Multi-Step Equations and Inequalities

Before
In previous chapters you’ve . . .
- Written and evaluated variable expressions
- Used the distributive property to simplify variable expressions
- Solved one-step equations

Now
In Chapter 3 you’ll study . . .
- Writing and solving two-step equations
- Using the distributive property to solve equations
- Writing and solving inequalities
- Graphing inequalities on a number line

Why?
So you can solve real-world problems about . . .
- rafting, p. 123
- shopping, p. 133
- astronauts, p. 140
- skiing, p. 142
- in-line skates, p. 147
- amusement parks, p. 151
- advertising, p. 152
**Wind Power** These wind turbines near Palm Springs, California, produce power when the wind turns their blades. In this chapter, you will use inequalities to describe quantities like wind speed.

**What do you think?** Suppose a turbine begins producing power when the wind speed is at least 10 miles per hour, and shuts down when the wind speed exceeds 65 miles per hour. Plot 10 and 65 on a number line. Shade the number line to show the speeds at which the turbine produces power.
Chapter Prerequisite Skills

PREREQUISITE SKILLS QUIZ

Preparing for Success To prepare for success in this chapter, test your knowledge of these concepts and skills. You may want to look at the pages referred to in blue for additional review.

1. **Vocabulary** Explain what an equation is. Then give an example of an equation with a variable in it.

Use the distributive property to write an equivalent variable expression. (p. 71)

2. 9(x − 4)  
3. 8(z − 7)  
4. −6(−m + 12)  
5. −10(n − 5)

Simplify the expression. (p. 78)

6. c + 4 − c  
7. 9b − 12b + 3  
8. 4(a + 2) + a  
9. 2(2d + 5 + d)

Solve the equation. Check your solution. (pp. 91, 97, and 102)

10. x + 13 = 7  
11. \( \frac{h}{6} = -8 \)  
12. a − 9.6 = 2  
13. 65 = −13b

NOTETAKING STRATEGIES

**SUMMARIZING** At the end of each lesson, summarize the main idea of the lesson in your notes. Include important details.

Lesson 2.6 Solving Equations Using Multiplication or Division

Main Idea: Multiplying or dividing each side of an equation by the same nonzero number results in an equivalent equation.

Use division to solve multiplication equations. Use multiplication to solve division equations.

\[
\begin{align*}
2x &= 6 \\
\frac{2x}{2} &= \frac{6}{2} \\
x &= 3
\end{align*}
\]

\[
\begin{align*}
\frac{x}{5} &= 10 \\
5 \left( \frac{x}{5} \right) &= 5(10) \\
x &= 50
\end{align*}
\]

You may find this strategy helpful in Lesson 3.6 when you solve multi-step inequalities.
### 3.1 Modeling Two-Step Equations

#### Goal
Model and solve two-step equations.

#### Materials
- algebra tiles

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#### Investigate

**Use algebra tiles to solve** \(3x + 6 = 12\).

1. **Model** \(3x + 6 = 12\) using algebra tiles.
   
   ![](image1.png)

2. **Remove** six 1-tiles from each side.
   
   ![](image2.png)

3. **Divide** the remaining tiles into three equal groups. Each \(x\)-tile is equal to two 1-tiles. So, the solution is 2.
   
   ![](image3.png)

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#### Draw Conclusions

**Use algebra tiles to model and solve the equation.**

1. \(1 + 2x = 9\)  
2. \(4x + 1 = 5\)  
3. \(2x + 2 = 8\)  
4. \(9 = 2x + 5\)  
5. \(11 = 2 + 3x\)  
6. \(5x + 3 = 8\)

7. **Critical Thinking** What property of equality is used in Step 2? in Step 3?

8. **Writing** For each algebra-tile model shown above, write a corresponding algebraic equation.

9. **Interpret** Describe the steps you would take to solve \(2x + 1 = 5\) without using algebra tiles.
Solving Two-Step Equations

**Drum Set** You are buying a drum set that costs $495. The music store lets you make a down payment. You can pay the remaining cost in three equal monthly payments with no interest charged. You make a down payment of $150. How much is each monthly payment? In Example 4, you will see how to answer this question by writing and solving a **two-step equation**.

You can solve a two-step equation by using two inverse operations.

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**Example 1** Using Subtraction and Division to Solve

Solve \( 3x + 7 = -5 \). Check your solution.

\[
\begin{align*}
3x + 7 &= -5 \\
3x + 7 - 7 &= -5 - 7 \\
3x &= -12 \\
\frac{3x}{3} &= \frac{-12}{3} \\
x &= -4
\end{align*}
\]

**Answer** The solution is \(-4\).

**Check** \(3x + 7 = -5\)

\[
\begin{align*}
3(-4) + 7 &= -5 \\
-5 &= -5 \checkmark
\end{align*}
\]

Solution checks.

---

**Checkpoint**

Solve the equation. Check your solution.

1. \(4x + 1 = 5\) 
2. \(3n + 8 = 2\) 
3. \(1 = 2r + 9\) 
4. \(2 = 6h + 20\)

5. **Critical Thinking** How is solving \(3x - 7 = -5\) different from solving \(3x + 7 = -5\)?
Example 2  

**Using Addition and Multiplication to Solve**

Solve $\frac{x}{2} - 3 = 1$. Check your solution.

\[
\frac{x}{2} - 3 = 1 \quad \text{Write original equation.}
\]

\[
\frac{x}{2} - 3 + 3 = 1 + 3 \quad \text{Add 3 to each side.}
\]

\[
\frac{x}{2} = 4 \quad \text{Simplify.}
\]

\[
2\left(\frac{x}{2}\right) = 2(4) \quad \text{Multiply each side by 2.}
\]

\[
x = 8 \quad \text{Simplify.}
\]

Answer  The solution is 8.

✓ Check  \[
\frac{x}{2} - 3 = 1 \quad \text{Write original equation.}
\]

\[
\frac{8}{2} - 3 = 1 \quad \text{Substitute 8 for x.}
\]

\[
1 = 1 \quad \text{Solution checks.}
\]

**Checkpoint**

Solve the equation. Check your solution.

6. \[
\frac{b}{4} - 8 = 1
\]

7. \[
\frac{c}{6} - 2 = 6
\]

8. \[
2 = \frac{d}{5} - 1
\]

9. \[
12 = \frac{f}{2} - 8
\]

Example 3  

**Solving an Equation with Negative Coefficients**

Solve $7 - 4y = 19$. Check your solution.

\[
7 - 4y = 19 \quad \text{Write original equation.}
\]

\[
7 - 4y - 7 = 19 - 7 \quad \text{Subtract 7 from each side.}
\]

\[
-4y = 12 \quad \text{Simplify.}
\]

\[
\frac{-4y}{-4} = \frac{12}{-4} \quad \text{Divide each side by -4.}
\]

\[
y = -3 \quad \text{Simplify.}
\]

Answer  The solution is -3.

✓ Check  \[
7 - 4y = 19 \quad \text{Write original equation.}
\]

\[
7 - 4(-3) = 19 \quad \text{Substitute -3 for y.}
\]

\[
19 = 19 \quad \text{Solution checks.}
\]

✓ Checkpoint

Solve the equation. Check your solution.

10. \[
12 - 4s = -12
\]

11. \[
6 - 2m = 8
\]

12. \[
-2 = 5 - n
\]

Lesson 3.1  Solving Two-Step Equations  121
Example 4  Writing and Solving a Two-Step Equation

Find the monthly payment for the drum set described on page 120.

Solution

Let \( p \) represent the monthly payment. Write a verbal model.

\[
\text{Total cost of drum set} = \text{Amount of down payment} + \text{Number of months} \times \text{Monthly payment}
\]

\[
495 = 150 + 3p
\]

Substitute.

\[
495 - 150 = 150 + 3p - 150
\]

Subtract 150 from each side.

\[
345 = 3p
\]

Simplify.

\[
\frac{345}{3} = \frac{3p}{3}
\]

Divide each side by 3.

\[
115 = p
\]

Simplify.

Answer The monthly payment is $115.

3.1 Exercises

Guided Practice

Vocabulary Check

1. Copy and complete: You can use two \( \_ \_ \_ \_ \) operations to solve a two-step equation.

2. Describe the steps you would use to solve the equation \( 9 + 2s = 15 \).

Skill Check

Solve the equation. Check your solution.

3. \( 5c + 6 = 31 \)

4. \( -2 = \frac{t}{3} - 11 \)

5. \( -9z + 4 = -5 \)

6. \( -8 - 8d = 64 \)

Guided Problem Solving

7. Car Repair The total cost of repairing a car is the sum of the amount paid for parts and the amount paid for labor. You paid $78 for parts and $45 for each hour of labor. The total cost to repair the car was $168. How many hours did it take to repair the car?

1. Copy and complete the verbal model.

\[
\text{Total cost for repairs} = ? + \text{Cost for each hour of labor} \times ?
\]

2. Let \( h \) represent the number of hours spent on labor. Write an equation based on your verbal model.

3. Solve the equation to find how many hours it took to repair the car.
Solve the equation. Check your solution.

8. $12k + 7 = 31$
9. $13n + 42 = 81$
10. $56 = 17p - 29$
11. $\frac{w}{4} - 21 = -3$
12. $\frac{h}{9} - 19 = -10$
13. $\frac{d}{12} + 25 = 29$
14. $12 = \frac{a}{36} + 17$
15. $18 - r = 42$
16. $80 = 23 - 3\nu$
17. $-2q - 63 = 47$
18. $-\frac{x}{2} + 4 = 12$
19. $-5 = -19 - \frac{x}{7}$

20. **Driving** Your family is taking a long-distance car trip. You begin with 16 gallons of gasoline in the fuel tank. Your car uses 3 gallons of gasoline per hour of driving. You will stop to refuel when there is exactly 1 gallon of gasoline remaining in the tank.
   
   a. **Analyze** List the information you are given and the information you need to find.
   
   b. Write a verbal model. Then write an equation based on your verbal model.
   
   c. After how many hours will you need to stop to refuel? Justify your solution by making a table.

21. **Rafting** A group of 9 friends takes a white-water rafting trip. The total price of the trip before any discounts is $810. Each person in the group receives a student discount. The total price with the discount is $729. How much is the discount per person?

22. **Trains** A train consisting of 50 cars and one locomotive weighs a total of 4725 tons. The locomotive weighs 125 tons. All of the cars have the same weight. Find the weight (in tons) of one car.

Write the verbal sentence as an equation. Then solve the equation.

23. Five minus the product of 2 and a number is 7.

24. Thirty-two minus the product of 9 and a number is 140.

25. Thirteen plus the product of 6 and a number is 67.

26. Negative 8 minus the product of 3 and a number is 19.

27. **Extended Problem Solving** Your class has raised $755 for a hunger relief organization. The organization provides farm animals that people can use to produce food. Your class plans to buy animals for a family recovering from an earthquake.
   
   a. **Calculate** One heifer (a young cow) costs $500, and each flock of chicks costs $20. If your class buys one heifer, how many flocks of chicks can your class buy?
   
   b. **Calculate** Your class can also buy pigs for $120 each. If your class buys a heifer for $500, how many pigs can your class buy?
   
   c. **Interpret and Apply** If your class decides to buy the heifer and pigs as described in part (b), does your class have enough money to also buy a flock of chicks? Explain your reasoning.
Solve the equation. Check your solution.

28. \(54.7 = -9.3n + 8.2\) 
29. \(-5.7 + 2.6d = -14.02\)

30. \(3.2r + 14.7 = -6.74\) 
31. \(9.1 = \frac{k}{3.7} + 4.1\)

32. \(11.3 - \frac{P}{2.8} = 1.5\) 
33. \(-6.8 - \frac{c}{1.2} = -2.9\)

34. **Compare and Contrast** Your friend solved the equation \(18 - 2x = -36\) by first adding \(2x\) to each side of the equation. You solved the equation by subtracting 18 from each side as the first step. Compare and contrast the two methods. What do you notice?

35. **Class Trip** You are saving money for a class trip to Washington, D.C. You need $850 for the trip. You have saved $278. You can save an additional $50 each month.

   a. Write a variable expression to represent the total amount of money you have saved after \(m\) months. Evaluate your expression for whole-number values of \(m\). Record your results in a table like the one shown.

   - **Number of months from now**
   - **Amount of money saved**
     - 0: $278
     - 1: ?
     - 2: ?
     - 3: ?
     - 4: ?

   b. **Analyze** Use the data in your table to make a scatter plot. Put months on the horizontal axis and savings on the vertical axis. What pattern do you notice in your graph? How can you use the graph to find the number of months it will take you to save enough money for the trip?

c. Write and solve an equation to find the number of months it will take you to save enough money for the trip.

d. **Compare** List some advantages and disadvantages of the methods you used in parts (a), (b), and (c).

36. **Challenge** Solve \(\frac{x + 2}{4} = 2\). Explain how you solved the equation and how you know your solution is correct.

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**Mixed Review**

Use the distributive property to write an equivalent variable expression. 

(Lesson 2.2)

37. \(11(6z + 14)\) 
38. \(-9(2x + 12)\) 
39. \(12(3 - 5y)\) 
40. \(8(4 - 7w)\)

**Algebra Basics** Solve the equation. Check your solution. 

(Lesson 2.5)

41. \(c + 12 = 23\) 
42. \(b + 14 = 91\) 
43. \(x - 17 = -45\) 
44. \(d - 22 = -43\)

**Standardized Test Practice**

45. **Multiple Choice** What is the solution of the equation \(15y - 63 = 57\)?
   - A. \(-8\) 
   - B. 8 
   - C. 9 
   - D. 10

46. **Short Response** You purchase a video game system for $150. You make a down payment of $25. You pay the rest of the money you owe in 5 equal monthly payments with no interest. How much is each monthly payment? Show how you found your answer.
Solving Equations Having Like Terms and Parentheses

**School Spirit** Your school’s basketball team is playing in the championship game. For the game, the cheerleaders want to buy a banner that costs $47. They also want to buy small items to give to students in the stands. Pompoms cost $5.20 each. Noisemakers cost $0.80 each. The cheerleaders have a total budget of $377 for the game. If they buy equal numbers of pompoms and noisemakers, how many can they afford to buy?

### Example 1

**Writing and Solving an Equation**

Find how many pompoms and noisemakers the cheerleaders can afford to buy, as described above.

**Solution**

Let $n$ represent the number of pompoms and the number of noisemakers. Then $5.20n$ represents the cost of $n$ pompoms, and $0.80n$ represents the cost of $n$ noisemakers. Write a verbal model.

\[
\begin{array}{c|c|c|c}
\text{Cost of } n \text{ pompoms} & + & \text{Cost of } n \text{ noisemakers} & = & \text{Total budget} \\
5.20n & + & 0.80n & + & 47 = 377 \\
6.00n + 47 & = & 377 & \text{Substitute.} \\
6n + 47 - 47 & = & 377 - 47 & \text{Combine like terms.} \\
6n & = & 330 & \text{Subtract 47 from each side.} \\
\frac{6n}{6} & = & \frac{330}{6} & \text{Simplify.} \\
6n & = & 55 & \text{Divide each side by 6.} \\
\end{array}
\]

**Answer** The cheerleaders can afford to buy 55 pompoms and 55 noisemakers.
**Distributive Property** You can use the distributive property to solve equations involving parentheses.

### Example 2

#### Solving Equations Using the Distributive Property

Solve the equation.

a. $-21 = 7(3 - x)$

b. $-3(8 - 4x) = 12$

#### Solution

a. 

- Write original equation.
- $-21 = 7(3 - x)$
- Distributive property
- $-21 - 21 = 21 - 7x - 21$
- Subtract 21 from each side.
- $-42 = -7x$
- Simplify.
- $\frac{-42}{-7} = \frac{-7x}{-7}$
- Divide each side by $-7$.
- Simplify.
- $6 = x$

**Answer** The solution is 6.

b. 

- Write original equation.
- $-3(8 - 4x) = 12$
- Distributive property
- $-24 + 12x = 12$
- Add 24 to each side.
- $-24 + 12x + 24 = 12 + 24$
- Simplify.
- $12x = 36$
- Divide each side by 12.
- Simplify.
- $\frac{12x}{12} = \frac{36}{12}$
- $x = 3$

**Answer** The solution is 3.

### Example 3

#### Combining Like Terms After Distributing

Solve $5x - 2(x - 1) = 8$.

- Write original equation.
- $5x - 2(x - 1) = 8$
- Distributive property
- $5x - 2x + 2 = 8$
- Combine like terms.
- $3x + 2 = 8$
- Subtract 2 from each side.
- $3x = 6$
- Simplify.
- $\frac{3x}{3} = \frac{6}{3}$
- Divide each side by 3.
- Simplify.
- $x = 2$

### Checkpoint

Solve the equation. Check your solution.

1. $3n - 40 + 2n = 15$
2. $2(s - 1) = 6$
3. $13 = 2y - 3(y + 4)$
Guided Practice

**Vocabulary Check**

1. What property do you use when you rewrite the equation \(6(x + 1) = 12\) as \(6x + 6 = 12\)?

2. Identify the like terms you would combine to solve the equation \(-3x + 5 - 2x + 8 = 12\).

**Skill Check**

**Solve the equation. Check your solution.**

3. \(4 + x + 7 = 10\)

4. \(3x + 2x = 25\)

5. \(21 = 4x - 9 - x\)

6. \(3(x + 1) = 6\)

7. \(16 = 8(x - 1)\)

8. \(5 + 2(x - 2) = 19\)

9. **Geometry** The perimeter of the rectangle shown is 28 units. The length is 10 units. What is the width of the rectangle?

   1. Write an equation for the perimeter of the rectangle in terms of \(x\).
   2. Solve the equation to find the value of \(x\).
   3. Find the width of the rectangle using the value of \(x\).
   4. Check your answer.

Guided Problem Solving

9. Geometry The perimeter of the rectangle shown is 28 units. The length is 10 units. What is the width of the rectangle?

   1. Write an equation for the perimeter of the rectangle in terms of \(x\).
   2. Solve the equation to find the value of \(x\).
   3. Find the width of the rectangle using the value of \(x\).
   4. Check your answer.

Practice and Problem Solving

**Homework Help**

10. **Error Analysis** Describe and correct the error in solving the equation \(-2(5 - n) = 2\).

   \[-2(5 - n) = 2\]
   \[-10 - 2n = 2\]
   \[-10 - 2n + 10 = 2 + 10\]
   \[-2n = 12\]
   \[n = -6\]

   Solve the equation. Check your solution.

   11. \(13r - 7 - 10r = 2\)

   12. \(22 + 4y - 14 = 0\)

   13. \(2d + 24 + 3d = 84\)

   14. \(4(x + 5) = 16\)

   15. \(3(7 - 2y) = 9\)

   16. \(-2(z + 11) = 6\)

   17. \(-5(3n + 5) = 20\)

   18. \(-30 = 6(f - 5)\)

   19. \(12 = 3(m - 17)\)

   20. **Fishing** A family of five people has $200 to spend on fishing rods and fishing licenses. They spend a total of $20 on licenses. Assuming they buy 5 identical rods, what is the maximum amount they can spend on each rod?
21. **Karaoke** You want to organize a group of friends to go to a karaoke studio this Friday night. You must pay $30 to reserve a private karaoke room plus $5 for each person in the group. You also want to have snacks for the group at a cost of $2 per person. How many people can be in the group in order for the total cost to be $65?

**Solve the equation. Check your solution.**

22. \(-5(2w + 1) = 25\)

23. \(4(5 - p) = 8\)

24. \(-40 - (2x + 5) = -61\)

25. \(2 = 4(3k - 8) - 11k\)

26. \(42 = 18r + 4(t + 5)\)

27. \(-3(2z - 8) + 10z = 16\)

28. \(-5g - (8 - g) = 12\)

29. \(-5 = 0.25(4 + 20r) - 8r\)

30. \(2m + 0.5(m - 4) = 9\)

31. \(-12 = -2h + 0.2(20 - 6h)\)

32. **Photograph** The perimeter of a rectangular photograph is 22 inches. The length of the photograph is 1 inch more than the width. What are the dimensions of the photograph?

**Geometry** Find the value of \(x\) for the given triangle, rectangle, or square.

33. Perimeter = 40 units

34. Perimeter = 22 units

35. Perimeter = 104 units

36. Perimeter = 32 units

37. **Cell Phones** Your cell phone provider charges a monthly fee of $19.50 for 200 minutes. You are also charged $0.25 per minute for each minute over 200 minutes. Last month, your bill was $29.50.

a. Let \(m\) represent the total number of minutes you used last month. Use the verbal model below to write an equation.

\[
\text{Total phone bill} = \text{Monthly fee} + \text{Charge for each additional minute} \cdot \text{Number of minutes over 200}
\]

b. Solve the equation you wrote in part (a).

c. How many additional minutes did you use last month?

38. **Critical Thinking** Solve each equation by first dividing each side of the equation by the number outside the parentheses. Then solve each equation by first using the distributive property. What do you notice? When would you recommend using each method? Explain.

a. \(3(x + 7) = 42\)

b. \(4(6x - 8) = 14\)
39. **Challenge** The figure shown is composed of a triangle and a rectangle. The figure has a total area of 1258 square units. Find the value of $x$.

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**Mixed Review**

Plot the point in a coordinate plane. Describe the location of the point. *(Lesson 1.8)*

- 40. $J(-3, 8)$
- 41. $K(8, -3)$
- 42. $L(4, -4)$
- 43. $M(-1, -1)$
- 44. $N(0, 2)$
- 45. $P(5, 1)$
- 46. $Q(-9, 0)$
- 47. $R(-5, -8)$

Simplify the expression. *(Lesson 2.3)*

- 48. $a - 2 - (3 + a)$
- 49. $3b + 8 + 2(b - 4)$
- 50. $-2x + 5 - 7(x + 1)$
- 51. $2y - 4 + 3(y + 1)$
- 52. $-(2x + 3) + 4(x + 2)$
- 53. $3(2x - 7) + 8(4 - x)$
- 54. **Family Party** A family wants to hold a dinner party at a restaurant. The restaurant charges $150 to rent space for the party. The food cost for each person at the party is $18. How many people can come to the party if the family has $600 to spend? *(Lesson 3.1)*

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**Standardized Test Practice**

55. **Multiple Choice** What is the solution of the equation $-3(2x - 1) = -21$?
   - A. $-4$
   - B. $-3$
   - C. $3$
   - D. $4$

56. **Short Response** The length of a rectangle is 5 feet less than twice its width. The perimeter of the rectangle is 38 feet. Let $w$ represent the width. Write an equation for the perimeter of the rectangle in terms of $w$. Then solve the equation to find the length and width of the rectangle.

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**Brain GAME**

Solve each equation. In each group, there are two equations that have the same solution. Write the value of this solution in the corresponding letter’s blank to find the year blue jeans were patented.

- **A.** $10x + 7 = 17$
  - $2(7x + 6) = 40$
  - $-(x - 11) = 10$

- **B.** $8x - 15 = -47$
  - $6(2x - 1) = 90$
  - $-7x + 4x = -24$

- **C.** $-5x + 4x = -6$
  - $7x - (-12) = 61$
  - $7(x + 2) = 63$

- **D.** $2(6x + 7) = 50$
  - $-5x - 3x = -56$
  - $-11x - 9 = -42$

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**Patent Puzzle**

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**Lesson 3.2** Solving Equations Having Like Terms and Parentheses
3.3 Modeling Equations with Variables on Both Sides

**Goal**
Solve equations using algebra tiles.

**Materials**
- algebra tiles

**Investigate**

**Use algebra tiles to solve** $4x + 6 = 10 + 2x$.

1. Model $4x + 6 = 10 + 2x$ using algebra tiles.

2. Remove two $x$-tiles from each side.

3. Remove six 1-tiles from each side.

4. Divide the remaining tiles into two equal groups. Each $x$-tile is equal to two 1-tiles. So, the solution is 2.

**Draw Conclusions**

**Use algebra tiles to model and solve the equation.**

1. $9 + 2x = 1 + 3x$
2. $3x + 4 = 8 + x$
3. $5x + 2 = 3x + 14$

4. **Critical Thinking** In the activity above, would you find the correct solution if you performed Step 3 before Step 2? Explain.

5. **Writing** Explain how solving an equation with variables on both sides of the equal sign is different than solving an equation with the variable on one side.
Solving Equations with Variables on Both Sides

**Spanish Club** The Spanish club is arranging a trip to a Mexican restaurant in a nearby city. Those who go must share the $60 cost of using a school bus for the trip. The restaurant’s buffet costs $5 per person. How many students must sign up for this trip in order to limit the cost to $10 per student? In Example 2, you will see how to use an equation to answer this question.

Every equation in this lesson has variables on both sides of the equation. You can solve such an equation by getting the variable terms on one side of the equation and the constant terms on the other side.

### Example 1

**Solving an Equation with the Variable on Both Sides**

**Solve** \(7n - 5 = 10n + 13\).

1. \(7n - 5 = 10n + 13\)
2. \(7n - 5 - 7n = 10n + 13 - 7n\)
3. \(-5 = 3n + 13\)
4. \(-5 - 13 = 3n + 13 - 13\)
5. \(-18 = 3n\)
6. \(-18 / 3 = 3n / 3\)
7. \(-6 = n\)

**Answer** The solution is \(-6\).

### Checkpoint

Solve the equation. Check your solution.

1. \(5n - 2 = 3n + 6\)
2. \(8y + 4 = 11y - 17\)
3. \(m - 1 = 9m + 15\)
Example 2  Writing and Solving an Equation

How many students must go on the Spanish club trip to the Mexican restaurant, as described on page 131, in order for the cost per student to be $10?

Solution
Let \( s \) represent the number of students. Write a verbal model.

\[
\begin{array}{cc}
\text{Cost per student} & \times \text{Number of students} = \text{Cost of buffet} + \text{Cost of school bus} \\
10s & = 5s + 60 \\
10s - 5s & = 5s - 5s + 60 \\
5s & = 60 \\
\frac{5s}{5} & = \frac{60}{5} \\
s & = 12 \\
\end{array}
\]

Substitute.
Subtract 5s from each side.
Simplify.
Divide each side by 5.
Simplify.

Answer  The club needs 12 students to go on the trip.

Number of Solutions  When you solve an equation, you may find that it has no solution or that every number is a solution.

Example 3  An Equation with No Solution

Solve \( 5(2x + 1) = 10x \).

\[
\begin{align*}
5(2x + 1) & = 10x \\
10x + 5 & = 10x \\
\text{Write original equation.} & \quad \text{Distributive property}
\end{align*}
\]

Notice that \( 10x + 5 = 10x \) is not true because the number \( 10x \) cannot be equal to 5 more than itself. The equation has no solution. As a check, you can continue solving the equation.

\[
\begin{align*}
10x + 5 & - 10x = 10x - 10x \\
5 & = 0 \\
\text{Subtract } 10x \text{ from each side.} & \quad \text{Simplify.}
\end{align*}
\]

The statement \( 5 = 0 \) is not true, so the equation has no solution.

Example 4  Solving an Equation with All Numbers as Solutions

Solve \( 6x + 2 = 2(3x + 1) \).

\[
\begin{align*}
6x + 2 & = 2(3x + 1) \\
6x + 2 & = 6x + 2 \\
\text{Write original equation.} & \quad \text{Distributive property}
\end{align*}
\]

Notice that for all values of \( x \), the statement \( 6x + 2 = 6x + 2 \) is true. The equation has every number as a solution.
**Example 5**  
**Solving an Equation to Find a Perimeter**

**Geometry** Find the perimeter of the square.

1. A square has four sides of equal length. Write an equation and solve for x.
   
   \[2x = x + 4\]  
   Write equation.  
   
   \[2x - x = x + 4 - x\]  
   Subtract x from each side.  
   
   \[x = 4\]  
   Simplify.  

2. Find the length of one side by substituting 4 for x in either expression.
   
   \[2x = 2(4) = 8\]  
   Substitute 4 for x and multiply.  

3. To find the perimeter, multiply the length of one side by 4.
   
   \[4 \cdot 8 = 32\]  

**Answer** The perimeter of the square is 32 units.

---

**Guided Practice**

**Vocabulary Check**

1. Describe what steps you would take to solve \(8x + 5 = 2x - 7\).
2. Explain why the equation \(5z + 2 = 5z\) has no solution.

**Skill Check**

Solve the equation. Check your solution.

3. \(13m - 22 = 9m - 6\)
4. \(19c + 26 = 41 + 14c\)
5. \(15 - 4x = 42 - 7x\)
6. \(14 + 5y = 50 - 4y\)
7. \(18w - 2 = 10w + 14\)
8. \(-5a + 6 = 6a - 38\)

9. **Error Analysis** Describe and correct the error in solving the equation \(4x + 7 = x - 2\).

10. **Shopping** You spend $60 on clothes and buy 3 DVD movies. Your friend spends nothing on clothes but buys 8 DVD movies. You both spend the same amount of money. All the DVDs cost the same amount. How much does each DVD cost?
Solve the equation. Check your solution.

11. \(25u + 74 = 23u + 92\)
12. \(-5k - 19 = 5 - 13k\)
13. \(-11y + 32 = 104 - 5y\)
14. \(-15n + 16 = 86 - 29n\)
15. \(25t = 5(5t + 1)\)
16. \(13 - 3p = -5(3 + 2p)\)
17. \(-24s - 53 = 39 - s\)
18. \(14a - 93 = 49 - 57a\)
19. \(7(2p + 1) = 14p + 7\)
20. \(8v = 2(4v + 2)\)
21. \(3x + 6 = 3(2 + x)\)
22. \(2(-4h - 13) = 37 + 13h\)

Write the verbal sentence as an equation. Then solve the equation.

23. Nine plus 2 times a number is equal to 2 less than 3 times the number.
24. Three less than 11 times a number is equal to 9 plus 5 times the number.
25. Four minus 7 times a number is equal to 12 minus 5 times the number.
26. Twelve less than \(-9\) times a number is equal to 8 minus 4 times the number.

27. Toll Booth You lose your electronic tag that you use to pay tolls on the highway in your city. It costs you $24 to replace the tag. The cost of one toll when you don’t use the tag is $3. The cost of the same toll when you do use the tag is $1.50. How many times will you have to use the tag to pay for the tolls in order for the total cost to be the same as not using the tag?

Find the perimeter of the square.

28. \(4x\)
29. \(36 - 5x\)
30. \(7x + 30\)
31. Driving Your family is driving to Houston, Texas. A sign indicates that you are 700 miles from Houston. Your car’s trip odometer indicates that you are 400 miles from home. You are traveling at an average speed of 60 miles per hour.

a. Write an expression for the distance (in miles) you will be from Houston in \(x\) hours.

b. Write an expression for the distance (in miles) you will be from home in \(x\) hours.

c. Use the expressions from parts (a) and (b) to write and solve an equation to find the number of hours you will drive until you are exactly halfway between Houston and your home.

d. Suppose you travel by local roads instead of the highway. You travel the 700 miles at a speed of 45 miles per hour. How long will you drive before you are exactly halfway between Houston and your home?
32. **Pasta Machine** A pasta machine costs $33. The ingredients to make one batch of pasta cost $3.33. The same amount of pasta purchased at a store costs $0.99. How many batches of pasta will you have to make for the cost of the machine and ingredients to equal the cost of buying the same amount of pasta at the store?

33. **Writing** Describe a real-life situation that can be modeled by the equation \(11x + 5 = 8x + 23\). Then solve the equation and interpret your solution.

**Solve the equation. Check your solution.**

34. \(3x - 7 = 8 + 6(x + 2)\)

35. \(13y + 19 = 6(9 + y) + 14\)

36. \(8(z + 4) = 5(13 + z)\)

37. \(8a - 2(a + 5) = 2(a - 1)\)

38. **Geometry** The perimeter of the square is equal to the perimeter of the triangle. The sides of the triangle are equal in length.

   a. **Estimate** Without doing any calculations, estimate which figure has the greater side length. Explain your choice.

   b. What is the side length of each figure?

   c. What is the perimeter of each figure?

**Use a calculator to solve the equation. Check your solution.**

39. \(0.75m + 14 = 1.87m - 10.3936\)

40. \(19.5 + 0.5t = 10.6206 - 0.4t\)

41. \(-9.39 - 3.4d = -1.1d + 11.08\)

42. \(-130.5 - 9b = -55.104 + 3.2b\)

43. **Challenge** Consider the equation \(ax + 6 = 2(x + 3)\).

   a. For what value(s) of \(a\) does the equation have all numbers as a solution?

   b. For what value(s) of \(a\) does the equation have just one solution?

**Mixed Review**

**Algebra Basics** Solve the equation. Check your solution. (Lesson 2.5)

44. \(c - 20 = 14\)

45. \(d + 9 = -12\)

46. \(x - 3 = 17\)

47. \(y - 21 = -15\)

48. **Gym Membership** To join a gym, your friend pays a one-time fee of $75 and $45 per month for the duration of the membership. Your friend has paid a total of $345. How long has your friend been a member of the gym? (Lesson 3.1)

49. The perimeter of the square shown is 32 units. Find the value of \(x\). (Lesson 3.2)

**Standardized Test Practice**

50. **Multiple Choice** What is the solution of the equation \(2(3x + 4) = 6x + 5\)?

   A. 1  \hspace{1cm}  B. 3  \hspace{1cm}  C. All numbers  \hspace{1cm}  D. No solution

51. **Multiple Choice** For which equation is 6 a solution?

   F. \(-2y - 7 = 11 - 5y\)  \hspace{1cm}  G. \(11y - 32 = 7y - 12\)

   H. \(18y - 16 = 13y + 19\)  \hspace{1cm}  I. \(-7y - 24 = -8 - 9y\)
### 3.3 Solving Equations

**Goal** Use a table to solve an equation with the variable on both sides.

**Example**

1. **Use a table to solve** $5x - 1 = 4x + 3$.

   Enter the expressions on each side of the equal sign into a graphing calculator. The expression on the left is called Y1, and the expression on the right is Y2.

   **Keystrokes**
   
   \[
   \begin{array}{c}
   \text{Y= 5} \\
   \text{x} \\
   \text{=} \\
   \text{1 ENTER} \\
   \text{4} \\
   \text{x} \\
   \text{+ 3}
   \end{array}
   \]

   Use the calculator’s **table** feature to find the value of each expression for different values of $x$. Press **2nd** [TBLSET] and enter the settings shown on the first screen below. ($\Delta$Tbl represents the increment the calculator uses to go from one $x$-value to the next in the table.) Then, press **2nd** [TABLE] to display the table shown on the second screen.

   Compare the values of the expression in the Y1 column with the values of the expression in the Y2 column. The values are the same when $x = 4$. So, the solution of the equation $5x - 1 = 4x + 3$ is $4$.

2. **Draw Conclusions**

   **Use a table to solve the equation.**

   1. $x - 2 = 2x - 6$  
   2. $3x + 1 = x + 7$  
   3. $12 - x = x - 4$  
   4. $7x = 16 - x$  
   5. $5x + 2 = 8x - 1$  
   6. $4x - 6 = 2x + 4$  

   **Critical Thinking** Solve the equation $3x + 6 = 13x + 2$ using paper and a pencil. Explain how you would change the settings in the TABLE SETUP menu so that you could solve the equation using a calculator.
Write the verbal sentence as an equation. Then solve the equation.

1. Twice a number plus 5 is equal to 27.
2. Seven times the sum of 4 and a number is −14.
3. Three more than 4 times a number is equal to 9 less than twice the number.

Solve the equation. Check your solution.

4. $11k + 9 = 42$
5. $\frac{a}{3} + 11 = -5$
6. $\frac{w}{2} - 18 = -7$
7. $2 + 5t - 3 = 34$
8. $-3y + 15 - y = 39$
9. $5(n + 2) = 10$
10. $2 - 5(h + 3) = -28$
11. $5s = 7s + 1 - 2s$
12. $4d - 5 = -d$
13. $17 - 5m = 50 + 6m$
14. $3f - 12 = 3(f - 12)$
15. $8(4p + 1) = 32p + 8$

16. **Income** Your friend works as a waitress at a local restaurant. Her income consists of an hourly wage plus tips. On Wednesday, your friend earned $25 in tips over a 5 hour period. On Friday, your friend earned $30.76 in tips over a 3 hour period. How much is your friend’s hourly wage if your friend earned the same amount of money on Wednesday as on Friday?

17. All three sides of the triangle shown are equal in length. Find the perimeter of the triangle.

Two people are packing equal numbers of small boxes into large boxes. One person has 3 large boxes that are full of smaller boxes and 24 small boxes that are not yet packed. The other person has 5 large boxes that are full of smaller boxes and 10 small boxes that are not yet packed. Each large box holds the same number of small boxes. How many small boxes can each large box hold? What is the total number of small boxes each person will pack? How many large boxes will each person need in order to pack all of his or her small boxes?
Solving Inequalities Using Addition or Subtraction

**Vocabulary**
- **inequality**, p. 138
- solution of an inequality, p. 138
- equivalent inequalities, p. 139

**Before**
You solved one-step equations.

**Now**
You’ll solve inequalities using addition or subtraction.

**Why?**
So you can find the weight a truck can tow, as in Ex. 14.

An **inequality** is a statement formed by placing an inequality symbol between two expressions. For example, \( y + 5 \leq -6 \) is an inequality.

The **solution of an inequality** with a variable is the set of all numbers that produce true statements when substituted for the variable. You can show the solution of an inequality by graphing the inequality on a number line. When you graph an inequality of the form \( x > a \) or \( x < a \), use an open circle at \( a \). When you graph an inequality of the form \( x \geq a \) or \( x \leq a \), use a closed circle at \( a \).

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Words</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x &lt; 3 )</td>
<td>All numbers less than 3</td>
<td>![Graph of x &lt; 3]</td>
</tr>
<tr>
<td>( y &gt; 2 )</td>
<td>All numbers greater than 2</td>
<td>![Graph of y &gt; 2]</td>
</tr>
<tr>
<td>( z \leq 4 )</td>
<td>All numbers less than or equal to 4</td>
<td>![Graph of z ≤ 4]</td>
</tr>
<tr>
<td>( n \geq 2 )</td>
<td>All numbers greater than or equal to 2</td>
<td>![Graph of n ≥ 2]</td>
</tr>
</tbody>
</table>

**Example 1**  
**Writing and Graphing an Inequality**

**Science**  
The freezing point of water is \( 0 \)°C. At temperatures at or below the freezing point, water is a solid (ice). Write an inequality that gives the temperatures at which water is a solid. Then graph the inequality.

**Solution**  
Let \( t \) represent the temperature of water. Water is a solid at temperatures less than or equal to \( 0 \)°C.

**Answer**  
The inequality is \( t \leq 0 \). The graph is shown below.

![Graph of t ≤ 0]
Solving Inequalities  You can use the following properties to find the solutions of inequalities involving addition and subtraction. Using these properties, you can write equivalent inequalities. Equivalent inequalities are inequalities that have the same solution.

**Study Strategy**

The addition and subtraction properties of inequality are also true for inequalities involving ≤ and ≥.

**Addition and Subtraction Properties of Inequality**

**Words** Adding or subtracting the same number on each side of an inequality produces an equivalent inequality.

**Algebra** If \(a < b\), then \(a + c < b + c\) and \(a - c < b - c\).

If \(a > b\), then \(a + c > b + c\) and \(a - c > b - c\).

---

**Example 2**  **Solving an Inequality Using Subtraction**

**Solve** \(m + 5 \geq 10\). Graph and check your solution.

\[
egin{align*}
\text{Write original inequality.} \\
\text{Subtract 5 from each side.} \\
\text{Simplify.}
\end{align*}
\]

**Answer** The solution is \(m \geq 5\).

**Check** Choose any number greater than or equal to 5. Substitute the number into the original inequality.

\[
\begin{align*}
\text{Write original inequality.} \\
\text{Substitute 8 for } m. \\
\text{Solution checks.}
\end{align*}
\]

---

**Example 3**  **Solving an Inequality Using Addition**

**Solve** \(-10 > x - 12\). Graph your solution.

\[
\begin{align*}
\text{Write original inequality.} \\
\text{Add 12 to each side.} \\
\text{Simplify.}
\end{align*}
\]

**Answer** The solution is \(2 > x\), or \(x < 2\).

**Checkpoint**

Solve the inequality. Graph and check your solution.

1. \(n + 7 > 3\)  
2. \(10 \geq y + 4\)  
3. \(-6 \leq x - 9\)  
4. \(z - 5 < 1\)

---

Lesson 3.4  Solving Inequalities Using Addition or Subtraction
Example 4  Writing and Solving an Inequality

**Triathlon**  You are competing in a triathlon, a sports competition with three events. Last year, you finished the triathlon in 85 minutes. The table shows your times for this year’s first two events. What possible times can you post in the running event and still beat last year’s finishing time?

<table>
<thead>
<tr>
<th>Triathlon Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event</td>
</tr>
<tr>
<td>Swimming</td>
</tr>
<tr>
<td>Biking</td>
</tr>
<tr>
<td>Running</td>
</tr>
</tbody>
</table>

**Solution**

Let \( t \) represent this year’s running time. Write a verbal model.

\[
17 + 45 + t < 85
\]

Substitute.

\[
62 + t < 85
\]

Simplify.

\[
62 + t - 62 < 85 - 62
\]

Subtract 62 from each side.

\[
t < 23
\]

Simplify.

**Answer**  To beat last year’s finishing time, you must post a time in the running event that is less than 23 minutes.

### 3.4  Exercises

**Guided Practice**

**Vocabulary Check**

1. What are equivalent inequalities?
2. Explain how the graph of \( x > 5 \) is different from the graph of \( x \geq 5 \).

**Skill Check**

Tell whether the given number is a solution of \(-5 < n\).

3. 8  
4. -8  
5. -4  
6. 4

Solve the inequality. Graph and check your solution.

7. \( x + 2 > -3 \)  
8. \( 1 \geq x - 9 \)  
9. \( x + 4 < 3 \)  
10. \( x + 3 > 7 \)

**Guided Problem Solving**

11. **Astronauts**  To become a NASA pilot astronaut, a NASA pilot must log at least 1000 hours as pilot-in-command of a jet aircraft. A NASA pilot has completed all other qualifications and has 250 hours logged. How many more hours must the pilot log to become a pilot astronaut?

1. Write an inequality to represent the situation.
2. Solve the inequality. Then graph and check the solution.
3. Interpret the solution in terms of the real-life situation.
Write an inequality to represent the situation.
12. The greatest weight that a forklift can raise is 2500 pounds.
13. The speed limit is 55 miles per hour.
14. A truck can tow a maximum weight of 7700 pounds.
15. You must be at least 48 inches tall to ride the roller coaster.
16. You can save up to $50 on DVD players this week.

Write an inequality represented by the graph.
17. 
\[ -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \]
18. 
\[ -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \]
19. 
\[ -2 \quad 0 \quad 2 \quad 4 \quad 6 \quad 8 \]
20. 
\[ -40 \quad -30 \quad -20 \quad -10 \quad 0 \quad 10 \]

Solve the inequality. Graph your solution.
21. \[ x + 4 < 5 \]
22. \[ m + 8 \geq 12 \]
23. \[ -11 < y + 5 \]
24. \[ -8 \geq d - 7 \]
25. \[ -45 > g - 16 \]
26. \[ z - 15 \leq 72 \]
27. \[ f + 1 \geq -8 \]
28. \[ h + 19 \leq 15 \]
29. \[ 18.1 \leq p - 7 \]
30. \[ t - 7 < 3.4 \]
31. \[ b + 2.5 \leq 2.5 \]
32. \[ a - 10.2 > 5.3 \]
33. **Neon** The lowest temperature at which neon is a gas, called its boiling point, is \(-411^\circ F\). Write and graph an inequality to show the temperatures at which neon is a gas.

Solve the inequality. Graph your solution.
34. \[ 5 + m + 8 \geq 14 \]
35. \[ 13 + n - 26 < 38 \]
36. \[ 2.35 + p + 14.9 > 49.25 \]
37. \[ q + 4 + 16 \geq 30 \]
38. **Bacteria** In 1969, Apollo 12 astronauts found a small colony of *Streptococcus* bacteria that had apparently traveled unprotected to the moon on the Surveyor 3 spacecraft’s TV camera about three years earlier. The bacteria survived at temperatures as low as \(-280^\circ F\). Write and graph an inequality to show the temperatures at which the bacteria survived.

39. **Writing** Is it possible to check *all* the numbers that are solutions of an inequality? Explain. Does checking just *one* number guarantee that a solution is correct?

40. **Train Travel** You are traveling by train. You are allowed two carry-on bags, each with a maximum weight of 50 pounds. You have two bags: one that weighs 14 pounds and one that weighs 21 pounds.
   
a. Write and solve an inequality that represents the weight \(w\) (in pounds) of personal belongings you can add to the first bag without exceeding the weight limit.
   
b. Write and solve an inequality that represents the weight \(w\) (in pounds) of personal belongings you can add to the second bag without exceeding the weight limit.
In Exercises 41 and 42, graph the compound inequality. A compound inequality consists of two inequalities joined by the word and or or.

**Example** Graphing Compound Inequalities

Graph the compound inequality \( x > 3 \) and \( x < 10 \).
Include numbers that are both greater than 3 and less than 10.

41. \( x \geq -1 \) and \( x \leq 4 \)  
42. \( x < 3 \) and \( x \geq 0 \)

43. **Skiing** The ski wax you use keeps your skis performing well at temperatures from \(-6^\circ C\) to \(15^\circ C\). Express the lower limit of the ski wax as an inequality, and express the upper limit as an inequality. Then write the inequalities as a compound inequality and graph it.

44. **Challenge** Explain how you can graph the compound inequality \( x \leq 8 \) or \( x \geq 10 \). How does this graph look different from the graph of \( x \geq 8 \) and \( x \leq 10 \)?

**Mixed Review**

45. **Geometry** Find the length of a side of a square with a perimeter of 36.6 meters. *(Lesson 2.7)*

46. **Fundraising** A basketball team is raising money for uniforms and equipment. So far, the team has raised \$1275. The team plans to spend \$450 on equipment and buy as many uniforms as possible. Each uniform costs \$55. How many uniforms can the team buy with the money it has raised? *(Lesson 3.1)*

Write the verbal sentence as an equation. Then solve the equation. *(Lesson 3.3)*

47. Five plus 4 times a number is equal to the sum of 7 times the number and 11.

48. Eight less than 3 times a number is equal to \(-3\) plus twice the number.

**Standardized Test Practice**

49. **Multiple Choice** Which inequality is represented by the graph shown?

   - A. \( y < -2 \)
   - B. \( y > -2 \)
   - C. \( y \leq -2 \)
   - D. \( y \geq -2 \)

50. **Multiple Choice** Which number is a solution of the inequality \( b + 2 > 2 \)?

   - F. 4
   - G. 0
   - H. \(-1\)
   - I. \(-2\)
### 3.5 Multiplication and Division Properties of Inequality

**Goal**
Perform multiplications and divisions on inequality statements.

**Materials**
- pencil
- paper

---

#### Investigate

**Determine how multiplication or division affects an inequality.**

1. Choose two different integers and insert an inequality symbol between them to make a true statement.
   
   \(-2 < 4\)

2. Multiply each number in the original inequality by 2. Is the new inequality a true statement?
   
   \[2 \cdot (-2) \overset{?}{=} 2 \cdot 4\]
   
   \(-4 < 8 \ \checkmark\)

   Yes, \(-4\) is less than 8.

3. Multiply each number in the original inequality by \(-2\). Is the new inequality a true statement?
   
   \[-2 \cdot (-2) \overset{?}{=} -2 \cdot 4\]
   
   \[4 < -8 \ \times\]

   No, 4 is not less than \(-8\).

4. Divide each number in the original inequality by 2. Is the new inequality a true statement?
   
   \[\frac{-2}{2} \overset{?}{=} \frac{4}{2}\]
   
   \[-1 < 2 \ \checkmark\]

   Yes, \(-1\) is less than 2.

5. Divide each number in the original inequality by \(-2\). Is the new inequality a true statement?
   
   \[\frac{-2}{-2} \overset{?}{=} \frac{4}{-2}\]
   
   \[1 < -2 \ \times\]

   No, 1 is not less than \(-2\).

---

#### Draw Conclusions

1. **Critical Thinking** Repeat the steps above with a new pair of integers. In Steps 3 and 5, what could you do to the inequality symbols to make the statements true?

2. **Given that** \(a > b\), copy and complete using < or > to make a true statement.
   
   2. \(\frac{a}{2} \overset{?}{=} \frac{b}{2}\)
   
   3. \(\frac{a}{-2} \overset{?}{=} \frac{b}{-2}\)
   
   4. \(-a \overset{?}{<} -b\)
   
   5. \(3a \overset{?}{<} 3b\)
Solving Inequalities Using Multiplication or Division

*Geese Migration* Some flocks of Canada geese can fly nonstop for up to 16 hours. In this time, a flock can migrate as far as 848 miles. At what average speeds can such a flock fly during migration? In Example 3, you will see how to answer this question by solving an inequality.

As shown below, when each side of the inequality $2 < 8$ is multiplied by a positive number, the inequality remains true. When each side is multiplied by a negative number, the inequality sign must be reversed.

\[
\begin{align*}
2 &< 8 \\
4 \cdot 2 &< 4 \cdot 8 \\
8 &< 32 \\
-4 \cdot 2 &< -4 \cdot 8 \\
-8 &> -32
\end{align*}
\]

Reverse inequality sign.

These examples suggest the following rules for solving inequalities.

**Multiplication Property of Inequality**

*Words* Multiplying each side of an inequality by a *positive* number produces an equivalent inequality.

Multiplying each side of an inequality by a *negative* number and *reversing the direction of the inequality symbol* produces an equivalent inequality.

*Algebra* If $a < b$ and $c > 0$, then $ac < bc$.

If $a < b$ and $c < 0$, then $ac > bc$.

**Example 1** Solving an Inequality Using Multiplication

\[
\begin{align*}
\frac{m}{-3} &> 3 \\
-3 \cdot \frac{m}{-3} &< -3 \cdot 3 \\
m &< -9
\end{align*}
\]
**Division** The rules for solving an inequality using division are like the rules for solving an inequality using multiplication.

**Division Property of Inequality**

**Words** Dividing each side of an inequality by a *positive* number produces an equivalent inequality.

Dividing each side of an inequality by a *negative* number and *reversing the direction of the inequality symbol* produces an equivalent inequality.

**Algebra** If \( a < b \) and \( c > 0 \), then \( \frac{a}{c} < \frac{b}{c} \).

If \( a < b \) and \( c < 0 \), then \( \frac{a}{c} > \frac{b}{c} \).

---

**Example 2**  
**Solving an Inequality Using Division**

\[
-10r \geq 34  \quad \text{Original inequality}
\]

\[
-10r \leq 34  \quad \text{Divide each side by } -10.
\]

\[
-10 \cdot \frac{r}{10} \leq 34 \quad \text{Reverse Inequality symbol.}
\]

\[
-10 \cdot \frac{r}{10} = -3.4  \quad \text{Simplify.}
\]

---

**Checkpoint**

Solve the inequality. Graph your solution.

1. \( \frac{n}{6} > 7 \)
2. \( \frac{t}{4} \leq 8 \)
3. \( 2x > -8 \)
4. \( -7s \leq 14 \)

---

**Example 3**  
**Writing and Solving an Inequality**

Find the average speeds at which the flock of Canada geese described on page 144 can fly during migration.

**Solution**

Let \( s \) represent the average flight speeds. Write a verbal model.

<table>
<thead>
<tr>
<th>Flight time</th>
<th>Average speeds</th>
<th>Maximum flight distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>( s )</td>
<td>( 848 )</td>
</tr>
</tbody>
</table>

Substitute.

\[
16s \leq 848  \quad \text{Divide each side by } 16.\]

\[
\frac{16s}{16} \leq \frac{848}{16}  \quad \text{Simplify.}
\]

\[
s \leq 53  \quad \text{Answer}
\]

The flock of Canada geese can fly at average speeds of 53 miles per hour or less during migration.
Guided Practice

**Vocabulary Check**
1. Which property would you use to solve the inequality \(-7y \leq 49\)?
2. Explain how solving \(2x > -14\) is different from solving \(-2x > 14\).

**Skill Check** Solve the inequality. Graph and check your solution.

3. \(\frac{v}{2} < -8\)
4. \(8b > 32\)
5. \(\frac{u}{6} \geq 3\)
6. \(-6s \leq 54\)
7. \(5a < -35\)
8. \(\frac{p}{7} > 6\)
9. \(3r \geq 21\)
10. \(\frac{t}{4} \leq -9\)

**Guided Problem Solving**

11. **Training** While training for a marathon, you try to consume at least 2400 Calories each day. For one meal, you like to eat at least 500 Calories. You choose to eat pasta that has 200 Calories per cup. How many cups of pasta should you eat?

   1. Let \(c\) represent the number of cups of pasta. Write an inequality based on the verbal model given below.

<table>
<thead>
<tr>
<th>Calories per cup</th>
<th>Number of cups</th>
<th>Total calories for meal</th>
</tr>
</thead>
</table>

   2. Solve the inequality.

   3. Explain what the solution tells you about the situation.

Practice and Problem Solving

Solve the inequality. Graph your solution.

12. \(\frac{a}{2} < -9\)
13. \(\frac{b}{7} > 7\)
14. \(\frac{c}{8} \geq 3\)
15. \(-16y > 48\)
16. \(5z < 65\)
17. \(\frac{d}{-11} \leq 6\)
18. \(12x \geq -60\)
19. \(4w \leq 68\)
20. \(\frac{t}{9} < -12\)
21. \(\frac{h}{6} \leq 13\)
22. \(-16k \geq 96\)
23. \(6q > -84\)
24. \(-7s \geq -84\)
25. \(4m < -60\)
26. \(\frac{v}{5} > -2\)
27. \(\frac{n}{3} \geq -5\)
28. **Error Analysis** Describe and correct the error in solving the inequality \(9x > -45\).
29. **In-Line Skates** You want to use in-line skates. You can either rent in-line skates for $12 per day or purchase them for $60. How many times will you have to use the in-line skates in order for the cost of purchasing them to be less than the total cost of renting them?

**Write the verbal sentence as an inequality. Then solve the inequality.**

30. Five times a number is at least 45.
31. A number divided by 4 is at most 8.
32. A number divided by $-3$ is less than 6.
33. Seven times a number is greater than $-35$.
34. A number divided by 2 is no more than 5.
35. Three times a number is more than $-18$.

36. **Extended Problem Solving** The weight limit for freight loaded onto a freight elevator is 7500 pounds. The elevator is being used to move 50 heavy crates. Each crate weighs 375 pounds.
   
   a. **Interpret** Write and solve an inequality to determine how many crates you can move in one trip on the elevator. Assume that weight is the only factor affecting how many crates you can move at one time.
   
   b. **Apply** How many times will you need to load the elevator to move all of the crates? Explain.

37. **Reading** You need to read at least 105 pages of a book for your English class in the next 7 days. How many pages should you read each day?

38. **Biking** You want to bike at least 45 miles as part of a training program. If you bike for 5 hours, what average speeds will allow you to meet your goal?

**Use a calculator to solve the inequality. Graph your solution.**

39. $-8.9b \geq 40.94$
40. $\frac{x}{2.4} \geq 8.5$
41. $\frac{z}{7.2} < -3.4$

42. $6.3a > 10.71$
43. $-3.9c \leq 43.68$
44. $\frac{y}{-9.1} \leq 6.5$

45. **Water** Filling the bathtub uses 60 gallons of water. Taking a shower uses 2 gallons per minute. How many minutes can you be in the shower and still use less water than you would by filling the bathtub?

46. **Caribou** A herd of caribou can migrate as far as 36 miles in 24 hours.
   
   a. Write and solve an inequality to find the average speeds (in miles per hour) at which caribou can migrate.
   
   b. A caribou herd has been moving for three days. On a number line, graph the distances (in miles) the herd could have traveled.

47. **Carpeting** Your parents have decided to install new carpeting in your room, which is rectangular and measures 10 feet by 12 feet. They want to spend at most $200 on the carpeting. At the flooring store, carpeting is sold by the square foot. How much money will your parents spend per square foot for carpeting?

**In the Real World**

**Caribou** The caribou in North America are native to Europe, where they are called reindeer. Caribou calves weigh about 13 pounds at birth and may weigh as much as twice that after 10 days. Write an inequality that represents the average weight of a caribou after 10 days.
48. **Critical Thinking** The inequalities $2x < 3$ and $4x < 6$ are equivalent inequalities. Write a third inequality equivalent to $2x < 3$ and $4x < 6$.

49. **Challenge** An underwater camera can withstand pressures up to 1500 pounds per square inch. The formula $P = 14.7 + 0.45d$ can be used to find the water pressure $P$ (in pounds per square inch) at depth $d$ (in feet) underwater. Find the depths at which the camera can be used.

---

**Mixed Review**

**Algebra Basics** Solve the equation. Check your solution. *(Lesson 2.7)*

50. $x + 3.5 = 9.2$  51. $x - 6.7 = 5.8$  52. $44.72 = 5.2x$  53. $\frac{x}{7.6} = 9.5$

54. Find the perimeter of the square. *(Lesson 3.3)*

\[
3x - 5
\]

**Solve the inequality. Graph your solution.** *(Lesson 3.4)*

55. $x + 12 > 96$  56. $x + 17 \geq 44$  57. $x - 26 \leq 33$  58. $x - 14 < 29$

---

**Standardized Test Practice**

59. **Multiple Choice** Which number is not a solution of $\frac{t}{9} \geq 3$?

A. $-35$  B. $-30$  C. $-27$  D. $-25$

60. **Multiple Choice** Which number is a solution of $\frac{x}{-7} < 6$?

F. $-100$  G. $-56$  H. $-42$  I. $-14$

61. **Multiple Choice** Which inequality is equivalent to $-18 \leq 3p$?

A. $p \geq -54$  B. $-54 \geq p$  C. $p \geq -6$  D. $-6 \geq p$

---

**Brain Game**

Use the given information to list the six cousins in order from least to greatest age and give their ages. No two cousins are the same age.

- Erika is 4 years old.
- Charlie’s age is greater than 4 times Dawn’s age.
- All the girls’ ages are greater than Anthony’s age.
- All the cousins’ ages are less than or equal to 13.
- Matthew is older than exactly three cousins.
- Stephanie is 6 years older than Erika.
- All the cousins’ ages are greater than or equal to 2.
- Erika is 1 year older than Dawn.
- One boy is 6 years old.

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148  Chapter 3  Multi-Step Equations and Inequalities
Solving Multi-Step Inequalities

**Soccer** Your school’s soccer team is trying to break the school record for goals scored in one season. Your team has already scored 88 goals this season. The record is 138 goals. With 10 games remaining on the schedule, how many goals, on average, does your team need to score per game to break the record?

To solve a multi-step inequality like $2x + 1 > 5$, you should use the properties of inequality from Lessons 3.4 and 3.5 to get the variable terms on one side of the inequality and the constant terms on the other side.

**Example 1**  
**Writing and Solving a Multi-Step Inequality**

Find the average number of goals your team needs to score per game to break the school record, as described above.

**Solution**

Let $g$ represent the average number of goals scored per game. Write a verbal model.

\[
\begin{align*}
\text{Goals scored this season} & + \text{Number of games left} \times \text{Goals scored per game} > \text{School record} \\
88 + 10g & > 138 \\
88 + 10g - 88 & > 138 - 88 \quad \Rightarrow \quad 10g > 50 \\
\frac{10g}{10} & > \frac{50}{10} \\
g & > 5 \\
\end{align*}
\]

**Answer** Your team must score, on average, more than 5 goals per game.

**Checkpoint**

1. Look back at Example 1. Suppose the season goal record is 124 goals and your team has already scored 52 goals. With 12 games remaining on the schedule, how many goals, on average, does your team need to score per game to break the record?
Example 2  **Solving a Multi-Step Inequality**

\[
\frac{x}{-4} - 6 \geq -5 \\
\frac{x}{-4} - 6 + 6 \geq -5 + 6 \\
\frac{x}{-4} \geq 1 \\
-4 \cdot \frac{x}{-4} \leq -4 \cdot 1 \\
x \leq -4
\]

**Original Inequality**

**Add 6 to each side.**

**Simplify.**

**Multiply each side by -4.**

**Reverse inequality symbol.**

**Simplify.**

---

Example 3  **Combining like Terms in a Multi-Step Inequality**

**Ice Skating** You plan to go ice skating often this winter. The skating rink charges $4 for admission. You can either rent ice skates at the skating rink for $5 per day or buy your own pair for $45. How many times do you have to use the ice skates in order for the cost of buying them to be less than the total cost of renting them?

**Solution**

You have two options: buying skates or renting skates. Let \( v \) represent the number of visits to the skating rink. Write a variable expression for the cost of each option.

**Option 1: Buying Skates**

\[
\text{Cost of skates} + \text{Admission fee} \times \text{Number of visits} \quad \Rightarrow \quad 45 + 4v
\]

**Option 2: Renting Skates**

\[
\text{Skate rental fee} \times \text{Number of visits} + \text{Admission fee} \times \text{Number of visits} \quad \Rightarrow \quad 5v + 4v, \text{ or } 9v
\]

To find the values of \( v \) for which the cost of option 1 is less than the cost of option 2, write and solve an inequality.

\[
\text{Cost of option 1} < \text{Cost of option 2}
\]

\[
45 + 4v < 9v \\
45 + 4v - 4v < 9v - 4v \\
45 < 5v \\
\frac{45}{5} < \frac{5v}{5} \\
9 < v
\]

**Substitute.**

**Subtract 4v from each side.**

**Simplify.**

**Divide each side by 5.**

**Simplify.**

**Answer** If you buy skates, the cost will be less after more than 9 visits.
Guided Practice

**Vocabulary Check**
1. Write and solve an inequality for the following verbal sentence: Five plus 2 times a number is less than 20.
2. List the steps you would take to solve the inequality $-5x + 12 < -8$.

**Skill Check**

**Solve the inequality. Graph and check your solution.**

3. $4x + 1 > 1$
4. $7 \geq 5x - 3$
5. $\frac{x}{-2} + 6 < -14$
6. $10 > 6 + \frac{y}{5}$
7. $5y + 2 \leq y + 34$
8. $6 + y \geq 2y - 3$

**Guided Problem Solving**

9. **Amusement Parks** You are trying to decide whether to pay $120 for a season pass to an amusement park. If you buy the pass, you get an unlimited number of visits to the park and reduced parking for $8. If you do not buy the pass, you pay $23 admission and $10 for parking each time you visit the park. After how many visits to the park will the cost of the season pass be less than the cost of visiting without the season pass?

1. Write a variable expression for the cost of making $v$ visits to the park if you don’t buy a season pass.
2. Write an inequality in terms of $v$ showing that the cost of visiting the park with a season pass is less than the cost of visiting the park without a season pass.
3. Solve the inequality.

Practice and Problem Solving

**Tell whether the given number is a solution of $5x - 10 > 2x + 4$.**

10. $8$
11. $5$
12. $4$
13. $-2$

**Solve the inequality. Graph your solution.**

14. $2y + 7 > 11$
15. $6n - 3 \leq -9$
16. $11 - 4z < -1$
17. $3m - 8 > -30 + 5m$
18. $19 \geq \frac{x}{90} - 25$
19. $3 + \frac{b}{3} < 7$
20. $14p - 5 \geq -3p + 114$
21. $-3x - 3 < 2x - 83$

22. **Movie Rental** At a video store, you have two options for renting movies. You can pay $4 per movie, or you can pay a one-time membership fee of $10 and then pay only $1.50 per movie. After how many movie rentals will the cost of the membership be less than the cost of renting movies without the membership?
23. **Advertising** A small company has an advertising budget of $15,000. The company plans to produce and air a television commercial. It will cost $500 to produce the commercial and an additional $50 each time the commercial is aired. How many days can the company afford to run the commercial if it is aired once a day?

24. **Error Analysis** Describe and correct the error in solving the inequality $4x > 6x + 3$.

\[
\begin{align*}
4x & > 6x + 3 \\
4x - 6x & > 6x + 3 - 6x \\
-2x & > 3 \\
\frac{-2x}{-2} & > \frac{3}{-2} \\
x & > -\frac{3}{2}
\end{align*}
\]

**Solve the inequality. Graph your solution.**

25. \(4(5 - 3b) > 4b + 4\)  
26. \(\frac{x - 2}{3} > 4\)  
27. \(3y - 5 < 2(17 - 5y)\)

28. \(\frac{x + 5}{3} \leq 2\)  
29. \(\frac{-5x - 8}{4} \geq -22\)  
30. \(-3 \leq \frac{2x + 4}{4}\)

31. **Fundraising** You are designing greeting cards on your computer to raise money for a charity. You buy card stock at a cost of $.50 per card and rent a table at the fundraiser for $20. You will sell the cards in sets of 12 for $10.20. How many sets of cards do you have to sell in order to make more than what you spend?

32. **Long-Distance Calls** The table gives information about three long-distance telephone companies. For each company, the table gives the monthly fee and the charge per minute for making long-distance calls.

<table>
<thead>
<tr>
<th>Long-Distance Rates by Company</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
</tbody>
</table>

**Long-Distance Rates by Company**

a. After how many minutes of long-distance calls is the cost of using company A for one month less than the cost of using company B for one month?

b. After how many minutes of long-distance calls is the cost of using company C for one month less than the cost of using company B for one month?

c. **Interpret and Apply** If you spend 150 minutes each month making long-distance calls, which company should you use? Explain why.

33. **Challenge** Find all the values of \(x\) that make both of the following inequalities true: \(2x + 4 < 10\) and \(5 - 3x \leq 17\). Show how you found your answer.
34. **Extended Problem Solving** You and a friend join different health clubs. You pay a one-time membership fee of $150 and a monthly fee of $35. Your friend pays a one-time membership fee of $100 and a monthly fee of $40.

   a. **Analyze** Let \( m \) be the number of months that you and your friend have been health club members. Make a table with a column for the number \( m \) of months, a column for the amount you have paid after \( m \) months, and a column for the amount your friend has paid after \( m \) months. Complete the table for whole-number values of \( m \) from 1 to 12 to represent one year of membership at each health club.

   b. Make a scatter plot of the data from part (a). Show months on the \( x \)-axis and the amount paid on the \( y \)-axis. Plot points representing the amount you have paid in blue and the amount your friend has paid in red.

   c. **Writing** Using the scatter plot, determine the number of months you and your friend need to be members of your health clubs before you have paid less than your friend. Explain your reasoning.

   d. Check your answer to part (c) by writing and solving an inequality.

---

**Mixed Review**

Give the coordinates of the point. *(Lesson 1.8)*

35. \( A \) 36. \( B \)
37. \( C \) 38. \( D \)
39. \( E \) 40. \( F \)

Simplify the expression. *(Lesson 2.3)*

41. \( 13(2a + 1) \) 42. \( 12 + c + 8 \) 43. \( 5a + a \)

Algebra Basics Solve the equation. Check your solution. *(Lesson 3.2)*

44. \( 3(x + 4) = 9 \) 45. \( 4(2d + 1) = 28 \) 46. \( -10 = 2(7 - 2x) \)

47. Write and solve an equation for the following verbal sentence: Nine more than 3 times a number is equal to 7 less than twice the number. *(Lesson 3.3)*

---

**Standardized Test Practice**

48. **Multiple Choice** Which graph shows the solution of the inequality \( 7 - 6x \geq 13 \)?

   - A. ![Graph A]
   - B. ![Graph B]
   - C. ![Graph C]
   - D. ![Graph D]

49. **Multiple Choice** Which number is a solution of the inequality \( -7x + 3 < -7.5 \)?

   - F. \(-3\)  
   - G. \(-1\)  
   - H. 1  
   - I. 3
Vocabulary Review

1. Copy and complete: The value of a variable that, when substituted into an inequality, makes a true statement is a(n) ___.

2. Give an example of an inequality.

3. Copy and complete: The inequalities $2x < 2$ and $x < 1$ are ____ inequalities.

4. Are $-2x > 6$ and $x > -3$ equivalent inequalities? Why or why not?

3.1 Solving Two-Step Equations

Example  Solve the following problem.

A one-year membership in a video rental club costs $10. Members pay $1.25 per video rental. You spend $45 in one year. How many videos did you rent?

Solution

Let $v$ represent the number of videos you rented. Write a verbal model.

$$45 = 10 + 1.25v$$

Substitute.

$$45 - 10 = 10 + 1.25v - 10$$

Subtract 10 from each side.

$$35 = 1.25v$$

Simplify.

$$\frac{35}{1.25} = \frac{1.25v}{1.25}$$

Divide each side by $1.25$.

$$28 = v$$

Simplify.

Answer: You rented 28 videos.

5. **Spaghetti** Your friend bought a box of spaghetti for $1.59 and 2 jars of spaghetti sauce. The total cost was $6.49. Find the cost of one jar of sauce.
3.2 Solving Equations Having Like Terms and Parentheses

**Goal**

Solve equations having like terms and parentheses.

**Example** Solve \(2x - x + 1 = 5\) and \(4(3r - 9) = 36\).

**a.** \(2x - x + 1 = 5\)

\[x + 1 = 5\]

\[x + 1 - 1 = 5 - 1\]

\[x = 4\]

Write original equation.

Combine like terms.

Subtract 1 from each side.

Simplify.

**b.** \(4(3r - 9) = 36\)

\[12r - 36 = 36\]

Write original equation.

Distributive property

\[12r - 36 + 36 = 36 + 36\]

Add 36 to each side.

\[12r = 72\]

Simplify.

\[\frac{12r}{12} = \frac{72}{12}\]

Divide each side by 12.

\[r = 6\]

Simplify.

**Solve the equation. Check your solution.**

6. \(17h - 47 + 6h = 160\)

7. \(2(4p + 8) = 128\)

8. \(6(w - 4) + 18 = 30\)

3.3 Solving Equations with Variables on Both Sides

**Goal**

Solve equations with variables on both sides.

**Example** Solve \(13n - 45 = 36 + 4n\).

\[13n - 45 = 36 + 4n\]

Write original equation.

\[13n - 45 - 4n = 36 + 4n - 4n\]

Subtract 4n from each side.

\[9n - 45 = 36\]

Simplify.

\[9n - 45 + 45 = 36 + 45\]

Add 45 to each side.

\[9n = 81\]

Simplify.

\[\frac{9n}{9} = \frac{81}{9}\]

Divide each side by 9.

\[n = 9\]

Simplify.

**Solve the equation. Check your solution.**

9. \(11t + 14 = 95 - 16t\)

10. \(9n + 64 = -144 - 17n\)

11. \(3 + 2x = 2(2 + x)\)

12. \(3(2 + 6b) = 18b\)
3.4 Solving Inequalities Using Addition or Subtraction

**Goal**

Solve inequalities using addition or subtraction.

**Example**

Solve \( x + 13 \leq 20 \). Graph your solution.

\[
x + 13 \leq 20 \quad \text{Write original inequality.}
\]

\[
x + 13 - 13 \leq 20 - 13 \quad \text{Subtract 13 from each side.}
\]

\[
x \leq 7 \quad \text{Simplify.}
\]

✔  **Solve the inequality. Graph your solution.**

13. \( y + 11 < 23 \)  
14. \( 15 \geq z + 9 \)  
15. \( x - 5 \leq 14 \)  
16. \( m - 8 < 26 \)

3.5 Solving Inequalities Using Multiplication or Division

**Goal**

Solve inequalities using multiplication and division.

**Example**

Solve \( 5x > 30 \) and \( \frac{t}{-8} \leq 5 \). Graph your solutions.

a. \( 5x > 30 \)  
   \[
   \frac{5x}{5} > \frac{30}{5} \quad \text{Write original inequality.}
   \]
   \[
   x > 6 \quad \text{Simplify.}
   \]

b. \( \frac{t}{-8} \leq 5 \)  
   \[
   -8 \cdot \frac{t}{-8} \geq -8 \cdot 5 \quad \text{Write original inequality.}
   \]
   \[
   t \geq -40 \quad \text{Multiply each side by \(-8\).}
   \]
   \[
   t \geq -40 \quad \text{Simplify.}
   \]

✔  **Solve the inequality. Graph your solution.**

17. \( 3 > \frac{a}{-9} \)  
18. \( \frac{b}{7} \geq 13 \)  
19. \( 12c \leq 96 \)  
20. \( -68 < -17d \)

21. \( -2 > \frac{r}{-6} \)  
22. \( 196 \leq 14z \)  
23. \( 7h < -56 \)  
24. \( \frac{p}{5} > -6 \)
### 3.6 Solving Multi-Step Inequalities

**Goal** Solve multi-step inequalities.

#### Example

Solve \(-8y + 5 \leq 29\) and \(3x - 5 > 6x + 13\). Graph your solutions.

**a.** \(-8y + 5 \leq 29\)

Write original inequality.

\[-8y + 5 - 5 \leq 29 - 5\]

Subtract 5 from each side.

\[-8y \leq 24\]

Simplify.

\[-\frac{8y}{-8} \geq \frac{24}{-8}\]

Divide both sides by \(-8\). Reverse inequality symbol.

\[y \geq -3\]

Simplify.

**b.** \(3x - 5 > 6x + 13\)

Write original inequality.

\[3x - 5 - 3x > 6x + 13 - 3x\]

Subtract \(3x\) from each side.

\[-5 > 3x + 13\]

Simplify.

\[-5 - 13 > 3x + 13 - 13\]

Subtract 13 from each side.

\[-18 > 3x\]

Simplify.

\[-\frac{18}{3} > \frac{3x}{3}\]

Divide each side by 3.

\[-6 > x\]

Simplify.

#### Solve the inequality. Graph your solution.

25. \(-8m - 6 < 10\)
26. \(8p + 1 \geq 17\)
27. \(24 \geq 5z - 6\)

28. \(8 > 2 + \frac{b}{3}\)
29. \(\frac{p}{28} + 3 \leq 9\)
30. \(\frac{n}{3} + 4 > 5\)

31. \(12 - 4q \geq 6q + 2\)
32. \(6x - 5 > 12x + 1\)
33. \(6(3 - a) \leq 8a - 10\)

34. **Snowboarding** A ski resort charges $45 for an all-day lift pass and $40 per day for renting boots and a snowboard. At a store, you can buy boots and a snowboard for $360. How many times must you go snowboarding at the ski resort for the cost of buying your own boots and snowboard to be less than renting them?
### Solve the equation. Check your solution.

1. \(7f + 5 = 68\)
2. \(14 - 3g = 32\)
3. \(\frac{h}{3} - 14 = -11\)
4. \(\frac{x}{-2} + 5 = 7\)
5. \(12 - 2m + 5 = -1\)
6. \(-6y + 4 + 11y = -16\)
7. \(3(8 - a) = 12\)
8. \(-6(3x + 15) = 18\)
9. \(5t + 5 = 5t - 4\)
10. \(2n - 6 = -8n + 14\)
11. \(8b + 4 = 4(b - 7)\)
12. \(16p + 8 = 2(8p + 4)\)

13. **Movie Tickets** A family of four goes to a movie theater and spends $26.50. They buy 2 tickets for children at $3.50 per ticket, 2 tickets for adults, and 3 boxes of popcorn at $2.50 per box. What is the cost of one adult movie ticket?

14. **Ocean Water** The more salt that ocean water contains, the lower the temperature at which it freezes. Some ocean water freezes at temperatures of \(-1.9\)°C or less. Write and graph an inequality to show the temperatures at which this ocean water freezes.

### Solve the inequality. Graph your solution.

15. \(x + 75 > -125\)
16. \(w - 18 < -10\)
17. \(\frac{f}{12} \geq 3\)
18. \(-3a - 6 \leq -9\)
19. \(4(2 - d) \geq -12\)
20. \(2c - 5 < -21 - 2c\)

21. **School Supplies** You go to the store to buy supplies for class. You want to buy 5 identical folders. The most you can spend is $5.75. What are the individual folder prices that you can afford?

22. Nine is greater than or equal to 15 minus a number.

23. Eight times the sum of 5 and a number is less than 56.

24. Fifteen is greater than 3 times the difference of a number and 4.

25. Seven times a number minus 5 is less than or equal to 16.

26. **Making Bread** A bread-making machine costs $99. The ingredients to make a one pound loaf of bread cost $0.45. At a store, you pay $2.19 for the same size loaf of bread. How many whole loaves of bread will you have to make in order for the cost of the machine and ingredients to be less than the cost of buying an equivalent amount of bread at the store?
1. What is the solution of the equation \( \frac{t}{5} - 12 = 10? \)
   - A. -14  
   - B. 10  
   - C. 38  
   - D. 110

2. What is the solution of the equation \(-4(n + 5) = -32?\)
   - F. -13  
   - G. -12  
   - H. 3  
   - I. 13

3. The perimeter of the triangle shown is 15 units. What is the value of \( x? \)
   - A. 1  
   - B. 2  
   - C. 3  
   - D. 4

4. Which equation has no solution?
   - F. \( 4t - 8 = 4(t - 2) \)
   - G. \( 3(r - 1) = -2(2 + r) \)
   - H. \( 6p + 2 = 9p - 4 \)
   - I. \( 7(s + 1) = -3 + 7s \)

5. Giants Stadium in New Jersey can seat up to 80,242 people. Which inequality represents the number \( n \) of people that the stadium can seat?
   - A. \( n < 80,242 \)  
   - B. \( n > 80,242 \)  
   - C. \( n \leq 80,242 \)  
   - D. \( n \geq 80,242 \)

6. What is the solution of the inequality \( \frac{z}{-4} + 3 < 15? \)
   - F. \( z < -48 \)  
   - G. \( z > -48 \)
   - H. \( z < -3 \)  
   - I. \( z > -3 \)

7. What is the solution of the inequality \(-12 > y + 6? \)
   - A. \( y < -18 \)  
   - B. \( y < -6 \)  
   - C. \( y > -18 \)  
   - D. \( y > -6 \)

8. Which value is not a solution of the inequality \(-5y - 2 \geq 30.5? \)
   - F. \(-162.5 \)  
   - G. \(-10 \)  
   - H. \(-6.5 \)  
   - I. \(-3 \)

9. **Short Response** Two of your friends go bowling. One friend rents a pair of bowling shoes for $3 and bowls 3 games. The other friend brings his own bowling shoes, bowls 4 games, and buys a soda for $0.50. Both friends spend the same amount of money. Show how you can write and solve an equation to find the cost of one game.

10. **Extended Response** The table below shows the cost of renting a moving van for 1 day from two companies. The daily charge and the charge per mile are given.

<table>
<thead>
<tr>
<th>Company</th>
<th>Daily charge</th>
<th>Charge per mile</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$80</td>
<td>$.35</td>
</tr>
<tr>
<td>B</td>
<td>$75</td>
<td>$.39</td>
</tr>
</tbody>
</table>

a. How many miles \( m \) will you have to drive before the cost of renting a van for one day from company A is less than the cost of renting a van for one day from company B? Express your answer as an inequality.

b. Graph your inequality from part (a).

c. Which company is less expensive if you drive 100 miles in one day? Explain.
Strategies for Answering Multiple Choice Questions

You can use the problem solving plan on page 14 to solve a problem. If you have difficulty solving a problem in multiple choice format, you may be able to use one of the strategies below to choose the correct answer. You may also be able to use these strategies and others to check whether your answer to a multiple choice question is reasonable.

**Strategy: Use Estimation**

**Problem 1** You and a friend share the cost of a pizza and a salad. The pizza costs $8.98. The salad costs $3.98. How much do each of you pay?

- **A. $6.48**
- **B. $8.47**
- **C. $10.97**
- **D. $12.96**

To solve this problem, you need to add the cost of the two items and divide by 2.

Estimate: $\frac{9 + 4}{2} = \frac{13}{2} = 6.5$, so the correct answer is **A**.

**Strategy: Use Visual Clues**

**Problem 2** The line graph shows the number of people (in millions) who participated in snowboarding in the United States from 1991 to 2001. What was the greatest increase in participation in any two-year period?

- **F. About 0.5 million**
- **G. About 1 million**
- **H. About 1.5 million**
- **I. About 2 million**

To solve this problem, you need to identify the period with the greatest increase, then find the amount of increase for that period.

The greatest change occurred between 1999 and 2001. The vertical distance between the points for 1999 and 2001 is about 2 units. Each unit represents 1 million people. So, the correct answer is **I**.
Strategy: Use Number Sense

Problem 3  The temperature in a freezer is \(-12^\circ\text{C}\). During one 8 minute period, the temperature drops about 0.5°C each minute. What is the temperature after those 8 minutes?

A. \(-16^\circ\text{C}\)  
B. \(-12.5^\circ\text{C}\)  
C. \(-11.5^\circ\text{C}\)  
D. \(-8^\circ\text{C}\)

To solve this problem, you need to find the total change in temperature and add it to the original temperature.

Eliminating Unreasonable Choices  The strategies used to find the correct answers for Problems 1–3 can also be used to eliminate answer choices that are unreasonable or obviously incorrect.

Problem 4  The length of a rectangle is 5 meters less than twice the width. The perimeter is 26 meters. What is the width of the rectangle?

A. 2 m  
B. 6 m  
C. 10.5 m  
D. 42 m²

To solve this problem, you need to use the perimeter formula \(P = 2l + 2w\) and the given information to write and solve an equation.

Watch Out  Some answers may appear correct at first glance, but they may be incorrect answers you would get by making common errors.

Checkpoint

1. You and 4 friends share the $29.95 cost of a lunch. How much does each person pay?
   A. $5.99  
   B. $7.49  
   C. $24.95  
   D. $34.95

2. What is the solution of the equation \(5x + 3 = -15\)?
   F. \(-3.6\)  
   G. \(-2.4\)  
   H. \(2.4\)  
   I. \(3.6\)

3. At 9 P.M., the temperature is 0°F. It drops 3°F each hour for 4 hours. What is the temperature at 1 A.M.?
   A. \(-12^\circ\text{F}\)  
   B. \(-7^\circ\text{F}\)  
   C. \(7^\circ\text{F}\)  
   D. \(12^\circ\text{F}\)
Multiple Choice

1. Which variable expression represents the number of inches in $y$ yards?
   A. $36y$  B. $\frac{y}{36}$  C. $\frac{36}{y}$  D. $36 - y$

2. What is the value of the expression $15 - 14 + 2 + 5$?
   F. $\frac{1}{7}$  G. $\frac{51}{2}$  H. 8  I. 13

3. During the month of January, the average high temperature in Montreal, Canada, is $-6^\circ C$, and the average low temperature is $-15^\circ C$. How much greater is the average high temperature than the average low temperature?
   A. $-21^\circ C$  B. $-9^\circ C$  C. $9^\circ C$  D. $21^\circ C$

4. Which point has coordinates $(1, -2)$?
   F. A  G. B  H. C  I. D

5. Which expression is equivalent to $3(0.9 + 7)$?
   A. $3(0.9) + 7$  B. $0.27 + 21$  C. $3(0.9) + 3(7)$  D. $2.7 + 7$

6. Which terms of the expression $4x + 9 + 9x - 4$ are like terms?
   F. $4x$ and $-4$  G. $4x$ and 4  H. $4x$ and $9x$  I. $9x$ and 9

7. What is the solution of $10 + x = -19$?
   A. $-29$  B. $-9$  C. $9$  D. $29$

8. Which expression is equivalent to $8(x + 2) - 5(x - 3)$?
   F. $3x + 31$  G. $3x + 1$  H. $3x - 1$  I. $13x + 31$

9. What is the solution of $3 - 8x = -141$?
   A. $-18$  B. $-12$  C. $\frac{1}{18}$  D. $18$

10. What is the solution of $7(x + 5) - 10 = 2x$?
    F. $-5$  G. $-1$  H. $5$  I. $7$

11. Which of the inequalities has the solution whose graph is shown?
    A. $2x + 10 < 16$  B. $-4x - 5 < 7$
    C. $-3x + 8 < -1$  D. $4 - 2x > 10$

12. A soccer goalie has made 175 saves so far this season. The school record is 236 saves in a season. There are 6 games left to play. Which inequality could you solve to find the average number of saves the goalie must make in each of the remaining games to break the school record?
    F. $175 + 6x < 236$  G. $175 + 6x > 236$
    H. $236 + 6x > 175$  I. $236 + 6x < 175$

13. What is the solution of the inequality $-3x + 14 > 2x - 11$?
    A. $x > -5$  B. $x > 5$
    C. $x < -5$  D. $x < 5$
**Short Response**

14. Tell whether the statement \(|a - b| = |a| - |b|\) is always, sometimes, or never true for integers \(a\) and \(b\). Give examples to explain your reasoning.

15. While waiting in the checkout line at the grocery store, you add the prices of the 3 items you are buying to make sure you have enough money. Explain how to use mental math and the properties of addition to find the total cost if the prices of the 3 items are $1.85, $2.74, and $4.15. Find the total cost.

16. Profit is the difference of income and expenses. The table shows one store's profit for each of its first 4 months. Find the mean profit. Explain your method.

<table>
<thead>
<tr>
<th>Month</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>March</td>
<td>$-670</td>
</tr>
<tr>
<td>April</td>
<td>$340</td>
</tr>
<tr>
<td>May</td>
<td>$320</td>
</tr>
<tr>
<td>June</td>
<td>$400</td>
</tr>
</tbody>
</table>

17. You have at most 3 hours to do homework. You spend 45 minutes on math. You want to divide the time remaining equally among 4 other subjects. Write and solve an inequality to find the number of minutes you can spend on each one. Explain your method.

**Extended Response**

18. You are painting a room that is 16 feet long, 14 feet wide, and 8 feet high. The room has two identical windows and two identical doors. One door leads to a closet 4 feet long, 4 feet wide, and 8 feet high. You do not plan to paint the closet.

a. The paint you choose is available in both 1 gallon cans that cover about 400 square feet each and 1 quart cans that cover about 100 square feet each. You plan to put 2 coats of paint on each wall, not including the doors or windows. How much paint should you buy?

b. A 1 gallon can of paint costs $13.90, and a 1 quart can of paint costs $8.90. How much will it cost to put 2 coats of paint on each wall?

c. You've budgeted $40 for paint. Can you afford to paint the inside walls of the closet, excluding the door, with 1 coat of paint?

19. The table shows the annual fee at two gyms and the fee each time members take an aerobics class.

<table>
<thead>
<tr>
<th>Gym</th>
<th>Annual fee</th>
<th>Class fee</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$540</td>
<td>$3</td>
</tr>
<tr>
<td>B</td>
<td>$360</td>
<td>$5</td>
</tr>
</tbody>
</table>

a. In a year's time, which gym is less expensive if you plan to take 4 aerobics classes each month?

b. Write and solve an inequality to determine the number of aerobics classes for which the total cost for 1 year at gym A is less than that at gym B.

c. How many aerobics classes should you average each month so that the total cost for 1 year at gym B is less than that at gym A?
Chapter 1

Multiple Choice In Exercises 1–11, choose the letter of the correct answer.

1. What is the value of 9 − x when x = 5? (Lesson 1.1)
   A. 1  B. 4  C. 14  D. 45

2. How can you write 7 × 7 × 7 as a power? (Lesson 1.2)
   F. 7 × 3  G. 3 × 7  H. 7³  I. 3³

3. What is the value of $6^2$? (Lesson 1.2)
   A. 24  B. 216  C. 1296  D. 4096

4. What is the value of $28 ÷ 7 + 16$? (Lesson 1.3)
   F. 4  G. 20  H. 28  I. 212

5. What is the value of $2(x + y)^2$ when $x = 3$ and $y = 4$? (Lesson 1.3)
   A. 28  B. 50  C. 98  D. 2401

6. Which list of integers is in order from least to greatest? (Lesson 1.4)
   F. −3, −5, −7, −9  G. −5, −3, 0, 4
   H. −2, 4, −5, 9  I. −8, 7, −6, 3

7. What is the value of $−15 + 9$? (Lesson 1.5)
   A. −24  B. −6  C. 6  D. 24

8. What is the value of $−27 − x$ when $x = −8$? (Lesson 1.6)
   F. −35  G. −19  H. 19  I. 35

9. In 24 hours, the temperature went from −8°C to 12°C. What was the change in temperature? (Lesson 1.6)
   A. −20°C  B. −4°C  C. 4°C  D. 20°C

10. What is the value of $\frac{x^2}{y}$ when $x = −4$ and $y = −2$? (Lesson 1.7)
    F. −8  G. −$\frac{1}{8}$  H. $\frac{1}{8}$  I. 8

11. In which quadrant is the point (−2, −3) located? (Lesson 1.8)
    A. Quadrant I  B. Quadrant II
    C. Quadrant III  D. Quadrant IV

12. Short Response You have $500 in a savings account. You make deposits of $30, $125, $10, $20, and $65, and you make withdrawals of $75, $89, $143, $15, and $20. Write a positive integer to represent each deposit and a negative integer to represent each withdrawal. Find the final balance in your savings account. (Lessons 1.5, 1.6)

13. Extended Response The table shows the number of cell phone subscribers (in millions) in the United States from 1996 to 2001. (Lesson 1.8)

<table>
<thead>
<tr>
<th>Years since 1996</th>
<th>Subscribers (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>44</td>
</tr>
<tr>
<td>1</td>
<td>55</td>
</tr>
<tr>
<td>2</td>
<td>69</td>
</tr>
<tr>
<td>3</td>
<td>86</td>
</tr>
<tr>
<td>4</td>
<td>109</td>
</tr>
<tr>
<td>5</td>
<td>128</td>
</tr>
</tbody>
</table>

a. Explain how to make a scatter plot of the data.
b. Make a scatter plot.
c. Does the scatter plot suggest any relationship between the number of years since 1996 and the number of cell phone subscribers? Explain.
Multiple Choice in Exercises 14–24, choose the letter of the correct answer.

14. Which property is illustrated by the statement \(2x + (y + 1) = (2x + y) + 1\)? (Lesson 2.1)
   A. Identity property of addition
   B. Commutative property of addition
   C. Associative property of addition
   D. Associative property of multiplication

15. Use a conversion factor to convert 1.5 miles to feet. (Lesson 2.1)
   F. \(\frac{1}{7290}\) foot  G. \(\frac{1}{7290}\) mile
   H. 7920 feet  I. 7920 miles

16. Which variable expression is equivalent to \(4x - 6\)? (Lesson 2.2)
   A. \(2(2x - 6)\)  B. \(4(x - 6)\)
   C. \(2(2x + 3)\)  D. \(2(2x - 3)\)

17. What is the value of \(x(y - z)\) when \(x = -2.5\), \(y = 4\), and \(z = 0.1\)? (Lesson 2.2)
   F. \(-10.25\)  G. \(-9.75\)
   H. 9.75  I. 10.25

18. Which terms of the expression \(-6k - 6 + 4 + 4k\) are like terms? (Lesson 2.3)
   A. \(-6k\) and \(-6\)  B. \(-6k\) and \(4k\)
   C. \(6k\) and \(4k\)  D. 4 and \(4k\)

19. Which expression is equivalent to \(15y - 2(y + 3)\)? (Lesson 2.3)
   F. \(13y - 6\)  G. \(13y + 3\)
   H. \(13y + 6\)  I. \(16y + 1\)

20. Whippets are among the fastest running dogs. Suppose a whippet can run at a rate of 52 feet per second for a short period of time. How long would it take the whippet to run 156 feet? (Lesson 2.4)
   A. 1 second  B. 2 seconds
   C. 3 seconds  D. 4 seconds

21. What is the solution of \(x + 11 = 20 - 7\)? (Lesson 2.5)
   F. 2  G. 13  H. 16  I. 24

22. What is the solution of \(\frac{x}{-12} = 24\)? (Lesson 2.6)
   A. \(-288\)  B. \(-2\)  C. 2  D. 288

23. You went shopping with \$42.60 and came home with \$3.33. How much money did you spend? (Lesson 2.7)
   F. \$3.33  G. \$38.70  H. \$39.27  I. \$45.93

24. What is the solution of \(-3y = 14.7\)? (Lesson 2.7)
   A. \(-44.1\)  B. \(-4.9\)  C. 4.9  D. 17.7

25. Short Response A square has a perimeter of 84 meters. Explain how to write an equation to find the side length of the square. Then find the side length. (Lesson 2.6)

26. Extended Response At store A, a wide-screen TV sells for \$1500 after a \$250 mail-in rebate. Store A charges \$50 for delivery and setup. Store B promises to sell any TV for \$75 less than any competitor’s original price and to include free delivery and setup. (Lesson 2.5)
   a. What is the original price of the TV at store A?
   b. Which store offers a better deal? Explain your reasoning.
Chapter 3

Multiple Choice In Exercises 27–36, choose the letter of the correct answer.

27. What is the solution of $-2x + 7 = 25$? (Lesson 3.1)
   A. $-16$   B. $-9$   C. $9$   D. $16$

   F. $1.99$   G. $2.50$   H. $3.00$   I. $5.00$

29. What is the solution of the equation $15 - 2(w + 5) = 11$? (Lesson 3.2)
   A. $-7$   B. $-3$   C. $8$   D. $12$

30. The area of the rectangle is 28 square units. What is the value of $x$? (Lesson 3.2)
    \[
    \begin{array}{c}
    \text{3x + 4} \\
    \text{4}
    \end{array}
    \]
   F. 1   G. 2
   H. 7   I. 8

31. Which statement about the equation $2(x - 1) = 3x - (x + 2)$ is true? (Lesson 3.3)
   A. The equation has no solution.
   B. The solution is $-0.5$.
   C. The solution is $-1$.
   D. The equation has every number as a solution.

32. At temperatures less than $-458^\circ F$, helium is a solid. Which inequality describes the temperatures $t$ (in degrees Fahrenheit) at which helium is a solid? (Lesson 3.4)
   F. $t > -458$   G. $t < -458$
   H. $t \geq -458$   I. $t \leq -458$

33. You and a friend have $25 to pay for your dinners at a restaurant. Your friend’s dinner costs $13.35. How much can you spend on your dinner? (Lesson 3.4)
   A. Less than $11.65$   B. More than $11.65$
   C. At most $11.65$   D. At least $11.65$

34. What is the solution of $\frac{h}{7} \geq 14$? (Lesson 3.5)
   F. $h \leq -2$   G. $h \geq -2$
   H. $h \leq -98$   I. $h \geq -98$

35. Which number is not a solution of $-4s < 42$? (Lesson 3.5)
   A. $-11$   B. $-10$   C. $10$   D. $11$

36. What is the solution of $4 + 6x \geq -8 + 4x$? (Lesson 3.6)
   F. $x \geq -6$   G. $x \leq -6$
   H. $x \geq -1$   I. $x \leq -1$

37. Short Response Describe the steps you would take to solve the equation $15z - 12 = 3(14 + 3z) - 12$. Then find the solution. (Lesson 3.3)

38. Extended Response Your school is having a fundraising dance. Your costs include $125 for a DJ and $47.50 for decorations. You plan to charge $4.50 for each ticket. (Lesson 3.6)
   a. How many tickets must you sell before you start making a profit?
   b. How many tickets must you sell to make a profit of at least $300?
   c. How would raising the ticket price to $5.00 affect your answers to parts (a) and (b)? Explain.