^7)(9 \times 10^5)  
46. (5 \times 10^{-3})(7.5 \times 10^8)  
47. (8.5 \times 10^{-2})(7 \times 10^{-7})

Order the numbers from least to greatest.
48. 2.6 \times 10^4; 3,500; 9.2 \times 10^4  
49. 8,700; 1.97 \times 10^3; 3.98 \times 10^4  
50. 9.1 \times 10^{-4}; 5.2 \times 10^{-2}; 0.0013  
51. 7.61 \times 10^{-3}; 0.00009; 8.4 \times 10^{-6}
52. **Compact Discs** The information stored on a compact disc is encoded in a series of pits. The spaces between the pits are called lands. Each land is about 0.000003 meter long, and the average pit length is 0.0000022 meter.

   a. Write each of these lengths in scientific notation. Then write the combined length of a pit and a land in scientific notation.

   b. Suppose a compact disc contains 2,000,000,000 pits and 2,000,000,000 lands. How long would this series of pits and lands be if laid out in a straight line? Give your answer in scientific notation.

53. **Extended Problem Solving** Plankton are microscopic organisms that drift in water. A right whale feeds by swimming through masses of plankton with its mouth open. Answer the following questions using scientific notation.

   a. **Analyze** When a right whale feeds, about 2.3 cubic meters of water pass through its mouth each second. Right whales feed in areas with about 9000 plankton per cubic meter. How many plankton does a right whale ingest each second?

   b. How many plankton does a right whale ingest in 1 hour of feeding?

   c. **Estimate** A right whale may feed for up to 15 hours a day. Use a calculator to find how many plankton a right whale ingests in a day.

   d. **Estimate** Suppose a right whale consumes 500,000 Calories per day. About how many calories does a single plankton contain?

54. **Challenge** Let $n$ be any positive integer. Consider the expressions $n \times 10^{n+1}$ and $(n + 1) \times 10^n$.

   a. Make a table of values for each expression when $n = 1, 2, 3, \text{and} 4$.

   b. Is the value of $n \times 10^{n+1}$ always, sometimes, or never greater than the value of $(n + 1) \times 10^n$? Explain.

---

**Mixed Review**

Order the integers from least to greatest. *(Lesson 1.4)*

55. $-16, 13, 11, -17$  
56. $-23, 24, -27, 25$  
57. $-119, 99, -114, -98$

**Algebra Basics** Solve the equation. *(Lesson 2.7)*

58. $x + 3.6 = -10.8$  
59. $y - 9.5 = 11.2$  
60. $2.5m = -5.1$

Solve the inequality. Graph and check your solution. *(Lesson 3.6)*

61. $3x - 7 > 8$  
62. $-4y + 16 < 36$  
63. $2 - 5x > 27$

---

**Standardized Test Practice**

64. **Extended Response** The table shows the 2001 populations of several countries.

   a. Which country has the greatest population?

   b. Which country has the least population?

   c. How many times greater is the population of the country in part (a) than the population of the country in part (b)? Explain.

<table>
<thead>
<tr>
<th>Country</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>$1.273 \times 10^9$</td>
</tr>
<tr>
<td>Fiji</td>
<td>844,000</td>
</tr>
<tr>
<td>Iceland</td>
<td>$2.78 \times 10^5$</td>
</tr>
<tr>
<td>Russia</td>
<td>142,300,000</td>
</tr>
</tbody>
</table>
4.7 Using Scientific Notation

**Goal** Use a scientific calculator to perform operations on numbers written in scientific notation.

**Example**

Use a calculator to solve the following problem.

Scientists have discovered over 100 exoplanets (planets outside of our solar system). One of these exoplanets orbits the star Epsilon Eridani. The mass of this exoplanet is about $2.3 \times 10^{27}$ kilograms. The star and the exoplanet are about 10.5 light-years from the Sun.

How many times more massive is the exoplanet than Earth, which has a mass of $6 \times 10^{24}$ kilograms? Given that 1 light-year is equal to $9.5 \times 10^{12}$ kilometers, what is the distance (in kilometers) from the Sun to the exoplanet?

**Solution**

To find how many times more massive the exoplanet is than Earth, divide the mass of the exoplanet by the mass of Earth.

**Keystrokes**

2.3  EE 27 ÷ 6 EE 24 =

The exoplanet is about 383 times more massive than Earth.

To find the distance (in kilometers) of the exoplanet from the Sun, multiply the distance in light-years by the number of kilometers in a light-year.

**Keystrokes**

9.5 EE 12 × 10.5 =

The exoplanet is about $9.98 \times 10^{13}$ kilometers from the Sun.

**Draw Conclusions**

Use a calculator to find the product or quotient.

1. $(6.13 \times 10^{17}) \times (8.92 \times 10^{-11})$
2. $(4.09 \times 10^{-9}) \div (5.31 \times 10^{23})$
3. **Tau Boo** The star Tau Boo has an exoplanet that is about $2.5 \times 10^2$ times as massive as Earth. What is the mass (in kilograms) of the Tau Boo exoplanet?
Chapter Review

Vocabulary Review
- prime number, p. 173
- composite number, p. 173
- prime factorization, p. 173
- factor tree, p. 173
- monomial, p. 174
- common factor, p. 177
- greatest common factor (GCF), p. 177
- relatively prime, p. 178
- equivalent fractions, p. 182
- simplest form, p. 183
- multiple, p. 187
- common multiple, p. 187
- least common multiple (LCM), p. 187
- least common denominator (LCD), p. 188
- scientific notation, p. 204

1. Give an example of a prime number and an example of a composite number.
2. What does it mean for two nonzero whole numbers to be relatively prime?
3. Write two equivalent fractions and explain why they are equivalent.
4. Is the number $0.32 \times 10^{-4}$ written in scientific notation? Why or why not?

4.1 Factors and Prime Factorization

Goal
Factor numbers and monomials.

Example
Write the prime factorization of 240.

- Write original number.
- Write 240 as $12 \cdot 20$.
- Write 12 as $3 \cdot 4$ and 20 as $4 \cdot 5$.
- Write 4 as $2 \cdot 2$, twice.

The prime factorization of 240 is $2^4 \cdot 3 \cdot 5$.

Example
Factor the monomial $42x^4y$.

- Write 42 as $2 \cdot 3 \cdot 7$.
- Write $x^4$ as $x \cdot x \cdot x \cdot x$.

Write the prime factorization of the number.

- 5. 75
- 6. 104
- 7. 129
- 8. 138

Factor the monomial.

- 9. $36a^2b^3$
- 10. $98x^3y^2$
- 11. $72w^6z$
- 12. $15r^2s^2$
4.2 Greatest Common Factor

**Goal**
Find the greatest common factor of 45, 18, and 90.

**Example**
Find the greatest common factor of 45, 18, and 90.

Write the prime factorization of each number.

\[
45 = 3 \cdot 3 \cdot 5 \\
18 = 2 \cdot 3 \cdot 3 \\
90 = 2 \cdot 3 \cdot 3 \cdot 5
\]

The common prime factors are 3 and 3. The GCF is the product \(3 \cdot 3 = 9\).

Find the greatest common factor of the numbers.

13. 26, 74  
14. 32, 64  
15. 12, 40, 68  
16. 15, 42, 63

4.3 Equivalent Fractions

**Goal**
Write fractions in simplest form.

**Example**
Write \(\frac{60}{75}\) in simplest form.

Write the prime factorization of the numerator and the denominator.

\[
60 = 2^2 \cdot 3 \cdot 5 \\
75 = 3 \cdot 5 \cdot 5
\]

The GCF of 60 and 75 is \(3 \cdot 5 = 15\).

\[
\frac{60}{75} = \frac{60 \div 15}{75 \div 15} \quad \text{Divide numerator and denominator by GCF.}
\]

\[
= \frac{4}{5} \quad \text{Simplify.}
\]

**Example**
Write \(\frac{21a^2}{49ab}\) in simplest form.

\[
\frac{21a^2}{49ab} = \frac{3 \cdot 7 \cdot a \cdot a}{7 \cdot 7 \cdot a \cdot b} \quad \text{Factor numerator and denominator.}
\]

\[
= \frac{3 \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot a \cdot a}{\frac{1}{1} \cdot \frac{1}{1} \cdot b} \quad \text{Divide out common factors.}
\]

\[
= \frac{3a}{7b} \quad \text{Simplify.}
\]

Write the fraction in simplest form.

17. \(\frac{4}{18}\)  
18. \(\frac{12}{21}\)  
19. \(\frac{17}{68}\)  
20. \(\frac{30}{72}\)

21. \(\frac{6ab}{4b^2}\)  
22. \(\frac{5cd}{2d}\)  
23. \(\frac{8xy}{2x^2y}\)  
24. \(\frac{22m^2n}{11mn^2}\)
4.4 Least Common Multiple

**Goal**

Use the LCD to compare \( \frac{5}{36} \) and \( \frac{17}{90} \).

1. Find the least common multiple of the denominators.

\[
36 = 2 \times 2 \times 3 \times 3 \\
90 = 2 \times 3 \times 3 \times 5
\]

The common factors are 2, 3, and 3.

Multiply all of the factors, using the common factors only once.

\[
\text{LCM} = 2 \times 3 \times 3 \times 2 \times 5 = 180, \text{ so the LCD} = 180.
\]

2. Write equivalent fractions using the LCD.

\[
\frac{5}{36} = \frac{5 \times 5}{36 \times 5} = \frac{25}{180} \quad \frac{17}{90} = \frac{17 \times 2}{90 \times 2} = \frac{34}{180}
\]

3. Compare the numerators: \( \frac{25}{180} < \frac{34}{180} \), so \( \frac{5}{36} < \frac{17}{90} \).

**Use the LCD to determine which fraction is greater.**

25. \( \frac{1}{12} \) \( \frac{3}{40} \) 26. \( \frac{4}{15} \) \( \frac{7}{27} \) 27. \( \frac{7}{30} \) \( \frac{11}{36} \) 28. \( \frac{4}{45} \) \( \frac{13}{60} \)

29. **Soccer**

You and your friend are on different soccer teams. This season, your team won 14 out of 20 games. Your friend’s team won 18 out of 24 games. Which team won a greater fraction of its games?

4.5 Rules of Exponents

**Goal**

Use rules of exponents to simplify products and quotients.

**Example**

Find the product. Write your answer using exponents.

a. \( 5^8 \cdot 5^6 = 5^{8+6} \) \[ \text{Product of powers property} \]

\[ = 5^{14} \] \[ \text{Add exponents.} \]

b. \( 7a^2 \cdot a^6 = 7 \cdot (a^2 \cdot a^6) \) \[ \text{Associative property of multiplication} \]

\[ = 7 \cdot a^{2+6} \] \[ \text{Product of powers property} \]

\[ = 7a^8 \] \[ \text{Add exponents.} \]

**Find the product. Write your answer using exponents.**

30. \( 2^{11} \cdot 2^3 \) 31. \( 3^5 \cdot 3^7 \) 32. \( 7^8 \cdot 7^9 \) 33. \( 10^4 \cdot 10^4 \)

34. \( 16b^4 \cdot b^2 \) 35. \( c^9 \cdot 8c^2 \) 36. \( 5x \cdot 4x^9 \) 37. \( y^4 \cdot y^3 \cdot y^2 \)
4.6 Negative and Zero Exponents

Goal
Rewrite expressions containing negative or zero exponents.

Example
Write $8^0 b^{-5}$ using only positive exponents.

$$8^0 b^{-5} = 1 \cdot b^{-5} \quad \text{Definition of zero exponent}$$

$$= \frac{1}{b^5} \quad \text{Definition of negative exponent}$$

Write the expression using only positive exponents.

38. $12^{-4}$
39. $6^0$
40. $7c^{-3}$
41. $15d^{-3}$

4.7 Scientific Notation

Goal
Write numbers in scientific notation.

Example
Write the number in scientific notation.

<table>
<thead>
<tr>
<th>Standard form</th>
<th>Product form</th>
<th>Scientific notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 41,800,000</td>
<td>$4.18 \times 10^7$</td>
<td>$4.18 \times 10^7$</td>
</tr>
<tr>
<td>b. 0.0000037</td>
<td>$3.7 \times 0.000001$</td>
<td>$3.7 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Example
Order $4.7 \times 10^{-5}$, 0.0000056, and $3.2 \times 10^{-6}$ from least to greatest.

1. Write each number in scientific notation if necessary.
   
   $0.000056 = 5.6 \times 10^{-5}$

2. Order the numbers with different powers of 10.
   
   Because $10^{-6} < 10^{-5}$, $3.2 \times 10^{-6} < 4.7 \times 10^{-5}$ and $3.2 \times 10^{-6} < 5.6 \times 10^{-5}$.

3. Then order the numbers with the same power of 10.
   
   Because $4.7 < 5.6$, $4.7 \times 10^{-5} < 5.6 \times 10^{-5}$.

4. Write the original numbers in order from least to greatest.
   
   $3.2 \times 10^{-6}$, $4.7 \times 10^{-5}$, 0.000056

Write the number in scientific notation.

42. 0.000000745  43. 67,000,000  44. 0.000000881  45. 4,280,000,000

Copy and complete the statement using $<$, $>$, or $=$.

46. $4.8 \times 10^{-5}$ $\underline{\neq} 4.8 \times 10^{-8}$
47. $1.08 \times 10^6$ $\underline{=}$ $1.09 \times 10^7$
Chapter Test

Write the prime factorization of the number.

1. 27
2. 60
3. 84
4. 260

Find the greatest common factor of the numbers. Then tell whether they are relatively prime.

5. 25, 75
6. 30, 49
7. 32, 90
8. 42, 108

Write the fraction in simplest form.

9. \(\frac{27}{90}\)
10. \(\frac{46}{60}\)
11. \(\frac{8xy}{16y}\)
12. \(\frac{12a^2}{2ab}\)

Use the LCD to determine which fraction is greater.

13. \(\frac{3}{5}, \frac{8}{15}\)
14. \(\frac{11}{12}, \frac{11}{20}\)
15. \(\frac{3}{35}, \frac{7}{45}\)
16. \(\frac{29}{50}, \frac{61}{100}\)

17. Basketball The table shows the points you scored and the total points your team scored for each game in the season playoff.

<table>
<thead>
<tr>
<th>Game</th>
<th>You</th>
<th>Your team</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>42</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>57</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>65</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>52</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
<td>60</td>
</tr>
</tbody>
</table>

   a. For each game, write the fraction of your team’s points that you scored. Give your answers in simplest form.
   b. In which game did you score the greatest fraction of points?

Find the product or quotient. Write your answer using exponents.

18. \(13^6 \cdot 13^4\)
19. \(4m^7 \cdot 5m^6\)
20. \(\frac{7^6}{7^5}\)
21. \(\frac{4w^{15}}{24w^3}\)

Write the expression using only positive exponents.

22. \(15^{-4}\)
23. \(16h^{-7}\)
24. \(12x^0\)
25. \(m^{-4}n^5\)

Write the number in scientific notation.

26. \(5,100,000,000\)
27. \(6,450,000,000,000\)
28. \(0.000000000897\)
29. \(0.00000093\)

Copy and complete the statement using <, >, or =.

30. \(9.0 \times 10^{17} \_ 5.2 \times 10^{18}\)
31. \(7.31 \times 10^{-2} \_ 7.31 \times 10^{-3}\)
32. \(1.25 \times 10^{-3} \_ 1.05 \times 10^{-9}\)
33. \(8.12 \times 10^5 \_ 8.18 \times 10^4\)
1. Which number is not prime?
   A. 7   B. 37   C. 53   D. 57

2. Which expression is the prime factorization of 168?
   F. $2^2 \cdot 3^2 \cdot 7$   G. $2 \cdot 3 \cdot 7^2$
   H. $2^3 \cdot 3 \cdot 7$   L. $2^3 \cdot 3 \cdot 7^2$

3. What is the greatest common factor of $14x^2$ and $38x^2$?
   A. $2x^3$   B. $2x^2$   C. $266x^3$   D. $532x^2$

4. Which numbers are relatively prime?
   F. 25, 36   G. 12, 20
   H. 24, 28   L. 45, 84

5. Which fraction is not in simplest form?
   A. $\frac{1}{2}$   B. $\frac{21}{32}$   C. $\frac{35}{54}$   D. $\frac{54}{72}$

6. In a florist’s window, 30 of 36 plants are flowering. Write the fraction of flowering plants in simplest form.
   F. $\frac{5}{6}$   G. $\frac{10}{12}$   H. $\frac{15}{18}$   I. $\frac{30}{36}$

7. What is the LCM of 16 and 80?
   A. 8   B. 80   C. 160   D. 320

8. Which fraction is greater than $\frac{17}{60}$?
   F. $\frac{4}{15}$   G. $\frac{7}{30}$   H. $\frac{19}{45}$   I. $\frac{29}{120}$

9. Which expression is equivalent to $8x^4 \cdot 5x^3$?
   A. $20x^7$   B. $40x^7$   C. $20x^{12}$   D. $40x^{12}$

10. Which expression is equivalent to $\frac{15b^9}{25b^3}$?
    F. $\frac{3b^3}{5}$   G. $\frac{3}{5b^3}$   H. $\frac{3b^6}{5}$   I. $\frac{3}{5b^6}$

11. Which expression is not equivalent to $\frac{1}{64}$?
    A. $2^{-6}$   B. $4^{-4}$   C. $8^{-2}$   D. $64^{-1}$

12. Which expression is equivalent to $3^{-4}x^0$?
    F. $\frac{1}{81}$   G. $3^4$   H. $\frac{1}{81x}$   I. $\frac{x}{81}$

13. Which list of numbers is in order from least to greatest?
    A. $1.4 \times 10^6, 3.28 \times 10^3, 6.3 \times 10^2, 8.2 \times 10^3$
    B. $6.3 \times 10^2, 8.2 \times 10^3, 3.28 \times 10^3, 1.4 \times 10^6$
    C. $1.4 \times 10^6, 6.3 \times 10^2, 3.28 \times 10^3, 8.2 \times 10^3$
    D. $6.3 \times 10^2, 3.28 \times 10^3, 8.2 \times 10^3, 1.4 \times 10^6$

14. **Short Response** A certain type of bacteria has been found in lengths 0.000018 meter, $7.5 \times 10^{-6}$ meter, and $2.5 \times 10^{-6}$ meter. Order these lengths from least to greatest.

15. **Extended Response** You have a wooden board that measures 54 centimeters by 90 centimeters. You want to cut the board into identical square pieces with integer side lengths and use all of the wood.
    a. Make a sketch of the board. Find three possible side lengths for the squares.
    b. What is the largest side length you can choose? Explain.
    c. How many square pieces will you have?