Chapter 1 **Variables, Expressions, and Integers**

- Write and evaluate variable expressions.
- Perform operations with integers.
- Plot points in a coordinate plane.

**Chapter 2 Solving Equations**

- Use mathematical properties to simplify variable expressions.
- Write and solve one-step equations.
- Perform operations with positive and negative decimals.

**Chapter 3 Multi-Step Equations and Inequalities**

- Write and solve multi-step equations.
- Write and solve inequalities.

From Chapter 2, p. 100

How long does it take a gray whale to migrate?
Variables, Expressions, and Integers

**Before**
In previous courses you’ve . . .
- Performed operations on whole numbers
- Read bar graphs and line graphs
- Found the mean of a set of data

**Now**
In Chapter 1 you’ll study . . .
- Evaluating and writing variable expressions
- Using the order of operations
- Comparing and ordering integers
- Performing operations on integers
- Locating points in a coordinate plane

**Why?**
So you can solve real-world problems about . . .
- DVD rentals, p. 8
- aquariums, p. 13
- basketball, p. 19
- volcanoes, p. 24
- hockey, p. 32
- avalanches, p. 37
- free diving, p. 45
- fuel economy, p. 50

How tall is an iceberg?
**Math In the Real World**

**Icebergs** Icebergs like this one in LeConte Bay, Alaska, typically float with most of their mass below sea level. In this chapter, you will use integers to describe distances above and below sea level.

**What do you think?** Suppose the highest point on an iceberg is 45 feet above the water’s surface. The lowest point is 357 feet below the surface. Find the vertical distance between these two points.
Chapter Prerequisite Skills

PREREQUISITE SKILLS QUIZ

Preparing for Success To prepare for success in this chapter, test your knowledge of these concepts and skills. You may want to look at the pages referred to in blue for additional review.

Vocabulary Copy and complete the statement using a review word.

1. In the multiplication equation $12 \cdot 5 = 60$, 12 and 5 are called _?_ and 60 is called the _?_.

2. When you divide one number by another, the result is called the _?_.

Find the sum or difference. (p. 771)

3. $7.2 + 13.7$
4. $2.41 + 34.6$
5. $10.5 - 7.3$
6. $27.1 - 18.6$

Find the product or quotient. (p. 772)

7. $3.2 \times 1.4$
8. $0.5 \times 27$
9. $27.88 \div 8.2$
10. $11.9 \div 1.7$

NOTETAKING STRATEGIES

KEEPING A NOTEBOOK Some useful items to put in your notebook include the following.

- assignments
- formulas
- rules and properties
- vocabulary
- symbols
- worked-out examples

When you copy examples into your notebook, you may find it helpful to draw a diagram. Include comments that make the solution process clear. For example, a diagram can help you to order the numbers $3.2, 3.09, 3, 3.15, 3.12,$ and $3.02$ from least to greatest.

Draw a number line and graph the numbers:

```
  3  3.02  3.09  3.12  3.15  3.2
1 3.1 3.05
  3
```

Write the numbers in the order in which they appear from left to right: $3, 3.02, 3.09, 3.12, 3.15, 3.2$.

In Lesson 1.8, you may want to include a diagram of a coordinate plane in your notebook.
Expressions *and* Variables

**Vocabulary**
- numerical expression, p. 5
- variable, p. 5
- variable expression, p. 5
- evaluate, p. 5
- verbal model, p. 6

**Before**
You evaluated numerical expressions.

**Now**
You’ll evaluate and write variable expressions.

**Why?**
So you can find the amount left on a gift card, as in Ex. 39.

**Blue Whales** During its summer feeding season, a blue whale eats about 4 tons of food every day. To find about how many tons of food a blue whale eats in a given number of days, you can multiply the number of days by 4, as shown in the table.

<table>
<thead>
<tr>
<th>Days</th>
<th>Tons of food eaten</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4 \cdot 1</td>
</tr>
<tr>
<td>2</td>
<td>4 \cdot 2</td>
</tr>
<tr>
<td>10</td>
<td>4 \cdot 10</td>
</tr>
<tr>
<td>( d )</td>
<td>4 \cdot d</td>
</tr>
</tbody>
</table>

A **numerical expression** consists of numbers and operations. In the table, the expression 4 \cdot 10 is a numerical expression. It can also be written as 4 \times 10 or 4(10).

A **variable** is a letter used to represent one or more numbers. A **variable expression** consists of numbers, variables, and operations.

One way you can use a variable expression is to generalize a pattern, as in the table. The variable expression 4 \cdot d represents the amount of food a blue whale can eat in \( d \) days. You can also write 4 \cdot d as 4\( d \).

To **evaluate** a variable expression, substitute a number for each variable and evaluate the resulting numerical expression.

**Example 1** *Evaluating a Variable Expression*

Evaluate the expression 4 \cdot d when \( d = 120 \) to find about how many tons of food a blue whale eats in a feeding season of 120 days.

**Solution**
\[
4 \cdot d = 4 \cdot 120 \quad \text{Substitute 120 for } d.
\]
\[
= 480 \quad \text{Multiply.}
\]

**Answer** A blue whale eats about 480 tons of food in 120 days.
Example 2  Evaluating Expressions with Two Variables

Evaluate the expression when \( x = 10 \) and \( y = 4 \).

a. \( x + y = 10 + 4 \)  Substitute 10 for \( x \) and 4 for \( y \).
   \[ = 14 \]  Add.

b. \( xy = 10(4) \)  Substitute 10 for \( x \) and 4 for \( y \).
   \[ = 40 \]  Multiply.

√ Checkpoint

Evaluate the expression when \( x = 6 \) and \( y = 12 \).

1. \( y + 8 \)
2. \( 9 - x \)
3. \( y - x \)
4. \( xy \)

Writing Variable Expressions You can solve a real-world problem by creating a verbal model and using it to write a variable expression. A verbal model describes a problem using words as labels and using math symbols to relate the words. The table shows common words and phrases that indicate mathematical operations.

<table>
<thead>
<tr>
<th>Common Words and Phrases that Indicate Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition</strong></td>
</tr>
<tr>
<td>plus</td>
</tr>
<tr>
<td>the sum of</td>
</tr>
<tr>
<td>increased by</td>
</tr>
<tr>
<td>total</td>
</tr>
<tr>
<td>more than</td>
</tr>
<tr>
<td>added to</td>
</tr>
</tbody>
</table>

Example 3  Writing a Variable Expression

Baseball You plan to divide the 120 players in a baseball league into teams with the same number of players. Use a verbal model to write a variable expression for the number of teams if you know the number of players on each team.

Solution

Let \( p \) represent the number of players on each team. The word divide indicates division.

\[
\text{Number of teams } = \frac{\text{Number of players in league}}{\text{Number of players on each team}}
\]

\[
= 120 \div p
\]

Answer The number of teams is \( 120 \div p \), or \( \frac{120}{p} \).

Watch Out

Order is important in subtraction and division expressions. "The difference of a number and 7" means \( n - 7 \), not \( 7 - n \). The quotient of a number and 5" means \( \frac{n}{5} \), not \( \frac{5}{n} \).

Reading Algebra

When you write a variable expression involving division, use a fraction bar instead of the division symbol \( \div \). For example, write "the quotient of \( n \) and 12" as \( \frac{n}{12} \).
Guided Practice

Vocabulary Check
1. Identify the variable in the expression $21 + d$.
2. Compare and contrast the expressions $2 + x$ and $2 + 3$.

Skill Check
Evaluate the expression when $x = 4$.
3. $10 - x$  
4. $x + 7$  
5. $2x$  
6. $\frac{32}{x}$

Evaluate the expression when $m = 5$ and $n = 6$.
7. $\frac{n}{2}$  
8. $m + n$  
9. $n - m$  
10. $mn$

Guided Problem Solving
11. Astronauts In 2002, astronauts Carl Walz and Dan Bursch spent 196 days in orbit. How many sunrises did they see?

1) An astronaut in orbit circles Earth every 90 minutes and sees 16 sunrises each day. Let $d$ be the number of days an astronaut is in orbit. Write a variable expression for the number of sunrises seen in $d$ days.

2) Identify the value of $d$ for Walz’s and Bursch’s 2002 space flight.

3) Find the number of sunrises Walz and Bursch saw.

Practice and Problem Solving

Evaluate the expression when $x = 6$.
12. $x + 3$  
13. $15 - x$  
14. $2x$  
15. $\frac{x}{3}$
16. $20x$  
17. $\frac{24}{x}$  
18. $30 - x$  
19. $15 + x$

Evaluate the expression when $a = 4$, $b = 2$, and $c = 16$.
20. $a + b$  
21. $c - a$  
22. $ab$  
23. $\frac{a}{b}$
24. $bc$  
25. $\frac{c}{a}$  
26. $a - b$  
27. $\frac{c}{b}$
28. $b + c$  
29. $c - b$  
30. $ac$  
31. $a + c$
Write a variable expression to represent the phrase.

32. The product of 72 and a number
33. The difference of a number and 1
34. 13 more than a number
35. The sum of a number and 9.4
36. The quotient of a number and 3
37. A number divided by 41
38. **Error Analysis** Describe and correct the error in writing a variable expression for the difference of a number and 31.

39. **Gift Card** You can evaluate the expression $50 - d$ to find the amount you have left on a $50 gift card after you have spent $d$ dollars. Find the amount left after you have spent $18.

40. **Music Competition** The double bar graph shows three students’ scores in a music competition. A student’s final score is the sum of the points for technique $t$ and for interpretation $i$.

   a. Write a variable expression for a student’s final score.
   b. Find each student’s final score.
   c. **Interpret** You earn 35 points for technique. At least how many points must you earn for interpretation to have a higher score than students A, B, and C?

Evaluate the expression when $a = 2.5$, $b = 15$, and $c = 3.5$.

41. $a + b$
42. $b - c$
43. $bc$
44. $a + c$
45. $\frac{b}{a}$
46. $c - a$
47. $\frac{c}{a}$
48. $ac$

Write a variable expression to represent the phrase.

49. The number of inches in $x$ feet
50. The number of pounds in $y$ ounces

51. **DVD Rentals** You belong to an online DVD rental service. Your yearly rental budget is $200. Each rental costs $4.

   a. Copy and complete the table.
   b. Write a variable expression for the cost of $r$ rentals.
   c. Write a variable expression for the amount of your budget left after $r$ rentals.
   d. **Writing** How many DVDs will you be able to rent before the $200 is spent? Explain how you found your answer.
52. **Extended Problem Solving**  
In football, each field goal (FG) is worth 3 points. Each kicked point after touchdown (PAT) is worth 1 point. The table shows career totals for three leading kickers.

<table>
<thead>
<tr>
<th>Player</th>
<th>FGs</th>
<th>PATs</th>
</tr>
</thead>
<tbody>
<tr>
<td>George Blanda</td>
<td>335</td>
<td>943</td>
</tr>
<tr>
<td>Nick Lowery</td>
<td>383</td>
<td>562</td>
</tr>
<tr>
<td>Norm Johnson</td>
<td>366</td>
<td>638</td>
</tr>
</tbody>
</table>

a. Let \( p \) be the number of points after touchdown that a kicker scored, and let \( f \) be the number of field goals. Write a variable expression for the total number of points.

b. **Evaluate** Find the total number of points for each kicker.

c. **Compare** List the players in order from least total number of points to greatest.

53. **Critical Thinking** Are there any values of the variable \( a \) for which the expressions \( 2 + a \) and \( 2a \) have the same value? Explain.

**Logical Reasoning** Describe the pattern shown in the table. Then write a variable expression involving \( n \) to complete the table. In the table, the three dots indicate that the pattern continues.

<table>
<thead>
<tr>
<th>Cost of item (dollars)</th>
<th>1.00</th>
<th>2.00</th>
<th>3.00</th>
<th>4.00</th>
<th>...</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost with tax (dollars)</td>
<td>1.05</td>
<td>2.10</td>
<td>3.15</td>
<td>4.20</td>
<td>...</td>
<td>?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cost of item (dollars)</th>
<th>1.00</th>
<th>1.50</th>
<th>2.00</th>
<th>2.50</th>
<th>...</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost with coupon (dollars)</td>
<td>0.50</td>
<td>1.00</td>
<td>1.50</td>
<td>2.00</td>
<td>...</td>
<td>?</td>
</tr>
</tbody>
</table>

55. **Challenge** The plastic tips on the ends of a shoelace are called aglets. Suppose a sneaker factory produces \( p \) pairs of shoes each hour and is in operation for \( h \) hours each day. Write a variable expression for the number of single aglets the factory uses each day. Evaluate the expression when \( p = 200 \) and \( h = 24 \). Explain what your answer means.

56. **Mixed Review**  
Find the sum or difference. \((p. 774)\)

57. \(3.2 + 4.7\)  
58. \(5.1 + 6.8\)  
59. \(7.3 - 2.1\)  
60. \(9.9 - 5.4\)

Find the product or quotient. \((pp. 775, 776)\)

61. \(8 \times (13.2)\)  
62. \(\frac{12.5}{5}\)  
63. \(\frac{24.32}{3.2}\)  
64. \((6.5) \times (4.3)\)

65. Order the decimals from least to greatest: 8.9, 8.79, 7.98, 9.87, 7.8, 9.78. \((p. 773)\)

66. **Multiple Choice** Write a variable expression for a length of time in minutes if you know the number \( s \) of seconds.

A. \( s + 60 \)  
B. \(\frac{s}{60} \)  
C. \(60s \)  
D. \(\frac{60}{s} \)

67. **Multiple Choice** Evaluate the expression \( x - y \) when \( x = 12.8 \) and \( y = 4 \).

F. 3.2  
G. 8.8  
H. 12.4  
I. 13.2
Powers and Exponents

You multiplied whole numbers and decimals. You’ll use powers to describe repeated multiplication. So you can find the total number of e-mails sent, as in Ex. 29.

A **power** is the result of a repeated multiplication of the same factor. For example, the number 125 is a power because $125 = 5 \cdot 5 \cdot 5$. A power can be written in a form that has two parts: a number called the **base** and a number called the **exponent**. The exponent shows the number of times the base is used as a factor.

$$5^3 = 5 \cdot 5 \cdot 5$$

The base 5 is used as a factor 3 times.

The table shows how to read and write powers. Numbers raised to the first power, such as $12^1$, are usually written without the exponent.

<table>
<thead>
<tr>
<th>Power</th>
<th>In words</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$12^1$</td>
<td>12 to the first power</td>
<td>$12^1 = 12$</td>
</tr>
<tr>
<td>$(0.5)^2$</td>
<td>0.5 to the second power, or 0.5 squared</td>
<td>$(0.5)(0.5) = 0.25$</td>
</tr>
<tr>
<td>$4^3$</td>
<td>4 to the third power, or 4 cubed</td>
<td>$4 \cdot 4 \cdot 4 = 64$</td>
</tr>
<tr>
<td>$8^4$</td>
<td>8 to the fourth power</td>
<td>$8 \cdot 8 \cdot 8 \cdot 8 = 4096$</td>
</tr>
</tbody>
</table>

**Example 1**

**Using Exponents**

Write the product using an exponent.

a. $13 \cdot 13 \cdot 13 \cdot 13 = 13^4$  The base 13 is used as a factor 4 times.

b. $(0.2)(0.2)(0.2) = (0.2)^3$  The base 0.2 is used as a factor 3 times.

c. $n \cdot n \cdot n \cdot n \cdot n = n^6$  The base $n$ is used as a factor 6 times.

d. $t \cdot t \cdot t \cdot t = t^5$  The base $t$ is used as a factor 5 times.

**Checkpoint**

Write the product using an exponent.

1. $10 \cdot 10 \cdot 10$

2. $(4.3)(4.3)$

3. $x \cdot x \cdot x \cdot x$

4. Critical Thinking  Evaluate each power: $0^2$, $0^3$, $0^4$. Use your results to write a rule for the value of 0 raised to any nonzero whole number exponent.
Example 2  Evaluating Powers with Variables

Evaluate the expression $x^4$ when $x = 0.5$.

$\begin{align*}
x^4 &= (0.5)^4 \\
&= (0.5)(0.5)(0.5)(0.5) \\
&= 0.0625
\end{align*}$

Substitute 0.5 for $x$.  
Use 0.5 as a factor 4 times.  
Multiply.

✓ Checkpoint

Evaluate the expression when $m = 3$.

5. $m^2$  
6. $m^3$  
7. $m^4$  
8. $m^5$

Using Formulas  A formula describes a relationship between quantities. Some formulas involve powers. For example, you can use a formula to find the area of a square or the volume of a cube.

Area of a square  
$A = s^2$

Volume of a cube  
$V = s^3$

Area is measured in square units, such as square feet (ft²) or square centimeters (cm²). Volume is measured in cubic units, such as cubic inches (in³) or cubic meters (m³).

Example 3  Using Powers in Formulas

Ice Sculpture  An artist uses a cube-shaped block of ice to make an ice sculpture for a competition. Find the volume of the block of ice.

Solution

Use the formula for the volume of a cube.

$V = s^3$  
$= (20)^3$  
$= 8000$  
Write the formula.  
Substitute 20 for $s$.  
Evaluate power.

Answer  The volume of the block of ice is 8000 cubic inches.

✓ Checkpoint

Find the area of a square with the given side length.

9. 9 meters  
10. 11 inches  
11. 1.5 centimeters
Guided Practice

**Vocabulary Check**
1. Identify the base and the exponent in the expression $13^5$.
2. How are the expressions $3^4$ and $4^3$ different?

**Skill Check**
Write the power in words and as a repeated multiplication. Then evaluate the power.

3. $12^2$
4. $(0.3)^3$
5. $(1.2)^3$
6. $5^4$

Evaluate the expression when $k = 6$.

7. $k^2$
8. $k^3$
9. $k^4$
10. $k^5$

11. **Gift Box** A gift box has the shape of a cube with an edge length of 14 inches. Find the volume of the box.

12. **Error Analysis** Describe and correct the error in writing $2^3$ as a repeated multiplication.

Practice and Problem Solving

**Homework Help**

**Example**
1. 13–29
2. 30–33
3. 36

**Online Resources**
CLASSZONE.COM
- More Examples
- eTutorPlus

Write the product using an exponent.

13. $32 \cdot 32$
14. $11 \cdot 11 \cdot 11$
15. $6 \cdot 6 \cdot 6 \cdot 6$
16. $2 \cdot 2 \cdot 2 \cdot 2$
17. $(5.6)(5.6)(5.6)$
18. $(1.7)(1.7)(1.7)$
19. $z \cdot z \cdot z$
20. $n \cdot n \cdot n \cdot n$

Write the power in words and as a repeated multiplication. Then evaluate the power.

21. $8^3$
22. $2^5$
23. $10^6$
24. $12^3$
25. $9^3$
26. $4^4$
27. $(0.2)^2$
28. $(0.6)^4$

29. **Extended Problem Solving** You send an e-mail to 4 friends. Each friend sends the e-mail to 4 more friends. Each of those friends sends it to 4 friends, and so on.

a. Copy and complete the table.

<table>
<thead>
<tr>
<th>Stage</th>
<th>E-mails sent, as a power</th>
<th>Value of power</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$4^1$</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>$4^2$</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>4</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

b. **Calculate** Find the number of e-mails sent at stage 9.

c. **Estimate** Estimate the stage at which more than 1,000,000 e-mails will be sent. Use a calculator to check your estimate.
Evaluate the expression when \( n = 7 \) and when \( n = 0.4 \).

30. \( n^2 \)  \hspace{1cm} 31. \( n^3 \)  \hspace{1cm} 32. \( n^4 \)  \hspace{1cm} 33. \( n^5 \)

34. **Writing** The *square* of a number is the second power of the number. The *cube* of a number is the third power of the number. Explain why these names are reasonable.

35. **Critical Thinking** Explain why 1 raised to any power is equal to 1.

36. **Aquariums** An aquarium has a square base with a side length of 15 inches. You fill the aquarium with water to a height of 15 inches.
   a. Find the volume of the water in the aquarium.
   b. A cubic inch of water weighs approximately 0.036 pound. Find the approximate weight of the water in the aquarium.

37. **Patterns** The table shows sums of odd numbers.
   a. Copy and complete the table. Identify any pattern that you see.
   
<table>
<thead>
<tr>
<th>( n )</th>
<th>Sum of first ( n ) odd numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1 + 3 = 4</td>
</tr>
<tr>
<td>3</td>
<td>1 + 3 + 5 = ?</td>
</tr>
<tr>
<td>4</td>
<td>1 + 3 + 5 + 7 = ?</td>
</tr>
<tr>
<td>5</td>
<td>?</td>
</tr>
</tbody>
</table>

   b. Write a variable expression for the sum of the first \( n \) odd numbers.
   c. Use your expression from part (b) to find the sum of the first 100 odd numbers.

38. **Challenge** Find values of \( x \), \( y \), and \( z \) so that each of the expressions \( x^2 \), \( y^3 \), and \( z^8 \) has a value of 64.

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**Mixed Review**

Find the product or quotient.  \((pp. 775, 776)\)

39. \((2.5)(7.1)\)  \hspace{1cm} 40. \((2.3)(8.4)\)  \hspace{1cm} 41. \(1.2 \div 2.4\)  \hspace{1cm} 42. \(5.2 \div 1.25\)

43. **Olympics** The bar graph shows the number of gold medals won by the four countries with the most gold medals in the 2000 Olympic Summer Games. How many gold medals did the four countries win in all? \((p. 781)\)

---

Evaluate the expression when \( x = 15 \). \((Lesson\ 1.1)\)

44. \(x + 4\)  \hspace{1cm} 45. \(200 - x\)  \hspace{1cm} 46. \(x - 11\)  \hspace{1cm} 47. \(3x\)

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**Standardized Test Practice**

48. **Multiple Choice** Which expression has a value of 81?
   
   A. \(4^3\)  \hspace{1cm} B. \(3^4\)  \hspace{1cm} C. \(2^8\)  \hspace{1cm} D. \(27^3\)

49. **Short Response** Compare each number in the top row of the table with the number below it. Describe any pattern you see. Complete the table with a variable expression involving \( n \).

<table>
<thead>
<tr>
<th>( 1 )</th>
<th>( 2 )</th>
<th>( 3 )</th>
<th>( 4 )</th>
<th>...</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 )</td>
<td>( 8 )</td>
<td>( 27 )</td>
<td>( 64 )</td>
<td>...</td>
<td>( ? )</td>
</tr>
</tbody>
</table>
A Problem Solving Plan

You can use the following 4-step plan to solve a problem.

1. **Read and Understand** Read the problem carefully. Identify the question and any important information.

2. **Make a Plan** Decide on a problem solving strategy.

3. **Solve the Problem** Use the problem solving strategy to answer the question.

4. **Look Back** Check that your answer is reasonable.

Reading and Planning

**Example** You plan to ship 5 books to a friend. The table shows the masses of the books. Is it possible to ship the books in 2 boxes, each with a mass of 6 kilograms or less? Explain.

<table>
<thead>
<tr>
<th>Book</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (kg)</td>
<td>1.4</td>
<td>2.1</td>
<td>3.8</td>
<td>1.9</td>
<td>2.5</td>
</tr>
</tbody>
</table>

**Read and Understand**

**What do you know?**
The mass of each box must be 6 kilograms or less.
The table gives the mass of each book.

**What do you want to find out?**
Is it possible to put the books in 2 boxes so that each box has a mass of 6 kilograms or less?

**Make a Plan**

**How can you relate what you know to what you want to find out?**
Check that the total mass of the books doesn’t exceed 12 kilograms. If it does, you can’t divide the books as you want.
Use the strategy *guess, check, and revise* to choose books for each box.
Solving and Looking Back

To solve the problem from page 14 about shipping books, carry out the plan. Then check the answer.

Solve the Problem

The total mass of the books is $1.4 + 2.1 + 3.8 + 1.9 + 2.5 = 11.7$ kg, so it may be possible to ship the books as you want.

Now use the strategy guess, check, and revise. Put the 3 lightest books in one box and the 2 heaviest in the other. The mass of the second box is more than 6 kg.

<table>
<thead>
<tr>
<th>Books</th>
<th>Total mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, D, B</td>
<td>$1.4 + 1.9 + 2.1 = 5.4$</td>
</tr>
<tr>
<td>C, E</td>
<td>$3.8 + 2.5 = 6.3$</td>
</tr>
</tbody>
</table>

Try switching books B and E. The mass of each box is less than 6 kg.

<table>
<thead>
<tr>
<th>Books</th>
<th>Total mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, D, E</td>
<td>$1.4 + 1.9 + 2.5 = 5.8$</td>
</tr>
<tr>
<td>C, B</td>
<td>$3.8 + 2.1 = 5.9$</td>
</tr>
</tbody>
</table>

Answer It is possible to ship the books in 2 boxes, each with a mass of 6 kg or less. Put books A, D, and E in one box and books C and B in the other.

Look Back It makes sense that the box with 3 books contains the 2 lightest books and the box with 2 books contains the heaviest book. So, the answer is reasonable.

☑️ Checkpoint

1. **Pool Schedules** A community pool offers 3 swim sessions each Saturday morning. Each session lasts 35 minutes, with 10 minutes between sessions. The final session ends at 11:05 A.M. At what time does the first session begin?

2. **Theater Seating** The center section of a theater has 10 rows. There are 41 seats in row 10, 38 seats in row 9, 35 seats in row 8, and so on. How many seats are in row 1?

3. **Movie Marathon** You have invited friends over to watch movies. You have rented 4 movies: one action, one science fiction, one comedy, and one animated. In how many different orders can you watch the movies?
Order of Operations

Flower Flag There are about 2000 plants in each of the 50 stars of this flower flag. There are about 64,100 plants in each of the 7 short stripes and 106,700 plants in each of the 6 long stripes. The blue region contains about 198,900 plants. You can approximate the total number of plants by evaluating the expression

\[50 \cdot 2000 + 7 \cdot 64,100 + 6 \cdot 106,700 + 198,900.\]

To evaluate expressions involving more than one operation, mathematicians have agreed on a set of rules called the order of operations.

Order of Operations

1. Evaluate expressions inside grouping symbols.
2. Evaluate powers.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

Example 1 Using Order of Operations

To approximate the total number of plants in the flower flag described above, use the order of operations to evaluate the expression

\[50 \cdot 2000 + 7 \cdot 64,100 + 6 \cdot 106,700 + 198,900.\]

\[= 100,000 + 448,700 + 640,200 + 198,900\]

\[= 1,387,800\]

Answer There are approximately 1,387,800 plants in the flower flag.
**Grouping Symbols** Parentheses ( ), brackets [ ], and fraction bars are common grouping symbols. Grouping symbols indicate operations that should be performed first. For example, compare the expressions \(3 \cdot 2 + 5\) and \(3(2 + 5)\). To evaluate \(3 \cdot 2 + 5\), you multiply first, then add. To evaluate \(3(2 + 5)\), you add first, then multiply.

**Example 2** Using Grouping Symbols

Evaluate the expression.

a. \(8(17 - 2.3) = 8(14.7)\) Subtract within parentheses.
   
   \[= 117.6\] Multiply.

b. \(\frac{14 + 6}{12 - 7} = (14 + 6) ÷ (12 - 7)\) Rewrite fraction as division.
   
   \[= 20 ÷ 5\] Evaluate within parentheses.
   
   \[= 4\] Divide.

c. \(5 \cdot [36 - (13 + 9)] = 5 \cdot [36 - 22]\) Add within parentheses.
   
   \[= 5 \cdot 14\] Subtract within brackets.
   
   \[= 70\] Multiply.

**Checkpoint**

Evaluate the expression.

1. \(28 - 63 ÷ 7\)
2. \(52 + 12.5 \cdot 4\)
3. \(9 \cdot 6 + 27 ÷ 3\)
4. \(10(1.5 + 0.6)\)
5. \(\frac{70 - 9.2}{3 + 5}\)
6. \(72 ÷ [(11 - 7) \cdot 2]\)

**Study Strategy**

You can use the first letters of the words of the sentence *Please Excuse My Dear Aunt Sally* to help you remember the order of operations.

- **P** parentheses
- **E** exponents
- **M** multiplication and division
- **A** addition and subtraction

**Example 3** Evaluating Variable Expressions

Evaluate the expression when \(x = 2\) and \(y = 5\).

a. \(4(x + y) = 4(2 + 5)\) Substitute 2 for \(x\) and 5 for \(y\).
   
   \[= 4(7)\] Add within parentheses.
   
   \[= 28\] Multiply.

b. \(3(x + y)^2 = 3(2 + 5)^2\) Substitute 2 for \(x\) and 5 for \(y\).
   
   \[= 3(7)^2\] Add within parentheses.
   
   \[= 3(49)\] Evaluate power.
   
   \[= 147\] Multiply.

**Checkpoint**

Evaluate the expression when \(x = 4\) and \(y = 2\).

7. \(1.2(x + 3)\)
8. \(1.2x + 3\)
9. \(3x - 2y\)
10. \(0.5[y - (x - 2)]\)
11. \(x^2 - y\)
12. \(2(x - y)^2\)
Example 4  Using a Problem Solving Plan

**Sewing** You buy a pattern and enough material to make two pillows. The pattern costs $5. Each pillow requires $3.95 worth of fabric and a button that costs $.75. Find the total cost.

**Solution**

**Read and Understand** You buy one pattern plus fabric and buttons for two pillows. You are asked to find the total cost.

**Make a Plan** Write a verbal model.

\[
\text{Total cost} = \text{Cost of pattern} + \text{Number of pillows} \cdot \text{Cost of each pillow}
\]

**Solve the Problem** Write and evaluate an expression.

\[
\text{Total cost} = 5 + 2(3.95 + 0.75)
\]

\[
= 5 + 2(4.70)
\]

\[
= 5 + 9.40
\]

\[
= 14.40
\]

**Answer** The total cost is $14.40.

**Look Back** Use estimation to check that the answer is reasonable. The cost of materials for each pillow is about $4 + $1 = $5. The total cost is about $5 + 2($5) = $15. The answer is reasonable.

## Exercises

### 1.3

**Guided Practice**

**Vocabulary Check**

1. Give three examples of grouping symbols.

2. Describe in order the steps you would take to evaluate the expression \(12(x - 3)^2\) when \(x = 5\).

**Skill Check**

**Evaluate the expression.**

3. \(15 - 3 \cdot 4\)

4. \(48 \div 6 + 2\)

5. \(3 \cdot 8 + 5 \cdot 4\)

6. \(\frac{18 + 12}{7 - 2}\)

7. \(17 - (3^2 - 2)\)

8. \(4[15 - (2 + 5)]\)

9. **Twin Convention** The table shows the numbers of sets of twins, triplets, quadruplets, and quintuplets registered at a twin convention. Write and evaluate an expression for the total number of people who registered at the convention.

<table>
<thead>
<tr>
<th>Type</th>
<th>Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Twins</td>
<td>2697</td>
</tr>
<tr>
<td>Triplets</td>
<td>29</td>
</tr>
<tr>
<td>Quadruplets</td>
<td>2</td>
</tr>
<tr>
<td>Quintuplets</td>
<td>1</td>
</tr>
</tbody>
</table>

Chapter 1  Variables, Expressions, and Integers
Evaluate the expression.

10. \(47.7 - 12 \cdot 3\)
11. \(11 \cdot 7 - 9 \cdot 5\)
12. \(14 ÷ 7 + 36 ÷ 4\)
13. \(5.8(3) + 3(1.1)\)
14. \(\frac{36 - 12}{2 + 6}\)
15. \(\frac{9.8 + 2.2}{7 - 5}\)
16. \(5(21 - 3^2)\)
17. \(7[2.5 + 3(12 - 7)]\)
18. \(84 ÷ [(18 - 16) \cdot 3]\)

Evaluate the expression when \(x = 3\), \(y = 4\), and \(z = 5\).

19. \(0.25y + x\)
20. \(0.25(y + x)\)
21. \(4(z - x)\)
22. \(\frac{6.5y}{x - 1}\)
23. \(x + \frac{24.4}{y}\)
24. \(7z - x^2\)
25. \(x + 2[z - (y - 1)]\)
26. \((x + y)^2 - 3.6\)
27. \(y + (z - 1)^2\)

28. **Plants** A boojum is a very slow-growing cactus. One fifty-year-old boojum is 1.5 meters tall and has been growing about 0.03 meter each year. Assume the growth pattern continues.

   a. Write an expression for the height in meters of the boojum \(y\) years from now.

   b. **Apply** How tall will the boojum be in 50 years?

29. **Craft Fair** Your school is setting up a row of 5 tables for a craft fair. Each table is 72 inches long. The space between each pair of neighboring tables must be 48 inches. Write and evaluate an expression to find the length of the space needed for the tables from the beginning of the first table to the end of the last table.

30. **Basketball** In basketball, players score points by making free throws worth 1 point each, field goals worth 2 points each, and field goals worth 3 points each. A player scores 4 free throws, 7 two-point field goals, and 2 three-point field goals. Write and evaluate an expression for the total number of points the player scores.

31. **Movies** You buy 4 videotapes for $14.99 each and 3 DVDs for $19.99 each. Find the total cost of the movies.

Evaluate the expression when \(x = 4\) and \(y = 3\).

32. \(5x^2 + 2y\)
33. \(7(x^2 - 5y)\)
34. \(\frac{x^2 + 9}{y + 2}\)
35. \(\frac{6.5y + 2}{x + 1}\)

36. **Cell Phone** You and your sister share cell phone service. You divide the bill equally, including the monthly fee of $39 plus $.30 for each additional minute beyond your free minutes.

   a. Write an expression for your share of the bill in a month when you are charged for \(m\) extra minutes.

   b. **Apply** One month, you are charged for 125 additional minutes. Find your share of the bill.
37. **Extended Problem Solving** Digital cameras capture images in rows and columns of *pixels*, which are small rectangular colored dots. The more pixels in a given space, the greater the detail of the image.

   a. **Calculate** The total number of pixels in an image is the product of the number of pixels in a row and the number of pixels in a column. Your camera produces an image that has 1280 pixels in a row and 1024 pixels in a column. Find the total number of pixels.

   b. A megapixel is 1,000,000 pixels. Find, to the nearest tenth, the number of megapixels in the image in part (a).

   c. **Apply** Let \( m \) be the number of megapixels in an image, and let \( l \) and \( w \) be the length and width in inches of a printed photo. A print is clear if the value of the expression \( \frac{m}{lw} \) is 0.017 or greater. Can you make a clear 8 inch by 10 inch print of the image in part (a)? Explain.

38. **Museum Cost** A group of 20 members and 5 nonmembers visited a museum. The admission cost was $6 for members and $10 for nonmembers. The group decided to divide the total cost evenly among all 25 people. What did each person pay?

39. **Challenge** You are decorating a square mouse pad. You place a colored sticker at each corner. If you have \( r \) different colors available, the number of possible patterns is the value of the expression \( \frac{r^4 + 2r^2 + 3r + 2}{8} \). Two patterns are different if you cannot produce either pattern by turning the other around.

   a. **Writing** Tell whether the patterns shown are the same. Explain.

   b. You decide to use two colors. Find the number of possible patterns.

   c. Sketch all the possible patterns for two different colors.

---

**Mixed Review**

Copy and complete the statement using <, >, or =. (p. 773)

40. 1.99 < 1.98  
41. 0.56 > 0.65  
42. 0.32 < 0.23

43. **Color Monitor** The *bit depth* of a color monitor is the number of colors it can display and is expressed as a power of 2. A 32 bit monitor can display \( 2^{32} \) colors. Write and evaluate an expression for the number of colors an 8 bit monitor can display. *(Lesson 1.2)*

---

**Standardized Test Practice**

44. **Extended Response** The number of calories in a serving of food is the sum of the calories from carbohydrate, protein, and fat. A cup of whole milk has 11 g of carbohydrate, 8 g of protein, and 8 g of fat.

   a. How many calories are there in a cup of whole milk?

   b. If you drank enough whole milk to get 20 g of protein, how many calories would it provide? Explain how you found your answer.
1.3 Using Order of Operations

Goal Use a calculator to evaluate expressions using the order of operations.

Example

Baseball Alex Rodriguez played for the Texas Rangers during the 2002 baseball season. Use the following information to calculate his batting average for that season.

To find a baseball player's batting average, you divide the number of hits he made by the number of times he was at bat and round the quotient to the nearest thousandth. The table gives Alex Rodriguez's 2002 batting statistics.

<table>
<thead>
<tr>
<th>2002 season</th>
<th>Hits</th>
<th>At bats</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before All-Star Game</td>
<td>100</td>
<td>328</td>
</tr>
<tr>
<td>After All-Star Game</td>
<td>87</td>
<td>296</td>
</tr>
</tbody>
</table>

Solution

Divide the total number of hits by the total number of times at bat. Use parentheses around each sum.

Keystrokes

\[
\left(\frac{100}{328}\right) + \left(\frac{87}{296}\right) =
\]

Answer Alex Rodriguez's batting average for the entire season was 0.300, which is usually written as .300.

Tech Help

The keystrokes shown may not be the same as on your calculator. See your calculator's instruction manual for alternative keystrokes. For additional keystroke help, visit the website below.

Online Resources

CLASSZONE.COM

• Keystroke Help

Draw Conclusions

Use a calculator to evaluate the expression.

1. \(50 + 21 ÷ 3\)  
2. \(15 \times (24 + 8)\)  
3. \((8 + 10) ÷ 2\)  
4. \((5 + 2)^2 - 3^2\)  
5. \((24 - 16) ÷ 2\)  
6. \((12 - 7)^2 - 1\)  
7. \(38 ÷ (2 + 17)\)  
8. \((8 + 3)^2 + 2\)  
9. Critical Thinking What result would you get in the example above if you didn't use parentheses when entering the expression? Why?

10. Baseball Barry Bonds played for the San Francisco Giants during the 2002 season. Use the information in the table to calculate his batting average for the entire 2002 season.

<table>
<thead>
<tr>
<th>2002 season</th>
<th>Hits</th>
<th>At bats</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before All-Star Game</td>
<td>80</td>
<td>232</td>
</tr>
<tr>
<td>After All-Star Game</td>
<td>69</td>
<td>171</td>
</tr>
</tbody>
</table>
Comparing and Ordering Integers

**Vocabulary**
- integer, p. 22
- negative integer, p. 22
- positive integer, p. 22
- absolute value, p. 23

**BEFORE**
You compared and ordered decimals.

**Now**
You’ll compare and order integers.

**WHY?**
So you can compare two volcanoes, as in Ex. 17.

**Supercooled Insects** Water freezes at 0°C, but some animals can resist freezing by producing a chemical that lowers the temperature at which the water in their bodies freezes. This temperature is called the supercooling point. Which of the insects listed in the table has the lowest supercooling point?

<table>
<thead>
<tr>
<th>Insect</th>
<th>Supercooling point (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arctic beetle</td>
<td>−54</td>
</tr>
<tr>
<td>Gall beetle</td>
<td>−35</td>
</tr>
<tr>
<td>Goldenrod gallfly</td>
<td>−9</td>
</tr>
<tr>
<td>Snow flea</td>
<td>−19</td>
</tr>
<tr>
<td>Wooly bear caterpillar</td>
<td>−70</td>
</tr>
</tbody>
</table>

The numbers in the table are *negative integers*. The *integers* are the numbers . . . , −3, −2, −1, 0, 1, 2, 3, . . . (The dots indicate that the numbers continue without end in both the positive and negative directions). *Negative integers* are integers that are less than 0.

**Positive integers** are integers that are greater than 0.

---

**Example 1**

**Graphing and Ordering Integers**

To determine which insect in the table above has the lowest supercooling point, graph the integers on a number line.

Read the numbers from left to right: −70, −54, −35, −19, −9.

**Answer** At −70°C, the wooly bear caterpillar has the lowest supercooling point.

**Checkpoint**

1. Use a number line to order these integers from least to greatest: −8, 5, −4, 2, 0, 6.
**Absolute Value** The **absolute value** of a number is its distance from 0 on a number line. The absolute value of a number \(a\) is written as \(|a|\). You can use a number line to find the absolute value of a number.

**Example 2**  
**Finding Absolute Value**

State the absolute value of the number.

a. 5  
b. −7

**Solution**

a.  

\[
\begin{array}{c}
\text{5 units} \\
-2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\end{array}
\]

The distance between 5 and 0 is 5. So, \(|5| = 5\).

b.  

\[
\begin{array}{c}
\text{7 units} \\
-8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 \\
\end{array}
\]

The distance between −7 and 0 is 7. So, \(|-7| = 7\).

**Opposites** Two numbers are **opposites** if they have the same absolute value but different signs. For example, −10 and 10 are opposites. The expression −10 can be read as “the opposite of 10” or as “negative 10.” The expression “−a” is read as “the opposite of a.”

**Example 3**  
**Finding Opposites**

State the opposite of the number.

a. 6  
b. −15

**Solution**

a.  

\[
\begin{array}{c}
\text{6 units} \\
-6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\end{array}
\]

The opposite of 6 is −6.

b.  

\[
\begin{array}{c}
\text{15 units} \\
-20 & -15 & -10 & -5 & 0 & 5 & 10 & 15 & 20 \\
\end{array}
\]

The opposite of −15 is 15.

**Checkpoint**

State the absolute value and the opposite of the number.

2. 3  
3. −1  
4. 10  
5. −11
Example 4  **Evaluating Variable Expressions**

Evaluate the expression when $y = -5$.

a. $-y$

b. $17 - |y|$

**Solution**

a. $-y = -(-5)$  Substitute $-5$ for $y$.

   $= 5$  The opposite of $-5$ is $5$.

b. $17 - |y| = 17 - |-5|$

   $= 17 - 5$  Substitute $-5$ for $y$.

   $= 12$  The absolute value of $-5$ is $5$.

   Subtract.

**Checkpoint**

Evaluate the expression when $x = -4$.

6. $-x$

7. $12 - |x|$

8. $|x| + 9$

9. $|x| - 1$

### Guided Practice

**Vocabulary Check**

1. Which of these numbers is not an integer: $-31$, $74$, $22.5$, $-7$, or $19$?

2. Explain why the absolute value of a number is never negative.

**Skill Check**

3. Write the integers in order from least to greatest: $-9$, $12$, $6$, $-3$, $0$, $-5$.

**State the absolute value of the number.**

4. $1$

5. $-9$

6. $15$

7. $-12$

**State the opposite of the number.**

8. $14$

9. $-33$

10. $-24$

11. $81$

**Evaluate the expression when $x = -3$.**

12. $|x| + 8$

13. $|x| + |-1|$

14. $20 - |x|$

15. $50 - |x|$

16. **Error Analysis**  Describe and correct the error in evaluating the expression $|-17|$.

   $| -17 | = -17$

17. **Volcanoes**  The elevation of the top of a volcano relative to sea level is called the summit elevation. The summit elevation of Kilauea in Hawaii is 1222 meters. The summit elevation of the underwater volcano Loihi in the Pacific Ocean is $-980$ meters. Which is farther from sea level, the top of Kilauea or the top of Loihi?
Copy and complete the statement using < or >.
18. \(-8 \_ 3\) 19. \(-9 \_ -12\) 20. \(0 \_ -4\) 21. \(-15 \_ -7\)

Graph the integers on a number line. Then write the integers in order from least to greatest.
22. \(-12, 4, -6, 0, -1\) 23. \(15, -8, -4, 7, -5\)
24. \(35, 60, -10, -5, 40\) 25. \(-22, -30, -25, -16\)

State the absolute value of the number.
26. \(-22\) 27. \(7\) 28. \(21\) 29. \(-40\)
30. \(38\) 31. \(-42\) 32. \(-73\) 33. \(105\)

State the opposite of the number.
34. \(6\) 35. \(9\) 36. \(-2\) 37. \(-11\)
38. \(-31\) 39. \(-67\) 40. \(81\) 41. \(100\)

42. **Neptune’s Moons** In 1989, data collected by the Voyager spacecraft showed the surface temperature of Triton, Neptune’s largest moon, to be about \(-392\)°F. Eight years later, data from the Hubble telescope showed the temperature to be about \(-389\)°F. Did the Hubble data indicate a temperature less than or greater than the one based on the Voyager data?

43. \(-x\) 44. \(|x| - 1\) 45. \(32 - |x|\) 46. \(-x - 2\)
47. \(5|x|\) 48. \(-x - 3\) 49. \(5 + (-x)\) 50. \(|x| + 10\)

51. **Underwater Cities** Archaeologists have discovered underwater ruins of ancient cities. The table shows the elevation relative to sea level of the deepest point of ruins at several sites.

<table>
<thead>
<tr>
<th>Site</th>
<th>Elevation relative to sea level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helike, Greece</td>
<td>3 meters below</td>
</tr>
<tr>
<td>Heraklion, Egypt</td>
<td>8 meters below</td>
</tr>
<tr>
<td>Port Royal, Jamaica</td>
<td>12 meters below</td>
</tr>
<tr>
<td>Unnamed city, Bay of Bengal</td>
<td>37 meters below</td>
</tr>
</tbody>
</table>

a. Write an integer to represent each elevation in the table.

b. Graph the integers on a number line.

c. Identify the site whose deepest point is farthest from sea level.

d. **Compare** The elevation of the modern-day Greek city of Polónia is 1 meter above sea level. Is Polónia closer to or farther from sea level than the deepest point of the ruins of Helike?
52. **Writing** Explain why the absolute value of 0 is 0.

53. **Extended Problem Solving** The table shows the daily high temperature at Alaska’s Barrow Observatory over a seven-day period.

   **Day** | **Temperature**
   --- | ---
   Sunday | $-19^\circ C$
   Monday | $-17^\circ C$
   Tuesday | $-14^\circ C$
   Wednesday | $-9^\circ C$
   Thursday | $-13^\circ C$
   Friday | $-18^\circ C$
   Saturday | $-21^\circ C$

   a. Did the temperature increase or decrease from Sunday to Monday?
   b. Did the temperature increase or decrease from Friday to Saturday?
   c. **Compare** Which day’s high temperature was highest for the week? Which was lowest?
   d. **Interpret and Apply** Describe any periods of two or more days during the week when the daily high temperature consistently increased or decreased.

   **Evaluate the expression when $a = -2$ and $b = -13$.**
   54. $|a| + |b|$  
   55. $-a + (-b)$  
   56. $-a + |b|$  
   57. $|b| - |a|$  
   58. $-|b|$  
   59. $-|a|$  

   60. **Critical Thinking** Are there values of $x$ for which $-x$ is less than $x$? Are there values of $x$ for which $-x$ is greater than $x$? Explain.

   **Evaluate the expression when $x = -7$.**
   61. $|x|$  
   62. $|x| + |-x|$  
   63. $|x| - |-x|$  
   64. $-|-x|$  

   65. **Challenge** Copy and complete using $x$ or $-x$: If $x > 0$, then $|x| = \underline{?}$. If $x < 0$, then $|x| = \underline{?}$.

**Mixed Review**

**Estimation** Estimate the sum or difference by rounding each number to the place of its leading digit. (p. 771)

66. $278 + 119 + 602$  
67. $588 - 131$  
68. $112 + 193 + 583$

**Find a low and high estimate for the product or quotient.** (p. 772)

69. $62 \times 708$  
70. $31,217 \div 218$  
71. $371 \times 47$

**Evaluate the expression when $x = 2$ and $y = 8$.** (Lesson 1.3)

72. $6(x + y)$  
73. $xy + 1$  
74. $\frac{x + 22}{y}$

**Standardized Test Practice**

75. **Multiple Choice** Which list of integers is in order from least to greatest?
   a. 2, 16, $-17$, 21, $-35$  
   b. 2, $-16$, $-17$, $-21$, $-35$  
   c. $-35$, $-17$, 2, 16, 21  
   d. 21, 16, 2, $-17$, $-35$

76. **Multiple Choice** Which of the following is the value of the expression $|x| + |-5|$ when $x = 5$?
   f. $-5$  
   g. 0  
   h. 5  
   i. 10
Evaluate the expression when \( x = 2 \) and \( y = 14 \).

1. \( x + 5 \)
2. \( y - 2 \)
3. \( x + y \)
4. \( \frac{y}{x} \)

5. **Word Processing** Your computer’s word-processing program fits about 250 words on one page. Let \( p \) represent the number of pages in a report. Write a variable expression for the approximate number of words in the report.

Write the product using an exponent.
6. \( 11 \cdot 11 \cdot 11 \cdot 11 \)
7. \( (2.6)(2.6)(2.6) \)
8. \( s \cdot s \cdot s \cdot s \)
9. \( y \cdot y \cdot y \cdot y \cdot y \)

Evaluate the expression.
10. \( 18 - 3 \cdot 2 \)
11. \( 27 \div 3 + 6 \)
12. \( \frac{20 + 12}{11 - 3} \)
13. \( 4(20 - 3^2) \)

Evaluate the expression when \( x = 20 \) and \( y = 5 \).
14. \( 0.5x + y \)
15. \( \frac{x + 5}{y} \)
16. \( 3(x - y) \)
17. \( y^2 - x \)

18. Graph the integers \(-18, 4, -20, -2, -6, 0\) on a number line. Then write the integers in order from least to greatest.

State the absolute value and the opposite of the number.
19. \(-24\)
20. \(8\)
21. \(31\)
22. \(-17\)
23. Evaluate the expression \( 44 - |x| \) when \( x = -10 \).

---

**Brain Game**

Use each of the digits 1, 2, 3, 4, and 5 exactly once in each statement to make a true statement.

\[
(?) + (?) - (?) \times (?) \div (?) = 1
\]

\[
(?) + (?) \times (?) \div (?) - (?) = 0
\]

\[
(?) \div (?) + (?) - (?) \times (?) = 8
\]
1.5 Adding Integers on a Number Line

**Goal**
Add integers on a number line.

**Materials**
- paper
- pencil

### Investigate

Use a number line to find the sum of two integers.

1. **Add \(-3 + 7\).**

   Draw a number line. Place a pencil at 0 and move 3 units to the left to reach \(-3\). Then move 7 units to the right to show addition of 7. Find your final position on the number line.

   Copy and complete the statement:
   \[-3 + 7 = ?\]

2. **Add \(-1 + (-7)\).**

   Draw a number line. Place a pencil at 0 and move 1 unit to the left to reach \(-1\). Then move 7 units to the left to show addition of \(-7\). Find your final position on the number line.

   Copy and complete the statement:
   \[-1 + (-7) = ?\]

### Draw Conclusions

Use a number line to find the sum.

1. \(-6 + 13\)  
2. \(-5 + 10\)  
3. \(-8 + 4\)  
4. \(-1 + 6\)

5. \(10 + (-6)\)  
6. \(10 + (-12)\)  
7. \(9 + (-3)\)  
8. \(-9 + (-3)\)

9. \(10 + (-7)\)  
10. \(8 + (-11)\)  
11. \(-7 + (-8)\)  
12. \(4 + (-8)\)

13. **Writing** Suppose you are adding a positive integer and a negative integer. Explain how you can tell without actually adding whether the sum of the integers is positive, negative, or zero by considering the lengths of the arrows that represent the integers.

14. **Critical Thinking** Another way to add integers on a number line is to start at the first number in the sum and to move a distance and direction determined by the second integer. Identify the addition expression represented in the diagram and find the sum.
Adding Integers

**Vocabulary**
additive inverse, p. 30

**Before**
You added decimals. You’ll add integers.

**Why?**
So you can find a hockey player’s plus-minus rating, as in Ex. 40.

**Scuba Diver** A scuba diver studying marine life is 4 feet below sea level. From that depth, the diver descends 72 feet to the ocean floor and then rises 61 feet. The diver rests there to avoid decompression illness. Where is the diver relative to sea level? In Example 3, you will see how to answer this question by adding integers.

One way to add integers is to use a number line.

To add a positive integer, move to the right.

To add a negative integer, move to the left.

---

**Example 1**  Adding Integers Using a Number Line

**Use a number line to find the sum.**

**a.** $3 + (-9)$

Start at 3. Move 9 units to the left. End at -6.

**Answer** The final position is -6. So, $3 + (-9) = -6$.

**b.** $-5 + 3$

Start at -5. Move 3 units to the right. End at -2.

**Answer** The final position is -2. So, $-5 + 3 = -2$.

**Checkpoint**

Use a number line to find the sum.

1. $-11 + 6$
2. $-1 + (-8)$
3. $10 + (-5)$

Lesson 1.5  Adding Integers
Absolute Values You can use absolute values to find the sum of two or more integers.

### Adding Integers

<table>
<thead>
<tr>
<th>Words</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. Same Sign</strong> Add the absolute values and use the common sign.</td>
<td>$8 + 12 = 20$  $-6 + (-4) = -10$</td>
</tr>
<tr>
<td><strong>2. Different Signs</strong> Subtract the lesser absolute value from the greater absolute value and use the sign of the number with greater absolute value.</td>
<td>$5 + (-8) = -3$  $-11 + 13 = 2$</td>
</tr>
<tr>
<td><strong>3. Opposites</strong> The sum of a number and its opposite is 0.</td>
<td>$7 + (-7) = 0$</td>
</tr>
</tbody>
</table>

**Additive Inverse Property** The opposite of a number is also called its **additive inverse**. Item 3 in the notebook can be written algebraically as $a + (-a) = 0$ and is called the **additive inverse property**.

### Example 2 Adding Two Integers

**a.** Find the sum $-54 + (-28)$.

\[
-54 + (-28) = -82
\]

**Same sign:** Add $|-54|$ and $|-28|$.

**Both integers are negative, so the sum is negative.**

**b.** Find the sum $38 + (-17)$.

\[
38 + (-17) = 21
\]

**Different signs:** Subtract $|-17|$ from $|38|$.

\[
|38| > |-17|, \text{ so the sum has the same sign as } 38.
\]

### Example 3 Adding More Than Two Integers

To answer the question about the position of the scuba diver at the top of page 29, you can find the sum $-4 + (-72) + 61$.

\[
-4 + (-72) + 61 = -76 + 61 \quad \text{Add } -4 \text{ and } -72.
\]

\[
= -15 \quad \text{Add } -76 \text{ and } 61.
\]

**Answer** The sum is $-15$, so the diver is 15 feet below sea level.

**Checkpoint**

Find the sum.

4. $-41 + 26$
5. $-19 + (-11)$
6. $52 + (-30) + (-46)$
Example 4  Evaluating Variable Expressions

Evaluate the expression when \( x = -22 \) and \( y = -12 \).

a. \( x + (-9) \)
   
   Solution
   
   \[ x + (-9) = -22 + (-9) \]
   
   Substitute \( -22 \) for \( x \).
   
   \[ = -31 \]
   
   Add.

b. \( x + 17 + y \)
   
   Solution
   
   \[ x + 17 + y = -22 + 17 + (-12) \]
   
   Substitute for \( x \) and for \( y \).
   
   \[ = -5 + (-12) \]
   
   Add \(-22\) and \(17\).
   
   \[ = -17 \]
   
   Add \(-5\) and \(-12\).

Checkpoint

Evaluate the expression when \( a = -18 \) and \( b = -3 \).

7. \( a + (-8) \)  
8. \( 32 + a \)  
9. \( a + b + 30 \)

Guided Practice

1. Copy and complete: To add two integers without using a number line, you need to use the _ of each number.

2. How can you tell whether the sum of \(-71\) and \(43\) is positive or negative without actually finding the sum?

Skill Check

Use a number line to find the sum.

3. \( -9 + 11 \)  
4. \( -2 + (-13) \)  
5. \( 15 + (-7) \)

Find the sum.

6. \( 24 + (-16) \)  
7. \( -15 + 3 \)  
8. \( -11 + (-2) \)

Evaluate the expression when \( x = -9 \).

9. \( x + 3 \)  
10. \( -6 + x \)  
11. \( x + (-3) \)

12. Food Science  Food scientists tested the effects that freezing and thawing have on the texture of a cheese filling for ravioli. The filling was frozen to a temperature of \(-18\)\(^\circ\)C. The temperature was then raised \(108\)\(^\circ\)C. What was the final temperature of the filling?

13. Error Analysis

Describe and correct the error in using a number line to find the sum of \(-2\) and \(5\).
14. Match the correct sum with the addition shown on the number line.

A. \(-6 + 8\)  
B. \(6 + (-8)\)  
C. \(-6 + (-8)\)

Use a number line to find the sum.

15. \(1 + (-17)\)  
16. \(-4 + 13\)  
17. \(-7 + (-3)\)

18. \(13 + (-3)\)  
19. \(-9 + (-5)\)  
20. \(-6 + (-7)\)

21. \(8 + (-2)\)  
22. \(-3 + 6\)  
23. \(-5 + (-4)\)

Find the sum.

24. \(-54 + 40\)  
25. \(-20 + (-32)\)  
26. \(66 + (-16)\)

27. \(19 + (-45)\)  
28. \(-32 + 17\)  
29. \(-72 + (-30)\)

30. \(7 + (-9) + 15\)  
31. \(-40 + 33 + 12\)  
32. \(55 + (-28) + (-6)\)

Evaluate the expression when \(x = -8, y = 4,\) and \(z = -5.\)

33. \(x + 15\)  
34. \(y + (-75)\)  
35. \(-19 + z\)

36. \(x + y\)  
37. \(x + z\)  
38. \(y + z\)

39. Critical Thinking  Use a single arrow on a number line to represent the sum \(-4 + (-10).\) What does the length of the arrow indicate? What does the direction of the arrow indicate?

40. Hockey  In the National Hockey League, a player is assigned a positive point each time his team scores while he is on the ice. He is assigned a negative point each time the opposing team scores while he is on the ice. (No points are assigned if the scoring team has more players on the ice than the other team.) The sum of the positive and negative points is called the player’s plus-minus rating. The table shows the points awarded to a player in two games.

<table>
<thead>
<tr>
<th>Game</th>
<th>Positive points</th>
<th>Negative points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>-5</td>
</tr>
</tbody>
</table>

a. Find the player’s plus-minus rating for game 1.
b. Find the player’s plus-minus rating for game 2.
c. Find the total plus-minus rating for the two games.
d. Interpret and Apply  Did the player have a better plus-minus rating in game 1 or in game 2? Explain.

41. Overdraft  Your checking account shows an overdraft, or a negative balance. Your present balance is \(-\$25.\) You deposit \$100, then write a check for \$12. What is your new balance?

Use a calculator to find the sum. Estimate the sum by rounding to check that your answer is reasonable.

42. \(-345 + (-978)\)  
43. \(2172 + (-4087)\)  
44. \(-1117 + 539\)
45. **Lake Vostok** Lake Vostok, an unfrozen lake buried under Antarctic ice, is about 1200 meters deep. Scientists drilled down 3623 meters into the ice to test for signs of life, but stopped 120 meters above the top of the lake to avoid contaminating it.

a. Find the position of the top of the lake relative to the ice surface.

b. Find the position of the bottom of the lake relative to the ice surface.

46. **Writing** Write three different pairs of integers, each of which has a sum of \(-24\). Explain how you chose the integers.

**Find the sum.**

47. \(-35 + 16 + (-12) + 7\)  
48. \(-2 + 10 + (-3) + 5\)

49. \(90 + (-24) + (-6) + 5\)  
50. \(-9 + 16 + (-12) + 3\)

**Evaluate the expression when** \(a = -14\), \(b = 5\), and \(c = -8\).

51. \(a + b + c\)  
52. \(-15 + b + c\)  
53. \(8 + a + (-4) + c\)

**Critical Thinking** Using the given information and the fact that \(x\) and \(y\) are integers, tell whether the sum \(x + y\) is even or odd. Explain your reasoning.

54. \(x\) and \(y\) are even.  
55. \(x\) and \(y\) are odd.  
56. \(x\) is even; \(y\) is odd.

57. For what values of \(x\) is \(|x| + x = 0\)? Explain.

58. Absolute value bars are grouping symbols. Use the order of operations to evaluate the expression \(-3 + | -x + 2 |\) when \(x = 12\).

**Challenge** Find values of \(a\) and \(b\) for which the statement is true.

59. \(|a + b| = |a| + |b|\)  
60. \(|a + b| < |a| + |b|\)

**Mixed Review**

Write a variable expression to represent the phrase. (Lesson 1.1)

61. The sum of a number and 14.5

62. The difference of a number and 2.75

63. **Area** You are building a house on a square-shaped lot. The side length of the lot is 70 yards. Find the area of the lot. (Lesson 1.2)

Copy and complete the statement using \(<\), \(>\), or \(=\). (Lesson 1.4)

64. \(|15| \_ 15\)  
65. \(-12 \_ |12|\)  
66. \(|-2| \_ -2\)

**Standardized Test Practice**

67. **Multiple Choice** What is the value of the expression \(x + |y|\) when \(x = -3\) and \(y = 12\)?

A. \(-15\)  
B. \(-9\)  
C. 9  
D. 15

68. **Short Response** For what integer value(s) of \(x\) is the value of the expression \(-10 + |x|\) greater than 0? Explain your reasoning.
Subtracting Integers

Before
You subtracted decimals.

Now
You’ll subtract integers.

Why?
So you can find the difference in road elevations, as in Ex. 11.

Earth Science  Kick-’em-Jenny is an underwater volcano in the Caribbean Sea.

Eruptions have caused the volcano to grow. In 1962, the summit elevation of Kick-’em-Jenny was –235 meters. In 2002, the summit elevation was –182 meters. By how many meters did the elevation of the volcano change? Example 3 uses integer subtraction to answer this question.

As you can see from the number lines below, the expressions 5 – 4 and 5 + (–4) have the same value, 1.

These equivalent expressions suggest the following rule for subtracting integers.

Subtracting Integers

Words To subtract an integer, add its opposite.

Numbers 3 – 7 = 3 + (–7) = –4  Algebra a – b = a + (–b)

Example 1  Subtracting Integers

a. 4 – 10 = 4 + (–10)  To subtract 10, add its opposite, –10.
   = –6  Add 4 and –10.

b. 7 – (–5) = 7 + 5  To subtract –5, add its opposite, 5.
   = 12  Add 7 and 5.

c. –2 – (–9) = –2 + 9  To subtract –9, add its opposite, 9.
   = 7  Add –2 and 9.
Example 2  Evaluating Variable Expressions

Evaluate the expression when \( x = -9 \).

a. \( x - (-40) \)

b. \( 7 - x \)

Solution

a. \( x - (-40) = -9 - (-40) \) Substitute \(-9\) for \( x \).

\[ = -9 + 40 \] To subtract \(-40\), add \(40\).

\[ = 31 \] Add \(-9\) and \(40\).

b. \( 7 - x = 7 - (-9) \) Substitute \(-9\) for \( x \).

\[ = 7 + 9 \] To subtract \(-9\), add \(9\).

\[ = 16 \] Add \(7\) and \(9\).

Checkpoint

Find the difference.

1. \(2 - 6\)  
2. \(3 - (-8)\)  
3. \(-7 - 4\)  
4. \(-1 - (-13)\)

Evaluate the expression when \( y = -14 \).

5. \(y - 3\)  
6. \(25 - y\)  
7. \(y - 10\)  
8. \(-9 - y\)

Evaluating Change  You can use subtraction to find the change in a variable quantity such as elevation or temperature. Subtract the original value of the quantity from the value after the change.

Example 3  Evaluating Change

To answer the question on page 34 about the change in elevation of the volcano Kick-'em-Jenny, you can subtract the elevation in 1962 from the elevation in 2002. Write a verbal model.

\[
\text{Change in elevation} = \text{Elevation in 2002} - \text{Elevation in 1962}
\]

\[
= -182 - (-235) \quad \text{Substitute values.}
\]

\[
= -182 + 235 \quad \text{To subtract \(-235\), add \(235\).}
\]

\[
= 53 \quad \text{Add \(-182\) and \(235\).}
\]

Answer  The difference is 53, so the summit elevation of Kick-'em-Jenny increased by 53 meters from 1962 to 2002.

Checkpoint

Find the change in temperature.

9. From \(32^\circ\text{F}\) to \(-10^\circ\text{F}\)  
10. From \(-45^\circ\text{F}\) to \(-80^\circ\text{F}\)

11. From \(8^\circ\text{C}\) to \(-3^\circ\text{C}\)  
12. From \(-2^\circ\text{C}\) to \(15^\circ\text{C}\)
Guided Practice

**Vocabulary Check**

1. Write the phrase as a variable expression: the difference of $-15$ and a number $x$.

2. Explain how you would find the difference of $-45$ and $-60$.

**Skill Check**

Find the difference.

3. $3 - 8$

4. $6 - (-2)$

5. $-9 - 4$

6. $-5 - (-1)$

Evaluate the expression when $m = -6$.

7. $m - 4$

8. $m - 16$

9. $7 - m$

10. $-7 - m$

**Guided Problem Solving**

11. **The Big Dig** Boston's Central Artery Project, called “The Big Dig,” is one of the most complex highway projects in American history. The project includes an underground highway and a tunnel. The lowest point of the highway is 110 feet below sea level. The lowest point of the tunnel is 90 feet below sea level. What is the difference in these two elevations?

   1. Write an integer to represent the elevation of the lowest point of the highway.
   2. Write an integer to represent the elevation of the lowest point of the tunnel.
   3. Find the difference of the elevations in Steps 1 and 2.

Practice and Problem Solving

Find the difference.

12. $8 - 9$

13. $1 - (-8)$

14. $-10 - 6$

15. $-5 - (-17)$

16. $0 - 15$

17. $2 - (-37)$

18. $-20 - 4$

19. $-1 - (-53)$

20. $24 - 41$

21. $-39 - 32$

22. $79 - (-98)$

23. $-86 - (-34)$

Evaluate the expression when $m = -6$.

24. $17 - m$

25. $4 - m$

26. $m - 7$

27. $-16 - m$

28. $m - 19$

29. $m - 3 - 10$

30. $20 - m - 5$

31. $14 - 30 - m$

32. **Error Analysis** Describe and correct the error in finding the difference of $-2$ and $-5$.

$$-2 - (-5) = -2 + (-5)$$

$$= -7$$

33. **Temperatures** The most extreme temperature change in Canadian history occurred when the temperature in Pincher Creek, Alberta, rose from $-19^\circ C$ to $22^\circ C$ in one hour. Find the change in temperature.
34. **Extended Problem Solving** There are four stages in the production of ice cream. First, the mix is pasteurized to destroy bacteria. Next, the temperature of the mix is lowered for aging. Flavors are added and the temperature is lowered even more to harden the ice cream. Finally, the ice cream is stored in a freezer. The graph shows the temperature at each stage.

a. **Calculate** Find the change in temperature between each two consecutive stages. Then find the absolute value of each change.

b. **Compare** Between which two consecutive stages is the absolute value of the temperature change the greatest?

c. **Estimate** You can use the formula \( C = \frac{5(F - 32)}{9} \) to convert a temperature \( F \) in degrees Fahrenheit to a temperature \( C \) in degrees Celsius. Suppose the temperature in your mouth is about 99°F. Use mental math to estimate the temperature in your mouth in degrees Celsius. About how much greater is the temperature in your mouth than the temperature of ice cream just out of a freezer?

**Find the change in temperature or elevation.**

35. From \(-15^\circ C\) to \(10^\circ C\)  
36. From \(-5^\circ F\) to \(-13^\circ F\)

37. From \(-120\) feet to \(-90\) feet  
38. From 30 meters to \(-70\) meters

**Find the value of the expression.**

39. \(-15 - 75 - 100\)  
40. \(-402 + 74 - 281\)

41. \(-10 - (-525) - 280\)  
42. \(118 - (-2) - 315\)

**Evaluate the expression when** \(x = -5\), \(y = 14\), and \(z = -7\).

43. \(-3 - y - x\)  
44. \(y - (-9) - z\)  
45. \(z - y - x\)  
46. \(x - y - z\)

47. **Chemistry** Ethylene glycol is a chemical that can be added to water to lower its freezing point, the temperature at which it freezes. The freezing point of solution A, which is one part ethylene glycol and three parts water, is \(-12^\circ C\). The freezing point of solution B, which is two parts ethylene glycol and two parts water, is \(-36^\circ C\). Which solution has a lower freezing point? How much lower is it?

48. **Avalanches** An avalanche may occur when the temperature keeps snow crystals from sticking together. The room temperature in an avalanche research lab is \(-30^\circ C\). Scientists study changes in snow crystals by melting snow on a hot plate that heats to only \(-1^\circ C\). In a regular lab, the room temperature is about \(18^\circ C\) and a hot plate heats to about \(300^\circ C\). How many degrees warmer is the hot plate than the room temperature in each lab? Which difference is greater? How much greater?
49. **Critical Thinking** Let $a$ and $b$ be integers. Is the value of the expression $a + b$ always greater than the value of the expression $a - b$? Explain.

**Evaluate the expression** $3 - (-x) + 8 - 10$ **for the given value of** $x$.

50. 18  
51. 5  
52. $-2$  
53. $-3$  

54. **Challenge** If $a$ is a negative integer and $b$ is a positive integer, tell whether the expression represents a *positive* or a *negative* integer. Explain your thinking.

a. $a - b$  
b. $b - a$  
c. $|a| + |b|$  
d. $-|a| - |b|$  

**Mixed Review**

55. **Work Backward** You want to arrive at school at 7:45 A.M. It takes you half an hour to shower and get dressed, 15 minutes to eat breakfast, and 20 minutes to walk to school. What is the latest you can get up and still arrive at school on time? *p. 801*

**Evaluate the expression when** $x = 6$ **and** $y = 12$. *(Lesson 1.3)*

56. $5x - y$  
57. $3x + y$  
58. $3(x + y)$  

59. $\frac{x + y}{3}$  
60. $x + \frac{y}{3}$  
61. $7x - (y + 1)$

**Find the sum.** *(Lesson 1.5)*

62. $89 + (-14)$  
63. $-104 + 53$  
64. $-67 + (-303)$

**Standardized Test Practice**

65. **Multiple Choice** Which expression has a value closest to 0?

A. $23 - 25$  
B. $23 - (-22)$  
C. $-23 - 23$  
D. $23 - (-25)$

66. **Multiple Choice** The top of a cliff overlooking the ocean is 1250 feet above sea level. The sea floor at the foot of the cliff is 40 feet below sea level. A rock falls off the cliff and drops to the sea floor. Which expression represents the change in elevation of the rock?

F. $-40 - 1250$  
G. $40 - 1250$  
H. $1250 - 40$  
I. $1250 - (-40)$

---

**-10 and Counting**

In the expression $-1 + (-2) + (-3) + (-4)$, each of the integers $-1$, $-2$, $-3$, and $-4$ appears exactly once. The value of the expression is $-10$.

Use each of the integers $-1$, $-2$, $-3$, and $-4$ exactly once to write an expression that involves addition or subtraction or both and has a value of $-8$. You may use grouping symbols as needed.

Use the same rules to write four more expressions with values of $-6$, $-4$, $-2$, and 0.
Mean, Median, Mode, and Range

**Mean**

*Data* are numbers or facts. The *mean* of a data set is the sum of the values divided by the number of values. The mean is a *measure of central tendency*, that is, an average.

**Example** The numbers of trails at ten Colorado ski resorts are listed below. Find the mean of the data.

76, 61, 112, 146, 65, 139, 43, 125, 85, 28

First, add the ten values. Then divide by 10, the number of values.

Mean = \[\frac{76 + 61 + 112 + 146 + 65 + 139 + 43 + 125 + 85 + 28}{10}\]

= \[\frac{880}{10}\] = 88

**Answer** The mean of the data is 88.

**Median**

The *median* is another measure of central tendency. The *median* of a data set is the middle value when the values are written in numerical order. If a data set has an even number of values, the median is the mean of the two middle values.

**Example** The numbers of ski lifts at ten Colorado ski resorts are listed below. Find the median of the data.

8, 5, 4, 13, 2, 25, 7, 23, 14, 5

First, write the values in order from least to greatest.

2 4 5 5 7 8 13 14 23 25

The data set has an even number of values, so the median is the mean of the two middle values, 7 and 8.

Median = \[\frac{7 + 8}{2}\] = \[\frac{15}{2}\] = 7.5

**Answer** The median of the data is 7.5.

*Student Reference* Mean, Median, Mode, and Range
Mode

The mode is another measure of central tendency. The mode of a data set is the value that occurs most often. A data set can have no mode, one mode, or more than one mode.

Example  Ten Colorado ski resorts have scheduled openings in the following months. Find the mode of the data.

November, November, December, November, October, October, December, November, November, December

The month that occurs most often is November.

Answer  The mode of the data is November.

Range

The range of a data set is a measure of dispersion, that is, an indicator of how spread out the data are. The range of a data set is the difference of the greatest value and the least value.

Example  The average annual snowfall amounts (in inches) for ten Colorado ski resorts are listed below. Find the range of the data.

300, 367, 300, 307, 410, 300, 200, 280, 300, 240

Range = Greatest value − Least value = 410 − 200 = 210

Answer  The range of the data is 210 inches.

 Cheat

Test your knowledge of mean, median, mode, and range by solving these problems.

Find the mean, median, mode(s), and range of the data.

1. 85, 96, 72, 88, 95, 80, 86  
2. 7, 11, 13, 9, 7, 8, 9, 12  
3. 0.8, 0.5, 0.5, 0.7, 0.3, 0.9, 0.5  
4. 90, 112, 105, 118, 96, 128, 110, 133  

5. Birds  The table shows the numbers of bird species observed in five national parks. Find the mean, median, mode(s), and range of the data.

<table>
<thead>
<tr>
<th>National Park</th>
<th>Species</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joshua Tree</td>
<td>239</td>
</tr>
<tr>
<td>Mesa Verde</td>
<td>216</td>
</tr>
<tr>
<td>North Cascades</td>
<td>178</td>
</tr>
<tr>
<td>Yosemite</td>
<td>147</td>
</tr>
<tr>
<td>Nez Perce</td>
<td>135</td>
</tr>
</tbody>
</table>
1.7 Multiplying Integers

Investigate

Use patterns to multiply integers.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>3(3)</td>
<td>9</td>
</tr>
<tr>
<td>3(2)</td>
<td>6</td>
</tr>
<tr>
<td>3(1)</td>
<td>3</td>
</tr>
<tr>
<td>3(0)</td>
<td>?</td>
</tr>
<tr>
<td>3(−1)</td>
<td>?</td>
</tr>
<tr>
<td>3(−2)</td>
<td>?</td>
</tr>
</tbody>
</table>

Identify a pattern in the second column of the table. Copy the table and use the pattern to complete the table. Then copy and complete the following statement:

The product of a positive integer and a negative integer is _?_.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>2(−3)</td>
<td>?</td>
</tr>
<tr>
<td>1(−3)</td>
<td>?</td>
</tr>
<tr>
<td>0(−3)</td>
<td>?</td>
</tr>
<tr>
<td>−1(−3)</td>
<td>?</td>
</tr>
<tr>
<td>−2(−3)</td>
<td>?</td>
</tr>
<tr>
<td>−3(−3)</td>
<td>?</td>
</tr>
</tbody>
</table>

Copy the table. Apply your results from Step 1 to complete rows 1–3. Identify a pattern in the second column. Use that pattern to complete the table. Then copy and complete the following statement:

The product of two negative integers is _?_.

Draw Conclusions

Find the product.

1. 3(−3)  
2. 3(−4)  
3. −3(5)  
4. −5(6)  
5. 10(−2)  
6. 4(−7)  
7. −8(−5)  
8. −3(−12)

9. When one factor in a product is a positive integer, you can think of multiplication as repeated addition. For example, the product 3(−1) is equal to the sum −1 + (−1) + (−1). Use this idea to justify the statements 4(−2) = −8 and −3(5) = −15.

10. Critical Thinking Suppose that a and b are positive integers.
Exercise 9 showed that if you multiply a by the opposite of b, or b by the opposite of a, the result is the opposite of ab. That is, if a and b are positive integers, then (−a)b = −ab and a(−b) = −ab. Suppose you apply this rule to the product (−a)(−b) twice:

(−a)(−b) = −[(−a)b] = −(−ab).

What is the opposite of the opposite of ab? Use your answer to copy and complete this statement: (−a)(−b) = _?_.

Lesson 1.7 Multiplying and Dividing Integers
Multiplying and Dividing Integers

**Stock Market** You own shares of stock in a computer company and in a utility. The value of the shares changes over time. The table shows the number of shares of each type of stock you own and the change in the value of each share over a one-year period. What was the total change in the value of your shares of stock? In Example 2, you will see how to multiply integers to answer this question.

<table>
<thead>
<tr>
<th>Stock Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stock</strong></td>
</tr>
<tr>
<td>Computer</td>
</tr>
<tr>
<td>Utility</td>
</tr>
</tbody>
</table>

In the activity on page 41, you may have recognized patterns in the products of integers. These patterns suggest the following rules.

**Multiplying Integers**

<table>
<thead>
<tr>
<th>Words</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>The product of two integers with the same sign is positive.</td>
<td>$2(4) = 8$</td>
</tr>
<tr>
<td>The product of two integers with different signs is negative.</td>
<td>$2(-4) = -8$</td>
</tr>
<tr>
<td>The product of any integer and 0 is 0.</td>
<td>$2(0) = 0$</td>
</tr>
<tr>
<td>$-2(-4) = 8$</td>
<td>$-2(4) = -8$</td>
</tr>
<tr>
<td>$-2(0) = 0$</td>
<td>$2(0) = 0$</td>
</tr>
</tbody>
</table>

**Example 1**

- **a.** $-3(-12) = 36$  
  *Same sign: Product is positive.*
- **b.** $-7(9) = -63$  
  *Different signs: Product is negative.*
- **c.** $-24(0) = 0$  
  *The product of any integer and 0 is 0.*
### Example 2 \textit{Multiplying Integers}

To find the total change in the value of the shares of stock described on page 42, first multiply the number of shares of each type of stock by the change in the price of each share. Then add the results.

$$
\begin{align*}
\text{Total change} & = \text{Computer shares} \cdot \text{Change in 1 share} + \text{Utility shares} \cdot \text{Change in 1 share} \\
& = 200(-3) + 150(2) \quad \text{Substitute values.} \\
& = -600 + 300 \quad \text{Multiply.} \\
& = -300 \quad \text{Add.}
\end{align*}
$$

**Answer** The total change in value was $-300$. The value of the stocks decreased by $300$.

### Dividing Integers

Because $3(-4) = -12$, you know that $-12 \div 3 = -4$ and $-12 \div (-4) = 3$. This relationship between products and quotients suggests that the rules for dividing integers are like the rules for multiplying integers.

<table>
<thead>
<tr>
<th>Words</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>The quotient of two integers with the same sign is positive.</td>
<td>$8 \div 4 = 2$ \hspace{1cm} $-8 \div (-4) = 2$</td>
</tr>
<tr>
<td>The quotient of two integers with different signs is negative.</td>
<td>$-8 \div 4 = -2$ \hspace{1cm} $8 \div (-4) = -2$</td>
</tr>
<tr>
<td>The quotient of 0 and any nonzero integer is 0.</td>
<td>$0 \div 4 = 0$ \hspace{1cm} $0 \div (-4) = 0$</td>
</tr>
</tbody>
</table>

### Example 3 \textit{Dividing Integers}

\begin{align*}
\text{a. } & -48 \div (-6) = 8 \quad \text{Same sign: Quotient is positive.} \\
\text{b. } & 56 \div (-8) = -7 \quad \text{Different signs: Quotient is negative.} \\
\text{c. } & 0 \div 9 = 0 \quad \text{The quotient of 0 and any nonzero integer is 0.}
\end{align*}

### Checkpoint

Find the product or quotient.

1. $9(-11)$
2. $-6(-8)$
3. $0(-100)$
4. $-4(-8)$
5. $-24 \div 3$
6. $0 \div (-25)$
7. $-35 \div (-7)$
8. $24 \div (-6)$
Example 4  Finding a Mean

Antarctic Temperatures  The table shows record low monthly temperatures from June to November at McMurdo Station in Antarctica. Find the mean of the temperatures.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°F)</td>
<td>−42</td>
<td>−59</td>
<td>−57</td>
<td>−47</td>
<td>−40</td>
<td>−19</td>
</tr>
</tbody>
</table>

Solution

To find the mean of the temperatures, first add the temperatures. Then divide by 6, the number of temperatures.

\[
\text{Mean} = \frac{-42 + (-59) + (-57) + (-47) + (-40) + (-19)}{6} = \frac{-264}{6} = -44
\]

Answer  The mean of the temperatures is −44°F.

1.7 Exercises

Guided Practice

Vocabulary Check

1. Explain what the mean of a data set is.

2. If \(a\) and \(b\) are integers and the expression \(ab\) is positive, what do you know about the signs of \(a\) and \(b\)?

Skill Check  Tell whether the product or quotient is positive or negative.

3. \(-238(-17)\)  4. \(\frac{920}{-23}\)  5. \(465(-147)\)  6. \(-\frac{256}{32}\)

7. \(-1209 \div 31\)  8. \(-65(219)\)  9. \(-98 \div (-2)\)  10. \(-99(-716)\)

Guided Problem Solving  Electronics  An electronic device is tested to determine how it reacts to changes in temperature. The device is placed in a test chamber at 22°C. After each minute, the temperature in the chamber is lowered 3°C. What is the temperature in the chamber after 9 minutes?

1. Write an integer that represents the change in temperature in the chamber in one minute.

2. Write a product of integers that represents the total change in temperature in 9 minutes. Then evaluate the product.

3. Find the temperature in the chamber after 9 minutes.
Practice and Problem Solving

Find the product or quotient.

12. 12(5)  
13. 28 ÷ 14  
14. 65 ÷ (−5)  
15. 6(−22)  
16. −7(50)  
17. −26 ÷ 13  
18. −72 ÷ (−36)  
19. 12(−30)  
20. \[\frac{175}{−25} \]  
21. \[\frac{−51}{−3} \]  
22. −17(−20)  
23. \[\frac{−840}{7} \]

24. **Error Analysis** Describe and correct the error in multiplying −5 and −12, then dividing by −4.

\[\frac{−5(−12)}{−4} = \frac{−60}{−4} = 15\]

25. **Compare and Contrast** Tell how the rules for multiplying and dividing integers are alike and how they are different.

26. **Critical Thinking** The table below gives expressions involving the multiplication of integers.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Number of integers</th>
<th>Product</th>
<th>Sign of product</th>
</tr>
</thead>
<tbody>
<tr>
<td>−1(−2)</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>−1(−2)(−3)</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>−1(−2)(−3)(−4)</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

a. Copy and complete the table.

b. **Writing** Write a rule for the sign of the product of more than two negative integers.

c. **Number Sense** Suppose that in part (b) the product included positive integer factors as well. Would your rule change? Explain.

27. **MIR Submersible** A MIR submersible is a type of submarine. As a MIR dives, its elevation changes by −100 feet each minute.

a. From the surface, a MIR takes about 200 minutes to reach the lowest point to which it can dive. What is its elevation at that point?

b. How long would a MIR take to dive to 1000 feet below sea level?

28. **Free Diving** Free diving means diving without breathing equipment. The graph shows the position with respect to sea level for five record free dives.

a. Find the mean of the positions.

b. Find the median of the positions.

c. **Compare** Does the mean or the median represent a lower position?
Simplify.
29. \(-5(-10)(-25)\) 
30. \(16(-4)(-8)\)
31. \(360 \div (-36) \div (-2)\) 
32. \(-72 \div 12 \div 3\)
33. \(-2(-14) \div (-7)\) 
34. \(20(-45) \div (-9)\)

**Number Sense** Without performing the indicated divisions, copy and complete the statement using >, <, or =.
35. \(-738 \div 82 \_ \_\_\_ -192 \div (-32)\) 
36. \(288 \div (-36) \_ \_\_\_ 756 \div 18\)
37. **Sports** A batter hits a baseball. The ball’s height \(h\) (in feet) above the ground \(t\) seconds after it is hit is given by the equation \(h = -16t^2 + 80t + 3\). Find the height of the ball 4 seconds after it is hit.

**In the Real World**

**Baseball** A baseball hit by a player in the major leagues can leave the bat at a speed of 110 miles per hour. Suppose a batter hits a ball at that speed directly back to the pitcher’s mound, about 61 feet from home plate. To the nearest tenth of a second, how long does it take the ball to reach the mound?

**In Exercises 38–41, evaluate the variable expression when \(x = -4\).**

**Example** Evaluating Variable Expressions

\[-7x^2 = -7(-4)^2\] 
Substitute \(-4\) for \(x\).

\[-7 = -7(16)\] 
Evaluate power.

\[-112\] 
Multiply.

**Answer** When \(x = -4\), \(-7x^2 = -112\).

38. \(-10x^2\) 
39. \(\frac{72}{x^2}\) 
40. \(-6x^2\) 
41. \(\frac{4x^2}{-10}\)

42. For what value of \(n\) is \(\frac{-4 + (-3) + 5 + 4 + (-3) + n}{-7}\) true?

43. **Explain** You know that for any positive integer \(n\), \(1^n = 1\). Is the statement \((-1)^n = -1\) true for any positive integer \(n\)? Explain.

44. **Challenge** Tell whether the statement is always, sometimes, or never true. Explain your answer.

a. If \(k\) is any integer and \(n\) is less than 0, then \(nk\) is less than \(n\).

b. If \(k\) is any integer and \(n\) is greater than 1, then \(nk\) is greater than \(n\).

**Mixed Review**

Write the integers in order from least to greatest. (Lesson 1.4)

45. \(-12, -21, 31, 0, -5, 13\) 
46. \(-45, -54, -22, -16, -70\)

Find the sum or difference. (Lessons 1.5, 1.6)

47. \(-27 + 51\) 
48. \(-17 + (-12)\) 
49. \(-18 - 33\) 
50. \(-41 - (-9)\)

**Standardized Test Practice**

51. **Multiple Choice** Which expression has a positive value?

A. \(\frac{-16(-5)}{4(-9)}\) 
B. \(-7^2 - 2\) 
C. \(5 - 4(-6)\) 
D. \(19 - 6(7)\)

52. **Short Response** Find the mean of these temperatures: \(-12^\circ F, 7^\circ F, -22^\circ F, -11^\circ F, 20^\circ F, -6^\circ F\). Describe the steps you used.
A **coordinate plane** is formed by the intersection of a horizontal number line called the **x-axis** and a vertical number line called the **y-axis**. The axes meet at a point called the **origin** and divide the coordinate plane into four **quadrants**.

Each point in a coordinate plane is represented by an **ordered pair**. The first number is the **x-coordinate**, and the second number is the **y-coordinate**.

**Note Worthy**

You may find it helpful to draw a coordinate plane in your notebook. Label the origin and the axes. Use colored arrows as in Example 1 to illustrate how to find the coordinates of a point.

**Example 1**  
**Naming Points in a Coordinate Plane**

Give the coordinates of the point.

a. \( A \)  
b. \( B \)

**Solution**

a. Point \( A \) is 2 units to the right of the origin and 4 units down. The \( x \)-coordinate is 2, and the \( y \)-coordinate is -4. The coordinates are \( (2, -4) \).

b. Point \( B \) is 3 units to the left of the origin and 2 units up. The \( x \)-coordinate is -3, and the \( y \)-coordinate is 2. The coordinates are \( (-3, 2) \).

**Checkpoint**

Use the coordinate plane in Example 1. Give the coordinates of the point.

1. \( C \)  
2. \( D \)  
3. \( E \)
Example 2  
Plotting Points in a Coordinate Plane

Plot the point in a coordinate plane. Describe the location of the point.

a. \( A(4, 1) \)  
b. \( B(0, -3) \)  
c. \( C(-2, -5) \)

Solution

a. Begin at the origin and move 4 units to the right, then 1 unit up. Point \( A \) is in Quadrant I.

b. Begin at the origin and move 3 units down. Point \( B \) is on the \( y \)-axis.

c. Begin at the origin and move 2 units to the left, then 5 units down. Point \( C \) is in Quadrant III.

Checkpoint

Plot the point in a coordinate plane. Describe the location of the point.

4. \( P(-1, 1) \)  
5. \( Q(4, -5) \)  
6. \( R(0, 0) \)  
7. \( S(-2, 0) \)

Scatter Plots  
A scatter plot uses a coordinate plane to display paired data. Each data pair is plotted as a point. A scatter plot may suggest whether a relationship exists between two sets of data.

Example 3  
Making a Scatter Plot

Fish  
A biologist measured the lengths and masses of eight rainbow trout. Make a scatter plot of the data shown in the table and describe any relationship you see.

<table>
<thead>
<tr>
<th>Length (millimeters)</th>
<th>405</th>
<th>360</th>
<th>413</th>
<th>395</th>
<th>247</th>
<th>280</th>
<th>265</th>
<th>351</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (grams)</td>
<td>715</td>
<td>557</td>
<td>754</td>
<td>584</td>
<td>184</td>
<td>248</td>
<td>223</td>
<td>506</td>
</tr>
</tbody>
</table>

Solution

1) Write the data as ordered pairs. Let the \( x \)-coordinate represent the length, and let the \( y \)-coordinate represent the mass: \((405, 715), (360, 557), (413, 754), (395, 584), (247, 184), (280, 248), (265, 223), (351, 506)\).

2) Plot the ordered pairs in a coordinate plane. You need only the first quadrant.

Notice that the points rise from left to right. You can conclude that as the lengths of the rainbow trout increase, their masses tend to increase.
**Guided Practice**

**Vocabulary Check**
1. What is the $x$-coordinate of the point $(-12, 7)$? What is the $y$-coordinate?
2. A point has one positive coordinate and one negative coordinate. Can you determine in which quadrant the point lies? Explain.

**Skill Check**
Plot the point in a coordinate plane. Describe the location of the point.
3. $J(2, 3)$
4. $K(-5, -1)$
5. $L(0, -3)$
6. $M(4, -4)$

**Guided Problem Solving**
7. **Earth Science** Scientists studying the Columbia River in Washington measured the speed of the water at one location in the river, but at different depths. The table shows the results. Is there any relationship between the depth of the water and its speed?

<table>
<thead>
<tr>
<th>Depth (inches)</th>
<th>8</th>
<th>24</th>
<th>31</th>
<th>71</th>
<th>88</th>
<th>103</th>
<th>119</th>
<th>127</th>
<th>134</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed (inches per second)</td>
<td>19</td>
<td>13</td>
<td>17</td>
<td>14</td>
<td>11</td>
<td>7</td>
<td>7</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

1. Write the data as ordered pairs. Let the $x$-coordinate represent depth, and let the $y$-coordinate represent speed.
2. Make a scatter plot of the data.
3. Does the scatter plot suggest any relationship between the depth of the water and its speed? Explain.

**Practice and Problem Solving**

**Give the coordinates of the point.**
8. $A$
9. $B$
10. $C$
11. $D$
12. $E$
13. $F$
14. $G$
15. $H$

16. **Writing** Explain how to plot the point $(6, -3)$ in a coordinate plane.

**Plot the point in a coordinate plane. Describe the location of the point.**
17. $P(5, 5)$
18. $Q(-1, 0)$
19. $R(8, -4)$
20. $S(2, -4)$
21. $T(-3, -6)$
22. $U(0, -5)$
23. $V(-4, -1)$
24. $W(6, -5)$
25. **Error Analysis**
Describe and correct the error in locating the point $(2, -8)$.

![Image of a hybrid car with a gas engine and electric motor]

In the **Real World**

**Fuel Economy** The hybrid car powered by the combined gas engine and electric motor shown has a fuel economy of 46 miles to the gallon in city driving and 51 miles to the gallon on the highway. Suppose you drive the hybrid car about 10,000 miles in the city each year. If you pay $1.75 for one gallon of gas, how much do you pay for gas in a year?

26. **Critical Thinking** How can you tell by looking at the coordinates of a point whether the point is on the $x$-axis or the $y$-axis?

27. **Fuel Economy** The table shows the engine sizes of several cars and the average highway mileage for each car.

<table>
<thead>
<tr>
<th>Engine size (liters)</th>
<th>3</th>
<th>6</th>
<th>2</th>
<th>4</th>
<th>1</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mileage (miles per gallon)</td>
<td>28</td>
<td>19</td>
<td>33</td>
<td>25</td>
<td>47</td>
<td>24</td>
<td>22</td>
</tr>
</tbody>
</table>

- **a.** Make a scatter plot of the data.
- **b. Interpret** Does the scatter plot suggest any relationship between the size of the engine in a car and the car’s average highway mileage? Explain.

28. **Geometry** Use a coordinate plane.

- **a.** Plot the points $(-3, -2)$, $(-3, 6)$, $(5, 6)$, and $(5, -2)$. Connect the points in order. Connect the last point to the first.
- **b.** Identify the figure. Explain your reasoning.

29. **Geometry** Use the variable expression $2x + 1$.

- **a.** Evaluate the expression when $x = -3$, $-2$, $-1$, $0$, $1$, $2$, and $3$.
- **b.** Use your results from part (a) to write a list of ordered pairs in the form $(x, 2x + 1)$.
- **c.** Plot the order pairs $(x, 2x + 1)$ from part (b) in a coordinate plane.
- **d. Writing** Describe what you notice about the points.

30. **Extended Problem Solving** In the game Go, each player begins with a supply of black or white stones. Each player in turn places a stone on a grid at the intersection of two grid lines. A player captures another player’s stone by surrounding it on four sides with his or her own stones. (Diagonals do not count.) For example, the diagram shows that a black stone has been captured. In the diagram, coordinate axes have been superimposed on part of a Go board.

- **a. Identify** Give the coordinates of each of the stones shown in the diagram.
- **b. Apply** A white stone is placed at $(-3, 2)$. Give the coordinates of the points that a player must cover with black stones to capture the white stone.
31. **Geometry**  Point $O$ is the origin of a coordinate plane. Choose points $P$, $Q$, and $R$ so that $O, P, Q,$ and $R$ are the corners of a square with a side length of 5 units. Identify the coordinates of $P, Q,$ and $R$. Explain your reasoning.

**Challenge**  The point $(a, b)$ is in Quadrant II of a coordinate plane. Describe the location of the point with the given coordinates.

32. $(b, a)$  
33. $(a, a)$  
34. $(b, b)$

**Mixed Review**

35. **Movie Tickets**  Let $c$ represent the cost in dollars of a ticket at the local movie theater. You use a $20$ bill to pay for two tickets. Write a variable expression for the amount of change you receive.  *(Lesson 1.1)*

Tell whether the sum is **always**, **sometimes**, or **never** negative.  *(Lesson 1.5)*

36. The sum of two negative integers

37. The sum of two positive integers

38. The sum of a negative integer and a positive integer

Find the **product** or **quotient**.  *(Lesson 1.7)*

39. $-15(3)$  
40. $-252 ÷ 12$  
41. $-63 ÷ (-3)$  
42. $9(-17)$

**Standardized Test Practice**

43. **Multiple Choice**  In which quadrant is the point $(-22, 35)$ located?
   
   A. Quadrant I  
   B. Quadrant II  
   C. Quadrant III  
   D. Quadrant IV

44. **Multiple Choice**  What are the coordinates of point $A$?
   
   F. $(-3, 1)$  
   G. $(1, -3)$  
   H. $(3, -1)$  
   I. $(-1, 3)$

**What Is It?**

Plot each pair of points on a coordinate grid and connect the two points to solve the riddle:

*What force and strength cannot get through, it with a gentle touch can do. And many in the street would stand, were it not a friend at hand. What is it?*

- $(-1, 3)$ and $(-1, -1)$
- $(3, 1)$ and $(4, 3)$
- $(-4, 1)$ and $(-2, -1)$
- $(-8, 3)$ and $(-6, -1)$
- $(-1, 3)$ and $(1, 3)$
- $(3, -1)$ and $(3, 1)$
- $(2, 3)$ and $(3, 1)$
- $(-1, 1)$ and $(1, 1)$
- $(-1, -1)$ and $(1, -1)$
- $(-4, 3)$ and $(-4, -1)$
- $(-9, 1)$ and $(-7, 1)$
- $(-4, 1)$ and $(-2, 3)$
- $(-10, -1)$ and $(-8, 3)$

Lesson 1.8  The Coordinate Plane
Chapter Review

Vocabulary Review

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- y-axis, p. 47
- origin, p. 47
- quadrant, p. 47
- ordered pair, p. 47
- x-coordinate, p. 47
- y-coordinate, p. 47
- scatter plot, p. 48

1. Draw a coordinate plane. Label the x-axis, the y-axis, the origin, and the quadrants.
2. Explain how these vocabulary terms are related: power, base, exponent.
3. Describe how to find the opposite of an integer.
4. What is a variable expression?

1.1 Expressions and Variables

Goal
Evaluate variable expressions.

Example
Evaluate the expression xy when x = 12 and y = 3.

\[ xy = 12 \cdot 3 \]
\[ = 36 \]

Substitute 12 for x and 3 for y. Multiply.

Evaluate the expression when p = 12 and q = 1.5.

5. \( 35 - p \)
6. \( q + 2 \)
7. \( \frac{60}{p} \)
8. \( 16q \)
9. \( p + q \)
10. \( p - q \)
11. \( \frac{p}{q} \)
12. \( pq \)

1.2 Powers and Exponents

Goal
Evaluate powers.

Example
Evaluate the power \((0.4)^2\).

\[ (0.4)^3 = 0.4 \cdot 0.4 \cdot 0.4 \]
\[ = 0.064 \]

Use 0.4 as a factor 3 times. Multiply.
1.3 Order of Operations

Goal
Use the order of operations to evaluate expressions.

Example
Evaluate the expression $800 - 7(2 + 3)^2$.

$$800 - 7(2 + 3)^2 = 800 - 7(5)^2$$
Add within parentheses.

$$= 800 - 7(25)$$
Evaluate power.

$$= 800 - 175$$
Multiply.

$$= 625$$
Subtract.

Evaluate the expression.

21. $20 \cdot 5 + 7 \cdot 3$
22. $\frac{5 + 4}{3} - 2$
23. $28 \div (5 - 1) \cdot 3$

1.4 Comparing and Ordering Integers

Goal
Compare and order integers.

Example
Graph the integers $-2, 3, 0, 2, -3$ on a number line. Then write the integers in order from least to greatest.

Write the integers from left to right: $-3, -2, 0, 2, 3$.

Example
State the absolute value and the opposite of $-2$.

The absolute value of $-2$ is 2.

The opposite of $-2$ is 2.

Example
Graph the integers on a number line. Then write the integers in order from least to greatest.

24. $4, 0, -3, 7, -6$
25. $2, -4, -3, 6, 5, -6$
26. $-8, -12, 4, -7, 1$

Example
State the absolute value and the opposite of the number.

27. $18$
28. $-9$
29. $4$
30. $-100$
1.5 Adding Integers

Goal
Add Integers.

Example
Find the sum.

a. \(-42 + (-17)\)

These integers have the same sign.

Add \(|-42| \) and \(|-17|\).

\[-42 + (-17) = 59\]

Both integers are negative, so the sum is negative.

b. \(-51 + 63\)

These integers have different signs.

Subtract \(|-51|\) from \(|63|\).

\[-51 + 63 = 12\]

\(|63| > |-51|\), so the sum is positive.

✓ Find the sum.

31. \(12 + (-18)\)  32. \(-8 + (-7)\)  33. \(-27 + 38\)  34. \(-11 + (-18)\)
35. \(61 + (-44)\)  36. \(-13 + (-21)\)  37. \(-21 + 9\)  38. \(-22 + (-7)\)
39. At 6:00 A.M., the temperature was \(-5^\circ\text{F}\). By 2:00 P.M., the temperature had risen \(22^\circ\text{F}\). What was the temperature at 2:00 P.M.?

1.6 Subtracting Integers

Goal
Subtract Integers.

Example
Find the difference.

a. \(7 - 15 = 7 + (-15)\)

To subtract 15, add its opposite, \(-15\).

\[-8\]

Add 7 and \(-15\).

b. \(-9 - (-11) = -9 + 11\)

To subtract \(-11\), add its opposite, \(11\).

\[2\]

Add \(-9\) and \(11\).

✓ Find the difference.

40. \(0 - 8\)  41. \(-2 - (-2)\)  42. \(-46 - 29\)  43. \(6 - (-13)\)
44. \(-15 - (-17)\)  45. \(31 - 40\)  46. \(-16 - 9\)  47. \(20 - (-11)\)
48. Find the difference of an elevation of 30 feet below sea level and an elevation of 118 feet above sea level.
1.7 Multiplying and Dividing Integers

Goal
Multiply and divide integers.

Example
Find the product or quotient.

a. \(-4(-15) = 60\)  
   Same sign: Product is positive.

b. \(-6(14) = -84\)  
   Different signs: Product is negative.

c. \(-42 \div (-7) = 6\)  
   Same sign: Quotient is positive.

d. \(20 \div (-5) = -4\)  
   Different signs: Quotient is negative.

Find the product or quotient.

49. \(-9(-12)\)  
50. \(52 \div (-4)\)  
51. \(-17(3)\)  
52. \(90 \div (-15)\)

53. \(-\frac{80}{16}\)  
54. \(20(-12)\)  
55. \(\frac{48}{16}\)  
56. \(-33(-3)\)

1.8 The Coordinate Plane

Goal
Identify and plot points in a coordinate plane.

Example
Give the coordinates of point \(P\).

Point \(P\) is 4 units to the left of the origin and 1 unit up. The \(x\)-coordinate is \(-4\), and the \(y\)-coordinate is 1. The coordinates of point \(P\) are \((-4, 1)\).

Use the coordinate plane shown in the example. Give the coordinates of the point.

57. \(Q\)  
58. \(R\)  
59. \(S\)  
60. \(T\)

Example
Plot the point \(A(1, -3)\) in a coordinate plane. Describe the location of the point.

Begin at the origin and move 1 unit to the right, then 3 units down. Point \(A\) is in Quadrant IV.

Plot the point in a coordinate plane. Describe the location of the point.

61. \(B(-2, 5)\)  
62. \(C(0, 4)\)  
63. \(D(-3, -1)\)  
64. \(E(4, -2)\)
Chapter Test

1. Evaluate the expression when \( y = 16 \) and \( z = 4 \).
   \[ \begin{align*}
   1. \ y + 9 & \quad 2. \ 11 - z & \quad 3. \ \frac{y}{z} & \quad 4. \ yz \\
   \end{align*} \]

2. Write the power in words and as a repeated multiplication. Then evaluate the power.
   \[ \begin{align*}
   5. \ 8^2 & \quad 6. \ 2^7 & \quad 7. \ (0.2)^5 & \quad 8. \ (0.7)^4 \\
   \end{align*} \]

3. **Sewing** You are making a beanbag footstool in the shape of a cube with an edge length of 50 centimeters. In order to fill the footstool with plastic beads, you need to know its volume. Find the volume of the footstool.

4. Evaluate the expression.
   \[ \begin{align*}
   10. \ 70.2 + 4(3.5) & \quad 11. \ \frac{75 - 39}{4 \cdot 3} & \quad 12. \ 90 \div 5 + 4 & \quad 13. \ 18 + 30 \div 6 \\
   \end{align*} \]

5. Evaluate the expression when \( r = 4 \) and \( s = 6 \).
   \[ \begin{align*}
   14. \ 3.5s + r & \quad 15. \ (r + 1)^2 - s & \quad 16. \ 4r + s^2 & \quad 17. \ 2(r^2 - 15) \\
   \end{align*} \]

6. State the absolute value and the opposite of the number.
   \[ \begin{align*}
   18. \ -78 & \quad 19. \ 121 & \quad 20. \ -33 & \quad 21. \ 19 \\
   \end{align*} \]

7. Find the sum or difference.
   \[ \begin{align*}
   22. \ 35 + (-11) & \quad 23. \ -28 + (-40) & \quad 24. \ -38 + (-8) & \quad 25. \ 43 + (-22) \\
   26. \ 5 - (-16) & \quad 27. \ -60 - 7 & \quad 28. \ -19 - 35 & \quad 29. \ -48 - (-72) \\
   \end{align*} \]

8. Find the product or quotient.
   \[ \begin{align*}
   30. \ -20(32) & \quad 31. \ \frac{-76}{4} & \quad 32. \ -25(-30) & \quad 33. \ 840 \div (-24) \\
   34. \ 18(-4) & \quad 35. \ 700 \div (-35) & \quad 36. \ -12(-16) & \quad 37. \ \frac{-270}{-18} \\
   \end{align*} \]

9. **Investments** The integers below represent the monthly gains and losses in the value of an investment over one year. Find the mean of the integers.
   \[ -$190, $75, -$65, $100, $72, -$54, -$62, -$87, $92, $81, -$73, $63 \]

10. **Geometry** Plot the points listed below in the same coordinate plane. Describe any pattern you see in the graph.
    \[ (-3, -6), (-2, -5), (-1, -4), (0, -3), (1, -2), (2, -1) \]
1. What is the value of the expression \( \frac{24}{x} \) when \( x = 8 \)?
   A. 3    B. 16    C. 32    D. 192

2. Which variable expression represents the following phrase?
   A number raised to the ninth power
   F. \( 9n \)    G. \( 9^n \)    H. \( n^9 \)    I. \( \frac{9}{n} \)

3. What is the first step in evaluating the expression \( 15 - 12 \div 3 \)?
   A. Subtract 12 from 15.
   B. Divide 12 by 3.
   C. Subtract 3 from 15.
   D. Divide 15 by 3.

4. What is the value of the expression \( x + y^2 \) when \( x = 3 \) and \( y = 5 \)?
   F. 64    G. 34    H. 28    I. 13

5. Which expression has a value of \(-4\)?
   A. \(-(-4)\)    B. \(-4\)    C. \(4\)    D. \(-|4|\)

6. Which list of integers is in order from least to greatest?
   F. \(-4, -7, 0, 2\)    G. \(0, 2, -4, -7\)
   H. \(-7, -4, 0, 2\)    I. \(2, 0, -7, -4\)

7. What is the value of the expression \(-11 + 24 + (-32)\)?
   A. \(-67\)    B. \(-19\)    C. \(19\)    D. \(67\)

8. Find the difference \(-32 - (-15)\).
   F. \(-47\)    G. \(-17\)    H. \(17\)    I. \(47\)

9. Which expression has a value of 12?
   A. \(\frac{-144}{-12}\)    B. \(\frac{36}{-3}\)
   C. \(2(-6)\)    D. \(-3(4)\)

10. In a coordinate plane, point \(P\) is 8 units to the left of the origin and 6 units up. What are the coordinates of point \(P\)?
    F. \((6, -8)\)    G. \((-6, 8)\)
    H. \((8, -6)\)    I. \((-8, 6)\)

11. **Short Response** The integers \(-6, 12, -2,\) and \(-16\) represent yards gained or lost by a football team on 4 plays. Describe the steps you would use to find the mean of the integers. Then find the mean.

12. **Extended Response** The table lists the thicknesses of the trunks of 6 loblolly pine trees and their heights.
    a. Make a scatter plot of the data.
    b. Describe any relationship the scatter plot suggests. Explain your thinking.

<table>
<thead>
<tr>
<th>Thickness (inches)</th>
<th>Height (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>9</td>
<td>44</td>
</tr>
<tr>
<td>10</td>
<td>46</td>
</tr>
<tr>
<td>12</td>
<td>52</td>
</tr>
<tr>
<td>15</td>
<td>61</td>
</tr>
<tr>
<td>16</td>
<td>67</td>
</tr>
</tbody>
</table>
Reading the Problem

Recognizing Relevant, Irrelevant, and Missing Information

When you read a problem, you should decide what information you need to solve the problem.

**Problem**  
In football, the turnover margin for a team is given by the expression \( t - g \), where \( t \) is the number of times the team takes the ball away from an opposing team through an interception or a fumble and \( g \) is the number of times the team gives the ball to an opposing team through an interception or a fumble. Information about a team is given in the table below. Did this team improve its turnover margin from 2000 to 2001? from 2001 to 2002?

<table>
<thead>
<tr>
<th>Season</th>
<th>Games played</th>
<th>Takes ball</th>
<th>Gives ball</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Intercep.</td>
<td>Fumbles</td>
</tr>
<tr>
<td>2000</td>
<td>16</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>2001</td>
<td>16</td>
<td>13</td>
<td>15</td>
</tr>
</tbody>
</table>

1. **What information is relevant?**
   To solve the problem, you need all the information given in the table except for the number of games played. That information is irrelevant for finding turnover margins.

2. **Is any information missing?**
   The table gives you enough information to find the turnover margins for 2000 and 2001 but not for 2002. So, you can determine whether the team improved its turnover margin from 2000 to 2001 but not whether it improved its turnover margin from 2001 to 2002.

3. **How is the problem solved?**
   Find the turnover margins for 2000 and 2001 by evaluating the expression \( t - g \). To find \( t \), add the number of interceptions and the number of fumbles for “Takes ball.” Likewise, to find \( g \), add the number of interceptions and the number of fumbles for “Gives ball.”

   **2000 season:**  
   \[ t - g = (11 + 8) - (20 + 13) \]
   \[ = 19 - 33 \]
   \[ = -14 \]

   **2001 season:**  
   \[ t - g = (13 + 15) - (26 + 11) \]
   \[ = 28 - 37 \]
   \[ = -9 \]

**Answer** Because \(-9 > -14\), you can conclude that the team improved its turnover margin from 2000 to 2001. You need information about the team’s turnovers for 2002 to determine whether the team improved its turnover margin from 2001 to 2002.
Problem Solving Practice

1. **Apples**  You buy 10 apples that weigh a total of 4 pounds for $3.92. What is the cost of a pound of apples? How much would 6 pounds of apples cost?

2. **Baking**  You are baking cookies that require 6 cups of cereal for each batch. Use the nutrition facts below to find the number of boxes of cereal you will need to make 3 batches of cookies. Then determine how many cookies you can make.

   **Nutrition Facts**
   
   Serving size: $1\frac{1}{2}$ cups (40 g)
   
   Servings per package: 10

3. **Lemonade**  You are making lemonade that requires 2 quarts of water. If you have already added 3 cups of water, how much more do you need to add? (If you do not know how many cups are in a quart, where can you find this information?)

4. **Vacation**  Your family is going on a vacation. Your destination is 880 miles away. Your family is driving there at an average speed of 55 miles per hour for 8 hours a day. The car gets 28 miles per gallon of gas, and the car's gas tank holds 10 gallons of gas. If you start your vacation with a full tank of gas, how many times will your family have to stop to fill the gas tank?

5. **Roller Coaster**  A roller coaster takes a group of 24 people every 5 minutes. The ride lasts 3 minutes. There are 52 people in front of you. It takes 10 minutes to walk from the roller coaster to the concert stage, where you have reservations for the 2:00 show. If it is 1:30 now, can you ride the roller coaster and still make it to the show on time?

6. **Football**  Use the information on the previous page along with the 2002 information below to determine if the team improved its turnover margin from 2001 to 2002. What was the team’s average points scored per game in 2002?

<table>
<thead>
<tr>
<th>Games played</th>
<th>Takes Ball Intercep.</th>
<th>Takes Ball Fumbles</th>
<th>Gives Ball Intercep.</th>
<th>Gives Ball Fumbles</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>7</td>
<td>12</td>
<td>25</td>
<td>12</td>
</tr>
</tbody>
</table>

7. **Temperature**  To find the departure from normal temperature, you can use the expression $a - n$, where $a$ is the actual average temperature for the day and $n$ is the normal, or average, temperature historically. Use the table below to find the departure from normal temperature for each day. Then find the mean departure from normal temperature for the week.

<table>
<thead>
<tr>
<th>Day</th>
<th>Normal temp.</th>
<th>Actual temp.</th>
<th>Precipitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunday</td>
<td>39°F</td>
<td>33°F</td>
<td>0.0 in.</td>
</tr>
<tr>
<td>Monday</td>
<td>38°F</td>
<td>42°F</td>
<td>0.1 in.</td>
</tr>
<tr>
<td>Tuesday</td>
<td>38°F</td>
<td>25°F</td>
<td>0.5 in.</td>
</tr>
<tr>
<td>Wednesday</td>
<td>37°F</td>
<td>24°F</td>
<td>0.0 in.</td>
</tr>
<tr>
<td>Thursday</td>
<td>37°F</td>
<td>38°F</td>
<td>0.0 in.</td>
</tr>
<tr>
<td>Friday</td>
<td>37°F</td>
<td>29°F</td>
<td>1.6 in.</td>
</tr>
<tr>
<td>Saturday</td>
<td>36°F</td>
<td>36°F</td>
<td>0.3 in.</td>
</tr>
</tbody>
</table>